Pergamon

PII: S0005-1098(97)00211-2

Automatica, Vol. 34, No. 4, pp. 445–450, 1998

Printed in Great Britain
0005-1098/98 $19.00 + 0.00

Brief Paper

Robust Control of Synchronous Motors with Non-linearities and Parameter Uncertainties*

P. CARAVANI† and S. DI GENNARO†

Key Words—Synchronous motors; robust control; nonlinear systems; uncertain dynamic systems; H-infinity control.

Abstract—Robust feedback control of a synchronous motor with model and parameter uncertainties and load disturbances is studied. The mathematical description of the motor in the (d, q) frame yields a bilinear model in state space form. The control strategy is based on a worst-case analysis and leads to the formulation of a min–max problem, whose solution provides an H^∞ bound for the noise-to-output transfer function. The design of the dynamic controller entails the solution of an appropriate Riccati equation. © 1998 Elsevier Science Ltd. All rights reserved.

1. *Introduction*

The control of synchronous motors has been widely investigated in a number of works under various points of view (Cerruto *et al.*, 1995; Ho *et al.*, 1995; Morimoto *et al.*, 1993; Rossi *et al.*, 1994; Caravani *et al.*, 1995; Di Gennaro *et al.*, 1994). One of the most frequently used mathematical descriptions is expressed in the (d, q) frame and the coupling between the angular velocity and the electrical quantities results in a bilinear model with the angular velocity as a natural output to be controlled. If model parameters are perfectly known, non-linearities can be canceled by proper selection of state feedback controls via exact feedback linearization (Isidori, 1995). When model parameters are imperfectly known, non-linearities can only be partially canceled by this technique and parameter uncertainty perturbs the equations of motion in a non-linear form. An important aspect of the control problem in this case is to obtain dynamic performances as far as possible insensitive to deviations of motor dynamic parameters and resistant torque from their nominal values. In this paper uncertainty sources are considered simultaneously. In particular, we consider variations of stator resistance and inductance, variations due to friction and resistant torque, presence of non-linear terms depending on these parameters.

The approach we take with regards to uncertainty does not presuppose a statistical description of the random variables. We design a feedback controller whose performance is guaranteed for all possible values of the input noise and of the uncertain parameters in a predefined class. All that is required is that the class of input noise be of bounded average power and that the possibly time-varying uncertain parameters be elements of a compact set.

The robustness result is based on the zero-sum differential game approach to H^∞ problems (Basar *et al.*, 1991) consisting of the definition of a quadratic functional providing an H^∞ bound for the output-to-noise ratio. Then a control law minimizing this functional in correspondence to the worst-case disturbance noise is computed, essentially solving a min–max problem.

Parameter and non-linear uncertainties can be fit into this context by an overbounding technique based on the performance of an auxiliary system with an augmented disturbance vector comprising the original noise plus the effects of the non-linearities and of the uncertain parameters.

In Section 2 the mathematical model of a permanent magnet synchronous motor is recalled. In Section 3 the overbound technique is presented and a procedure to calculate a state feedback controller is indicated. The application of this technique and simulation results are presented in Section 4, showing the performance of the controller in a number of different operating conditions. In Section 5 some final comments conclude the paper.

2. *Mathematical model of a synchronous motor*

In this section we briefly recall the equations of a permanent magnet (PM) synchronous motor, expressed in the so-called (d, q)-frame, deduced from the application of the Park transformation. This transformation associates to the physical variables in the three windings their components on the direct and quadrature axes, denoted by the d and q subscripts. The resulting equations, bilinear in the product angular velocity-current components, under the smooth-pole hypothesis can be written as follows (Leonard, 1985):

$$\frac{\mathrm{d}i_d}{\mathrm{d}t} = -\frac{R}{L} i_d + p\Omega i_q + \frac{1}{L} v_d, \tag{1}$$

$$\frac{\mathrm{d}i_q}{\mathrm{d}t} = -\frac{R}{L} i_q - p\Omega i_d - p\Phi\frac{1}{L}\Omega + \frac{1}{L} v_q, \tag{2}$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = \frac{p\Phi}{J} i_q - \frac{F}{J}\Omega - \frac{1}{J}c, \tag{3}$$

where R is the stator windings resistance, L the inductance, Φ is the flux of the permanent magnets, i_d, i_q are the currents and v_d, v_q are the applied voltages, and p is the number of pole pairs. In the mechanical equation (3), Ω is the rotor angular velocity, J is the rotor moment of inertia, F is the viscous friction coefficient, and c is the load torque. Equations (1)–(3) constitute the mathematical model of the PM synchronous motor.

