5 $f(x,y) = y^2 x (y-x+1)$

$$\frac{\partial f}{\partial x} = y^2(y-x+1) - y^2 x = y^2(y-2x+1)$$

$$\frac{\partial f}{\partial y} = 2yx(y-x+1) + y^2 x = yx(2y-2x+2+y) = yx(3y-2x+2)$$

$y=0 \quad x=\frac{1}{2}$ $\qquad (0,-1) \quad (x_0,0) \quad \left(\frac{1}{4},-\frac{1}{2}\right)$

$x=0 \quad y=-1$

$$\frac{\partial^2 f}{\partial x^2} = -2y^2 \qquad \frac{\partial^2 f}{\partial x \partial y} = 2y(y-2x+1)+y^2 = 3y^2-4xy+2y$$

$$\frac{\partial^2 f}{\partial y^2} = x(3y-2x+2)+3yx = 6xy-2x^2+2x$$

$$H_f(0,-1) = \begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix} \qquad \det H_f(0,-1) = -1 < 0$$

pto di sella

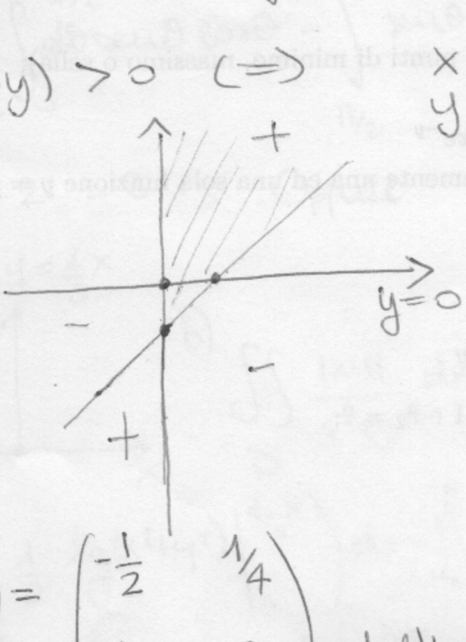
$$H_f(x_0,0) = \begin{pmatrix} 0 & 0 \\ 0 & -2x_0^2+2x_0 \end{pmatrix} \qquad \det H_f(x_0,0) = 0$$

non posso dire nulle

studio il segno della funzione

essendo $f(x,y)=0$ devo studiare il segno di f

$f(x,y) > 0 \iff \begin{matrix} x>0 \\ y>x-1 \end{matrix}$ oppure $\begin{matrix} x<0 \\ y<x-1 \end{matrix}$



$(x_0,0)$ $x_0<0$ pto di max

$(0,0)$ pto di sella

$(x_0,0)$ $0<x_0<1$ pto di min

$(1,0)$ pto di sella

$(x_0,0)$ $x_0>1$ pto di max

$$H_f\left(\frac{1}{4},-\frac{1}{2}\right) = \begin{pmatrix} -\frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & -\frac{3}{8} \end{pmatrix} \qquad \det H_f\left(\frac{1}{4},-\frac{1}{2}\right) = \frac{3}{16} - \frac{1}{16} > 0 \quad -\frac{1}{2} < 0 \Rightarrow \left(\frac{1}{4},-\frac{1}{2}\right) \max$$