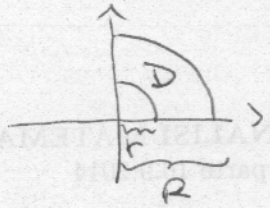
7) $I = \iint_D x e^{|y|}\, dx\,dy$

a) coordinate polari $\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \end{cases}$



$$I = \int_0^{\pi/2}\int_r^R \rho\cos\theta\, e^{\rho\sin\theta}\,\rho\,d\rho\,d\theta =$$

$$= \int_r^R \rho^2 \left[\int_0^{\pi/2} \cos\theta\, e^{\rho\sin\theta}\, d\theta\right] d\rho = \int_r^R \rho^2 \left[e^{\rho\sin\theta}\right]_0^{\pi/2} d\rho$$

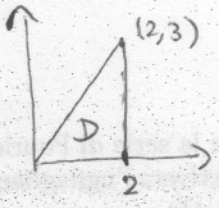
$$= \int_r^R \rho^2 (e^{\rho} - 1)\, d\rho = \rho^2 e^{\rho}\Big|_r^R - \int_r^R 2\rho e^{\rho} - \frac{1}{3}(R^3 - r^3)$$

$$= R^2 e^R - r^2 e^r - 2\rho e^{\rho}\Big|_r^R + 2e^{\rho}\Big|_r^R - \frac{1}{3}(R^3 - r^3)$$

$$= R^2 e^R - r^2 e^r - 2Re^R + 2re^r + 2(e^R - e^r) - \frac{1}{3}(R^3 - r^3)$$

$$= (R^2 - 2R + 2)e^R - (r^2 - 2r + 2)e^r - \frac{1}{3}(R^3 - r^3)$$

b)



$y = \frac{3}{2}x$

$$\int_0^2 \int_0^{3/2 x} x e^{|y|}\, dx\,dy = \int_0^2 x \left[\int_0^{3/2 x} e^{y}\, dy\right] dx =$$

$$= \int_0^2 x\left(e^{3/2 x} - 1\right) dx = \frac{2}{3}x e^{3/2 x}\Big|_0^2 - \int_0^2 \frac{2}{3} e^{3/2 x}\, dx - \frac{1}{2}x^2\Big|_0^2$$

$$= \frac{4}{3}e^3 - \frac{2}{3}\cdot\frac{2}{3}e^{3/2 x}\Big|_0^2 - \frac{1}{2}\,4 = \frac{4}{3}e^3 - \frac{4}{9}(e^3 - 1) - 2 =$$

$$= \frac{8}{9}e^3 - \frac{14}{9}$$