Parameters R, L and F are supposed to vary with respect to their nominal values R_0, L_0 and F_0 in some unknown fashion. Our aim is to design an appropriate control which guarantees robust performances in presence of parameter and load torque variations. To this aim we rewrite motor's equations putting in evidence these variations and disturbances in terms of the following parameters:

$$\frac{R}{L} = a_{10} + a_1, \tag{4}$$

$$\frac{1}{L} = a_{20} + a_2, \tag{5}$$

$$F = a_{30} + a_3, \tag{6}$$

* Received 11 October 1995; revised 29 July 1996; received in final form 7 October 1997. This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Kenko Uchida under the direction of Editor Tamer Başar. *Corresponding author* Prof. P. Caravani. Tel. + 39 862 434424; Fax + 39 862 434403; E-mail caravani@ing.univaq.it.

† Department of Electrical Engineering, University of L'Aquila, Monteluco di Roio, 67040 L'Aquila, Italy.

so that equations of motion (1)–(3) become

$$\frac{di_d}{dt} = -(a_{10}+a_1)i_d + p\Omega i_q + (a_{20}+a_2)v_d, \qquad (7)$$

$$\frac{di_q}{dt} = -(a_{10}+a_1)i_q - p\Omega i_d - p\Phi(a_{20}+a_2)\Omega + (a_{20}+a_2)v_q, \qquad (8)$$

$$\frac{d\Omega}{dt} = \frac{p\Phi}{J}i_q - \left(\frac{a_{30}}{J}+\frac{a_3}{J}\right)\Omega - \frac{1}{J}c. \qquad (9)$$

Introducing new controls

$$\tilde{v}_d = v_d + \frac{p\Omega i_q}{a_{20}}, \qquad (10)$$

$$\tilde{v}_q = v_q - \frac{p\Omega i_d}{a_{20}} \qquad (11)$$

non-linearities can be partially canceled,

$$\frac{di_d}{dt} = -(a_{10}+a_1)\,i_d - \frac{pa_2}{a_{20}}\Omega i_q + (a_{20}+a_2)\tilde{v}_d, \qquad (12)$$

$$\frac{di_q}{dt} = -(a_{10}+a_1)\,i_q + \frac{pa_2}{a_{20}}\Omega i_d - p\Phi(a_{20}+a_2)\Omega + (a_{20}+a_2)\tilde{v}_q, \qquad (13)$$

$$\frac{d\Omega}{dt} = \frac{p\Phi}{J}i_q - \left(\frac{a_{30}}{J}+\frac{a_3}{J}\right)\Omega - \frac{1}{J}c. \qquad (14)$$

Defining steady-state variables $i_{dr}=0$, i_{qr}, Ω_r, $\tilde{v}_{dr}$, $\tilde{v}_{qr}$, c_r under zero uncertainty ($a_1=0$, $a_2=0$, $a_3=0$), solutions of

$$0 = -a_{10}i_{dr} + a_{20}\tilde{v}_{dr}, \qquad (15)$$

$$0 = -a_{10}i_{qr} - p\Phi a_{20}\Omega_r + a_{20}\tilde{v}_{qr}, \qquad (16)$$

$$0 = \frac{\Phi}{J}i_{qr} - \frac{a_{30}}{J}\Omega_r - \frac{1}{J}c_r, \qquad (17)$$

and introducing state variables

$$x_1 = \frac{i_{dr}-i_d}{i_0}, \qquad x_2 = \frac{i_{qr}-i_q}{i_0}, \qquad x_3 = \frac{\Omega_r-\Omega}{\Omega_0},$$

controls

$$u_1 = \frac{\tilde{v}_{dr}-\tilde{v}_d}{v_0}, \qquad u_2 = \frac{\tilde{v}_{qr}-\tilde{v}_q}{v_0},$$

and normalized parameters

$$\beta, \alpha = \begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{pmatrix}$$

given by

$$\beta = \frac{c_r - c}{c_0}, \qquad \alpha_i = \frac{a_i}{a_{i0}}, \quad i=1,2,3,$$

where i_0, Ω_0, v_0, c_0 denote base values, equations (12)–(14) with conditions (15)–(17) can be put in matrix form

