⑤ $f(x,y) = 8(y-1)^2 + x^4 - 4yx^2$

$$\begin{cases} \dfrac{\partial f}{\partial x} = 4x^3 - 8xy = 4x(x^2-2y) = 0 \\[2ex] \dfrac{\partial f}{\partial y} = 16(y-1) - 4x^2 = 0 \end{cases}$$

Se $x = 0 \Rightarrow y = 1$

Se $2y = x^2 \Rightarrow y = \dfrac{x^2}{2} \Rightarrow 16\left(\dfrac{x^2}{2} - 1\right) - 4x^2 = 0 \quad 4x^2 - 16 = 0 \quad x^2 = 4 \quad x = \pm 2$

$\Rightarrow y = 2$

$P_1(0,1) \quad P_2(-2,2) \quad P_3(2,2)$ P.ti stazionari

$$\frac{\partial^2 f}{\partial x^2} = 12x^2 - 8y \qquad \frac{\partial^2 f}{\partial y^2} = 16 \qquad \frac{\partial^2 f}{\partial x \partial y} = -8x$$

$$H_f(0,1) = \begin{pmatrix} -8 & 0 \\ 0 & 16 \end{pmatrix}$$ P.to di sella

$$H_f(-2,2) = \begin{pmatrix} 32 & 16 \\ 16 & 16 \end{pmatrix} \quad \det H_f(-2,2) > 0 \quad 32 > 0 \Rightarrow P_2 \text{ min}$$

$$H_f(2,2) = \begin{pmatrix} 32 & -16 \\ -16 & 16 \end{pmatrix} \quad \det H_f(2,2) > 0 \quad 32 > 0 \Rightarrow P_3 \text{ minimo}$$

⑥ $f(x,y) = e^{(x^2+y^2)}$ f è definita su tutto $\mathbb{R}^2$

Studio la matrice Hessiana

$$\frac{\partial f}{\partial x} = e^{x^2+y^2}\, 2x \qquad \frac{\partial^2 f}{\partial x^2} = e^{x^2+y^2}(2+4x^2) \qquad \frac{\partial^2 f}{\partial x \partial y} = e^{x^2+y^2}\, 4xy$$

$$\frac{\partial f}{\partial y} = e^{x^2+y^2}\, 2y \qquad \frac{\partial^2 f}{\partial y^2} = e^{x^2+y^2}(2+4y^2)$$

$$H_f(x,y) = e^{x^2+y^2} \begin{pmatrix} 2+4x^2 & 4xy \\ 4xy & 2+4y^2 \end{pmatrix}$$

$$\det H_f(x,y) = e^{2(x^2+y^2)}\left((2+4x^2)(2+4y^2) - 16x^2y^2\right) =$$
$$= e^{2(x^2+y^2)}\left(4 + 8x^2 + 8y^2 + 16x^2y^2 - 16x^2y^2\right) = e^{2(x^2+y^2)}(4+8x^2+8y^2) > 0$$

$$2 + 4x^2 > 0 \Rightarrow d^2 f(\vec{x}_0) > 0 \quad \forall (x_0,y_0) \in \mathbb{R}^2$$

la funzione è convessa in tutto $\mathbb{R}^2$.
(strettamente convessa)