$$f(x,y) = xy - (x+1) + 2y^3(x-2)$$

$$xy - (x+1) + 2y^3(x-2) = 0 \qquad f \in C^1(\mathbb{R}^2)$$

$$2y - 3 + 2y^3 \cdot 0 = 0 \qquad y = \frac{3}{2}$$

$$\frac{\partial f}{\partial x} = y - 1 + 2y^3$$

$$\frac{\partial f}{\partial y} = x + 6y^2(x-2) \qquad \frac{\partial f}{\partial y}\left(2, \tfrac{3}{2}\right) = \cancel{\tfrac{3}{2}}\ 2 \neq 0$$

Posso applicare il teo del Dini in quanto $f \in C^1(\mathbb{R}^2)$ e $f(2,\tfrac{3}{2}) = 0$

$$\frac{\partial f}{\partial y}\left(2, \tfrac{3}{2}\right) \neq 0$$

$$\Rightarrow g'(2) = -\frac{\frac{3}{2} - 1 + 2\left(\frac{3}{2}\right)^3}{2} = -\frac{\frac{1}{2} + \frac{27}{4}}{2} = -\frac{29}{8}$$

(7)
$$I = \iint_D 3x^2\,dx\,dy$$

a)
$$\begin{cases} x = \frac{1}{\sqrt{2}}\rho\cos\vartheta \\ y = \frac{1}{2\sqrt{2}}\rho\sin\vartheta \end{cases} \qquad \begin{vmatrix} \frac{1}{\sqrt{2}}\cos\vartheta & \frac{1}{2\sqrt{2}}\sin\vartheta \\ -\frac{1}{\sqrt{2}}\rho\sin\vartheta & \frac{1}{2\sqrt{2}}\rho\cos\vartheta \end{vmatrix} = \frac{1}{4}\rho$$

$$I = \int_0^{\sqrt{2}}\int_0^{2\pi} \frac{3}{2}\rho^2\cos^2\vartheta\,\rho\,\frac{1}{4}\,d\rho\,d\vartheta = \frac{3}{8}\cdot\frac{1}{4}(\sqrt{2})^4\,\frac{2\pi}{2} = \frac{3}{8}\pi$$

b)
$$I = \int_0^2\int_0^{3x} 3x^2\,dx\,dy = \int_0^2 3x^2\,3x\,dx = \frac{9}{4}\cdot 16 = 36$$