⑤ $f(x,y) = 4\left[e^{-x^2+y}(1-y)+e^y\right]$

$f_x = 4\left[-2x\, e^{-x^2+y}(1-y)\right] = 0 \iff +2x(1-y)=0$

$f_y = 4\left[e^{-x^2+y}(\cancel{1}-y-\cancel{1})+e^y\right] \iff -ye^{-x^2}+1=0$

$x=0 \quad y=1 \quad (0,1)$

$y=1 \quad e^{-x^2}=1$

$\Rightarrow$ Ho un'unico pto stazionario

$\frac{\partial^2 f}{\partial x^2} = 4\left[+4x^2 e^{-x^2+y} - 2e^{-x^2+y}\right](1-y)$

$\frac{\partial^2 f}{\partial y^2} = 4\left[-e^{-x^2+y} - ye^{-x^2+y} + e^y\right] = 4e^y\left[-(y+1)e^{-x^2}+1\right]$

$\frac{\partial^2 f}{\partial x \partial y} = 4\,(+2x)\,e^{-x^2+y}\,y$

$H_f(0,1) = \begin{pmatrix} 0 & 0 \\ 0 & -4e \end{pmatrix}$ La matrice hessiana non dice nulla

Studiamo $\Delta = f(x,y) - f(0,1) = 4\left[e^{-x^2+y}(1-y)+e^y\right]-4e$

$\Delta > 0 \iff e^y\left[e^{-x^2}(1-y)+1\right] > e$

$e^{y-1}\left[e^{-x^2}(1-y)+1\right] > 1$

$e^{y-1} \simeq 1+y-1$ vicino a $(0,1)$ $\Rightarrow (1+y-1)\left[(1-x^2)(1-y)+1\right]>1$

$e^{-x^2} \simeq 1-x^2$

$y(1-y)+y-x^2(1-y)>1 \qquad -y^2+2y-1-x^2(1-y)>0$

$(1-y)^2+x^2(1-y) < 0$ mai

$\Delta = \begin{cases} 0 & y=1 \\ <0 & y\neq 1 \end{cases}$ Max.