SCHEMA SOLUZIONE ESAME 26.6.2014 2ª PARTE

(1) $y = \frac{1}{3}\cosh(3x) \qquad x \in [-\lg 3, \lg 3]$

Curva $\begin{cases} x = x \\ y = \frac{1}{3}\cosh(3x) \end{cases} \qquad x \in [-\lg 3, \lg 3]$

$ds = |\vec{r}\,'(x)|\,dx = \cosh(3x)\,dx$

$\vec{r}\,' = (1, \operatorname{senh}(3x))$

$|\vec{r}\,'| = \sqrt{1 + \operatorname{senh}^2 3x} = |\cosh(3x)| = \cosh 3x$

$L(\gamma) = \int_{-\lg 3}^{\lg 3} \cosh(3x)\,dx = \operatorname{senh} 3x \Big|_{-\lg 3}^{\lg 3} =$

$= \frac{e^{3x} - e^{-3x}}{2}\Big|_{-\lg 3}^{\lg 3} = \left(9 - \frac{1}{9}\right)\frac{1}{2} = \frac{80}{9}\cdot\frac{1}{2} = \frac{40}{9}$

$\rho = \text{costante} \qquad m = \rho L = \rho\frac{40}{9}$

$\bar{x} = \frac{\rho}{m}\int_\gamma x\,ds = \frac{\rho}{m}\int_{-\lg 3}^{\lg 3} x\,|\vec{r}\,'(x)|\,dx =$

$= \frac{1}{L}\int_{-\lg 3}^{\lg 3} x\cosh 3x\,dx = 0$

$\bar{y} = \frac{\rho}{m}\int_\gamma y\,ds = \frac{1}{L}\int_{-\lg 3}^{\lg 3} y(x)|\vec{r}\,'(x)|\,dx = \frac{1}{L}\int_{-\lg 3}^{\lg 3}\frac{1}{3}\cosh^2(3x)\,dx$

$= \frac{1}{3L}\int_{-\lg 3}^{\lg 3}\frac{e^{6x} + e^{-6x} + 2}{4}\,dx = \frac{1}{4\times 3L}\left[\frac{1}{6}e^{6x} - \frac{1}{6}e^{-6x} + 2x\right]_{-\lg 3}^{\lg 3} =$

$= \frac{1}{12L}\left[\frac{1}{6}\left(3^6 - 3^{-6} - 3^{-6} + 3^6 + 2\cdot 2\lg 3\right)\right] = \frac{1}{12}\cdot\frac{9}{40}\cdot\frac{1}{6}\cdot 2\left(3^6 - \frac{1}{3^6} + 2\lg 3\right)$

$= \frac{1}{160}\left(3^6 - \frac{1}{3^6} + 2\lg 3\right) = \frac{1}{160}\left(\frac{3^{12}-1}{3^6} + 2\lg 3\right)$