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{pmatrix} = \begin{pmatrix}-a_{10} & 0 & 0\\ 0 & -a_{10} & -p\Phi a_{20}\dfrac{\Omega_0}{i_0}\\ 0 & p\Phi\dfrac{i_0}{J\Omega_0} & -\dfrac{a_{30}}{J}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} + \begin{pmatrix}a_{20}\dfrac{v_0}{i_0} & 0\\ 0 & a_{20}\dfrac{v_0}{i_0}\\ 0 & 0\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix} + D(t), \qquad (18)$$

where non-linearities and variations with respect to the nominal dynamics are collected in the term:

$$D(t) = Pw + Hf(x) + \alpha_1 A_1 x + \alpha_2 A_2 x + \alpha_3 A_3 x + \alpha_2 B_1 u \qquad (19)$$

and

$$P = \begin{pmatrix}0 & 0 & -p\Omega_r\dfrac{i_{qr}}{i_0} & 0\\ 0 & a_{10}\dfrac{i_{qr}}{i_0} & (p\Phi\Omega_r - \tilde{v}_{qr})\dfrac{a_{20}}{i_0} & 0\\ -\dfrac{c_0}{J\Omega_0} & 0 & 0 & \dfrac{a_{30}\Omega_r}{J\Omega_0}\end{pmatrix},$$

$$H = \begin{pmatrix}-p\Omega_0 & 0\\ 0 & p\Omega_0\\ 0 & 0\end{pmatrix}, \qquad w = \begin{pmatrix}\beta\\ \alpha_1\\ \alpha_2\\ \alpha_3\end{pmatrix}, \qquad f(x) = \begin{pmatrix}\alpha_2 x_2 x_3\\ \alpha_2 x_1 x_3\end{pmatrix},$$

$$A_1 = \begin{pmatrix}-a_{10} & 0 & 0\\ 0 & -a_{10} & 0\\ 0 & 0 & 0\end{pmatrix}, \quad A_2 = \begin{pmatrix}0 & p\Omega_r & p\Omega_0\dfrac{i_{qr}}{i_0}\\ -p\Omega_r & 0 & -p\Phi a_{20}\dfrac{\Omega_0}{i_0}\\ 0 & 0 & 0\end{pmatrix},$$

$$A_3 = \begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\dfrac{a_{30}}{J}\end{pmatrix}, \qquad B_1 = \begin{pmatrix}\dfrac{a_{20}v_0}{i_0} & 0\\ 0 & \dfrac{a_{20}v_0}{i_0}\\ 0 & 0\end{pmatrix}.$$

In angular velocity control problems typical outputs of interest are the current i_d and the deviation of the angular rate Ω from its steady state value. In fact, the electromagnetic torque is generated by the i_q component of the current. Therefore, forcing the i_d component to zero tends to align the current vector along the q direction. This optimizes the use of all the available current for torque producing purposes. Hence, the i_q component need not to appear in the output vector, which will be taken as

$$y = \begin{pmatrix}x_1\\ x_3\\ u_1\\ u_2\end{pmatrix}. \qquad (20)$$

Although traditional synthesis methods (PID controllers, pole-placement, LQ-synthesis) can be used to control (18), the simultaneous presence of linear and non-linear perturbation terms in $D(t)$ makes it difficult to assess precisely their dynamics performance. It is well-known, for example, that while robustness criteria like gain and phase margins may prove inadequate (Doyle *et al.*, 1992), LQ methods describe (optimize) performance in terms of the probability distribution of the disturbance which, as in the present case, can be non-Gaussian or even unknown. A second serious drawback of traditional methods over our method, is that they establish no connection between controller and uncertainty structure. When design is based upon the nominal part of the dynamics only (equation (18) with $D(t)\equiv 0$) potential advantages that may accrue from the highly structured nature of the disturbance are foregone, and this results in a dynamic performance which is generally poorer, as will be shown in Section 4 with a simulation comparing our approach with LQ control.

3. *An overbound technique for non-linear uncertain systems*

The mathematical model (18), (20) derived in the previous section falls in the class of systems described by the equations:

$$\dot{x} = Hf(x) + \left[A_0 + \sum_{i=1}^{q} f_i A_i\right]x + \left[B_0 + \sum_{j=1}^{r} g_j B_j\right]u + Pw, \qquad (21)$$

$$y = \begin{pmatrix}z\\ u\end{pmatrix}, \qquad z = Cx, \qquad x(0) = 0, \qquad (22)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$ represent state, input and output at time t, and the vector $w \in \mathbb{R}^d$ denotes an unknown exogenous disturbance belonging to the space $\mathscr{P}$ of bounded power signals

