Stochastic Mechanics 6 CFU
Part II 15.7.2009

Exercise 1 Let W_t be a Brownian motion. Illustrate Ito's formula and use it to calculate
a $d(tX_t^{2n})$ for $n \geq 1$, where $dX_t = 2dt + t^2 dW_t$
b dX_t where $X_t = e^{t-W_t}$
c $\int_0^T (W_t^4 + W_t)dW_t$

Exercise 2 Given the equation

$$\dot{x} = \lambda x - x^3$$

a randomize the parameter $\lambda \to \lambda + \sigma\xi_t$ where ξ_t is a white noise and write the corresponding SDE
b solve the SDE (use the substitution $y = 1/x^2$)

Exercise 3 Given the SDE

$$dX_t = (1 + \beta X_t)dt + dW_t$$

with $X_{t=0} = X_0 \sim \mathcal{N}(0,9)$ and $\beta \in \mathbb{R}$, find $E(X_t)$ and $\sigma^2(X_t)$ and study their behavior as functions of β.

Exercise 4 Given the process X_t find the SDE that it solves in the following cases
a $X_t = \frac{W_t}{1+2t}$
b $X_t = e^{3W_t - t}$

Exercise 5 Write the Kolmogorov forward equation for Brownian motion.