

## Stochastic Mechanics 6 CFU

Part II 13.12.2007

**Exercise 1** let  $W_t$  be a Brownian motion. Calculate the Ito differential of the process

$$X_t = e^{3W_t}$$

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**Exercise 2** The Verhulst equation is used to model population dynamics

$$\dot{x} = \lambda x - x^2$$

**a** randomize the parameter  $\lambda \rightarrow \lambda + \sigma \xi_t$ , where  $\xi_t$  is a white noise and write the corresponding SDE.

**b** solve the SDE (use the substitution  $y = 1/x$ ).

**Exercise 3** Given the SDE

$$dX_t = (3 - X_t)dt + 5dW_t$$

with  $X_{t=0} = X_0 \sim \mathcal{N}(\mu, \sigma^2)$

**a** find the solution

**b** find  $E(X_t)$  and  $Var(X_t)$  and study their behavior when  $t \rightarrow \infty$ .

**Exercise 4** Give the definition of *diffusion process* and discuss an example.

**Exercise 5** Given the SDE

$$dX_t = -5X_t(1 + X_t^2) + 3X_t dW_t$$

**a** find the steady solution

**b** prove that it is uniformly stochastically asymptotically stable in the large (use Lyapunov function).