SCHEMA SOLUZIONE ESAME 12.6.2014 2 PARTE

1) $\gamma \begin{cases} x = 2\cos t \\ y = 3\sin t \end{cases} \quad t \in [0, \pi] \qquad \rho = \text{costante} \qquad m = \rho L$

$$\bar{x} = \frac{1}{m}\int_\gamma x\rho\, ds = \frac{1}{m}\int_0^\pi x(t)\rho\,|\vec{r}'(t)|\,dt$$

$$= \frac{1}{\rho L}\rho\int_0^\pi 2\cos t\,|\vec{r}'(t)|\,dt = \frac{2}{L}\int_0^\pi \cos t\sqrt{9-5\sin^2 t}\,dt = 0$$

nulli per simmetria delle funzioni rispetto a $\pi/2$

$\vec{r}'(t) = (-2\sin t, +3\cos t) \qquad |\vec{r}'(t)| = \sqrt{4\sin^2 t + 9\cos^2 t}$

$\Rightarrow |\vec{r}'(t)| = \sqrt{4+5\cos^2 t} = \sqrt{9-5\sin^2 t}$

$$\bar{y} = \frac{1}{m}\int_\gamma y\rho\, ds = \frac{1}{m}\int_0^\pi y(t)\rho\,|\vec{r}'(t)|\,dt$$

$$= \frac{1}{\rho L}\rho\int_0^\pi 3\sin t\sqrt{9-5\sin^2 t}\,dt \overset{\text{simmetria}}{\downarrow} = \frac{6}{L}\int_0^{\pi/2}\sin t\sqrt{9-5\sin^2 t}\,dt$$

$$= \frac{6}{L}\int_0^{\pi/2}\sin t\sqrt{4+5\cos^2 t}\,dt = \frac{6}{L}\int_0^1 (4+5z^2)^{1/2}\,dz$$

$z = \sqrt{\frac{4}{5}}\sinh u \Rightarrow \bar{y} = \frac{6}{L}\cdot 2\int_0^{u^*}\sqrt{1+(\sinh u)^2}\sqrt{\frac{4}{5}}\cosh u\,du$

$1 = \sqrt{4/5}\sinh u^*$

$$= \frac{24}{L\sqrt{5}}\int_0^{u^*}(\cosh u)^2\,du = \left(9 + \frac{12\,\log\left(\frac{1}{2}(3+\sqrt{5})\right)}{\sqrt{5}}\right)2$$