(2) $\lim_{(x,y)\to(0,0)} \left(\cos(xy)-1\right) \dfrac{2y^4+x^5}{(y^2+x^2)^2} = \lim_{(x,y)\to(0,0)} g_1(x,y)\,g_2(x,y)$

$\lim_{(x,y)\to(0,0)} g_1(x,y) = 0$

$\lim_{(x,y)\to(0,0)} g_2(x,y) \Rightarrow \frac{0}{0}$ forma indeterminata

$$\left|g_2(x,y)\right| \leq \left| \frac{2y^4}{y^4+x^4+2x^2y^2} + \frac{x^5}{y^4+x^4+2x^2y^2} \right| \leq 2+|x|$$

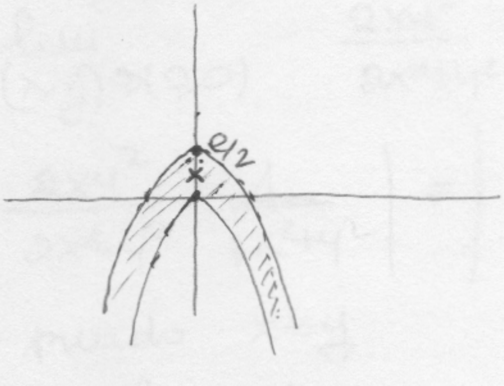
$$\Rightarrow \quad \left|g_1(x,y)\,g_2(x,y)\right| \leq \left|\cos xy - 1\right|(2+|x|) \xrightarrow[(x,y)\to(0,0)]{} 0$$

$$\Rightarrow \quad f(x,y) \xrightarrow[(x,y)\to(0,0)]{} 0$$

(3) $f(x,y) = \dfrac{(2y-3)\,\lg\left[x^2+(y-1)^2\right]}{\sqrt{1-\lg(2y+x^2)}}$

$x^2+(y-1)^2 > 0 \iff x^2+(y-1)^2 \neq 0 \quad \Rightarrow (x,y) \neq (0,1)$

$$\begin{cases} \lg(2y+x^2) < 1 \\ \lg(2y+x^2) \neq 1 \\ 2y+x^2 > 0 \end{cases} \qquad \begin{cases} 2y+x^2 < e \\ 2y+x^2 \neq e \\ y > -x^2/2 \end{cases} \qquad \begin{cases} y < (e-x^2)/2 \\ y \neq -\dfrac{x^2+e}{2} \\ y > -x^2/2 \end{cases}$$



$E = \left\{(x,y) \in \mathbb{R}^2 \; -\frac{x^2}{2} < y < e - \frac{x^2}{2} \; (x,y) \neq (0,1)\right\}$

aperto, illimitato, connesso