⑥ $f(x,y) = -3e^{-x}y + (2x+1)y^2 + 2e^{2x} \qquad f \in C^1(\mathbb{R})$

$f(x,y) = 0 \quad x_0 = 0$

$-3y + y^2 + 2 = 0 \quad y^2 - 3y + 2 = 0 \quad y = \dfrac{3 \pm \sqrt{9-8}}{2} = \begin{cases} 2 \\ 1 \end{cases}$

$(0,1) \quad (0,2)$ 2 soluzioni di $f(x,y) = 0$

Scelgo $(0,1)$ $\quad \dfrac{\partial f}{\partial y} = -3e^{-x} + 2y(2x+1) \quad \dfrac{\partial f}{\partial x} = 3e^{-x}y + 2y^2 + 4e^{2x}$

Dato che $\left.\dfrac{\partial f}{\partial y}\right|_{(0,1)} = -3+2 = -1 \neq 0 \quad f \in C^1(\mathbb{R})$

posso applicare teo Dini

$\Rightarrow g'(1) = -\dfrac{3+2+4}{-1} = 9$

⑦ $\displaystyle\iiint_P dx\,dy\,dz = \iint_{x^2+y^2 \le 4} 3\left(1 - \frac{x^2+y^2}{4}\right) dx\,dy$

Passiamo a coordinate polari

$\displaystyle 3 \int_0^2 \int_0^{2\pi} \left(1 - \frac{\rho^2}{4}\right) \rho\, d\rho\, d\theta = 3 \cdot 2\pi \left(\frac{1}{2}\rho^2 - \frac{1}{16}\rho^4\right)_0^2 =$

$= 6\pi\left(\frac{1}{2} \cdot 4 - \frac{1}{16} \cdot 16\right) = 6\pi$

$V_{cono} = \frac{1}{3}\pi R^2 h = \frac{1}{3}\pi\, 4 \cdot 3 = 4\pi = \frac{3}{2}\, 6\pi$