Stochastic Mechanics 6 CFU
Part I 13.12.2007

Exercise 1 A die is rolled twice. X is the modulus of the difference of outcomes, Y is the outcome of the first roll.
a Find an explicit formula for $E(X|Y)$.
b find and compare the σ-algebras generated by the random variables Y and $E(X|Y)$.

Exercise 2 Let X be a Bernoulli variable of parameter p, ($P(X=1)=p$ and $P(X=0)=1-p$) and Y a normal random variable $Y \sim \mathcal{N}(m,\sigma^2)$. X and Y are independent. Calculate the characteristic function of $Z=X+Y$.

Exercise 3 Let W_t and $\hat{W}_t$ be two independent Brownian motions, define $X_t = a(W_t + \hat{W}_t)$. Find $a \in R$ such that X_t is a Brownian motion.

Exercise 4 Give the definition of Ito integral for a step function and calculate

$$E\left[\int_0^t e^{3W_s} dW_s\right]^2$$