Stochastic Mechanics 6 CFU
Part II 8.6.2009

Exercise 1 Let W_t be a Brownian motion. Illustrate Ito's formula and use it to calculate
a $d(X_t^{2n})$ for $n \geq 1$, where $dX_t = 3t^3 dW_t$
b (dW_t^{2n+1}) for $n \geq 1$
c $\int_0^T W_t^4 dW_t$

Exercise 2 Solve the following stochastic differential equation

$$dX_t = [3\, e^{2X_t} + 5]dt + dW_t$$

using the substitution $y = h(x) = e^{-2x}$

Exercise 3 Given the SDE

$$dX_t = (6 - X_t)dt + 6dW_t$$

with $X_{t=0} = X_0 \sim \mathcal{N}(0, 4)$
a find the solution
b find $E(X_t)$ and $Var(X_t)$ and study their behavior when $t \to \infty$

Exercise 4 Suppose you have a SDE of the form

$$dX_t = (aX_t^n + bX_t)dt + cX_t dW_t$$

prove that the substitution $y = h(x) = x^{1-n}$ reduces the SDE to a linear SDE with multiplicative noise.

Exercise 5 Give the definition of *Diffusion process* and discuss an example.