

## Stochastic Mechanics 6 CFU

I Part 29.10.2007

**Exercise 1** Let  $X_1$  and  $X_2$  two Gaussian random variables with mean  $m_1, m_2$  and variance  $\sigma_1^2$  and  $\sigma_2^2$  respectively. Prove that  $Y = aX_1 + bX_2$  ( $a, b \in \mathbb{R}$ ) is a Gaussian random variable and calculate its mean value and its variance. (hint: use characteristic functions).

**Exercise 2** Three coins are tossed  $5c, 10c, 20c$ , let  $X$  be random variable which indicates the total amount shown and  $Y$  the number of tails, give the definition of  $E(X | Y)$  and calculate its possible values.

**Exercise 3** Give the definition of Brownian motion for a process  $W_t$  and verify that

$$\hat{W}_t = \sqrt{c}W_{\frac{t}{c}} \quad c > 0, t \geq 0$$

is a Brownian motion.

**Exercise 4** Let  $g = t(1 - t)$ ,  $t \in [0, 1]$  calculate

**a**  $\int_0^t g dW_t$  (give the formula)

**b**  $E(\int_0^t g dW_t)$

**c**  $E(\int_0^t g dW_t)^2$

**Exercise 5** Give the definition of Ito Stochastic integral for a step function and prove its properties.