Stochastic Mechanics 6 CFU

Part II 22.6.2010

Exercise 1 Let W_t be a Brownian motion. Illustrate Ito's formula and use it to calculate

a $d(X_t^{2n})$ for $n \geq 1$, where $dX_t = (t^2 + 1)dW_t$

b (dW_t^{2n+2}) for $n \geq 1$

c $\int_0^T W_t^5 dW_t$

Exercise 2 Solve the following stochastic differential equation

$$dX_t = [2\, e^{3X_t} + 5]dt + dW_t$$

using the substitution $y = h(x) = e^{-3x}$

Exercise 3 Given the SDE

$$dX_t = (\alpha + \beta X_t)dt + \gamma dW_t$$

with $X_{t=0} = X_0 \sim \mathcal{N}(0, 4)$ and $\alpha, \beta, \gamma \in \mathbb{R}$, $\alpha, \beta, \gamma \neq 0$.

a find the solution

b find $E(X_t)$ and $Var(X_t)$ and study their behavior when $t \to \infty$.

Exercise 4 Use Feynman-Kac formula to solve the following PDE with final condition, $x \in \mathbb{R}$ and $t \in [0, T]$,

$$\begin{cases} \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial x^2} = 2tf \\ f(x, T) = e^x \end{cases}$$

Exercise 5 Give the definition of *Diffusion process* and discuss an example.