Stochastic Mechanics 6 CFU
Part II 26.6.2009

Exercise 1 Let W_t be a Brownian motion. Illustrate Ito's formula and use it to calculate
a $d(X_t^{2n})$ for $n \geq 1$, where $dX_t = 2dt + 3t^2 dW_t$
b dX_t where $X_t = e^{4W_t}$
c $\int_0^T (W_t^3 + W_t^2) dW_t$

Exercise 2 Solve the following stochastic differential equation

$$\begin{aligned} dX_t &= \left[-X_t^3 + 3X_t\right] dt + 2X_t dW_t \\ X_{t=0} &= X_0 \end{aligned}$$

hint: use the substitution $y = h(x) = x^{-2}$.

Exercise 3 Given the SDE

$$dX_t = (1 + \beta X_t)dt + \sigma dW_t$$

with $X_{t=0} = X_0 \sim \mathcal{N}(0, 4)$ and β, $\sigma \in \mathbb{R}$, find β, σ such that $E(X_t) \to 2$ and $\sigma^2(X_t) \to 4$ when $t \to \infty$

Exercise 4 Given the process X_t find the SDE that it solves in the following cases
a $X_t = \frac{W_t}{2+t}$
b $X_t = e^{2W_t}$

Exercise 5 Verify the Chapman-Kolmogorov equation for Brownian Motion.