$$\| w \|^2 = \lim_{t_f \to \infty} \frac{1}{t_f} \int_0^{t_f} \| w(t) \|_2^2 \, dt < \infty$$

with $\| \cdot \|_2$ the euclidean norm of $\mathbb{R}^n$. The known matrices $A_i: \mathbb{R}^n \to \mathbb{R}^n$, $B_j: \mathbb{R}^m \to \mathbb{R}^n$ describe the uncertainty structure, and the scalars $f_i = f_i(t, x)$, $g_j = g_j(t, x)$ are uncertain parameters, generically depending on time and state, which can be regarded as unknown non-linearities. A further non-linearity is represented by the unknown function $f: \mathbb{R}^n \to \mathbb{R}^n$.

For this class of systems a simple control design procedure will be presented under the following assumptions:

(A1) Function $f(x)$ is such that $\|f(x)\|^2 \le k_1 \| x \|^2$, $\forall\, x \in \mathbb{R}^n$, for some k_1.

(A2) Functions f_i, g_j satisfy the bounds:

$$| f_i (t, x) | \le \bar{f}_i, \quad | g_j (t, x) | \le \bar{g}_j, \quad \forall t.$$

(A3) The pair (A_0, B_0) is stabilizable.

Remark 1. Assumption A1 expresses an energetic bound. In electric motors a non-linearity of this kind clearly stems from the coupling between current and angular velocity, and the bound expresses finite power operation of the machine under normal loading conditions. Notice that A1 implies $f(0) = 0$, so the origin of $\mathbb{R}^n$ is an equilibrium point of the unforced system. Moreover, in our case A1 becomes

$$\int_0^\infty \alpha_2^2 x_3^2 (x_1^2 + x_2^2)\, dt \le \bar{\alpha}_2^2 \bar{x}_3^2 \int_0^\infty (x_1^2 + x_2^2 + x_3^2) dt$$

and it is satisfied with $k_1 = \bar{\alpha}_2^2 \bar{x}_3^2$, where $\bar{\alpha}_2$ is the maximum ratio a_2/a_{20} and $\bar{x}_3$ the maximum difference between Ω_r and Ω expressed in base variable units. Assumption A2 is an instant bound on the parameter variations, requiring at each time t the uncertain parameters to take values inside an interval ($|f_i(t, x)| = |\alpha_i| \le \bar{\alpha}_i$, $i = 1, 2, 3$, and $|g_1(t, x)| = |\alpha_2| \le \bar{\alpha}_2$, with $\bar{\alpha}_i$ the maximum ratios a_i/a_{i0}). Assumption A3 is a natural (and minimal) requirement in state feedback control and can be easily verified in equation (18).

Control law u guarantees robust attenuation γ_0 in equations (21) and (22) if:

$$\frac{\| y \|}{\| w \|} \le \gamma_0, \tag{23}$$

for all $w \ne 0$ in $\mathscr{P}$ and all values of the uncertain parameters. Therefore, the robust control problem is to *find a state feedback controller such that the attenuation condition is satisfied for all t*.

Let us introduce next an auxiliary system:

$$\dot{x}_a = A_a x_a + B_a u + P_a v, \tag{24}$$

$$z = \begin{pmatrix} x_a \\ u \end{pmatrix}, \quad x_a(0) = 0, \tag{25}$$

where (A_a, B_a) is stabilizable and $v \in P$. This is a linear time-invariant system for which the following H^∞ result holds (Basar *et al.*, 1991).

Theorem 1. The feedback control law $u = F_\gamma x_a$ with:

$$F_\gamma = -B_a' S \tag{26}$$

$$\dot{S} + A_a' S + S A_a + I - S\left(B_a B_a' - \frac{1}{\gamma^2} P_a P_a'\right) S = 0, \qquad S(t_f) = I, \tag{27}$$

provides finite attenuation γ over $[0, t_f]$, that is it satisfies:

$$\frac{\| z \|}{\| v \|} \le \gamma, \quad \forall\, v \ne 0, \tag{28}$$

in system (24), (25) if and only if equation (27) has no conjugate points in $[0, t_f]$.

If this assumption holds for any t_f, under the hypothesis that the pair (A_a, B_a) is stabilizable, for $t_f \to \infty$ the solution S converges to a positive-semidefinite symmetric matrix S, and the resulting asymptotic controller:

$$F_\gamma = -B_a' S \tag{29}$$

makes the controlled system asymptotically stable (that is, $A_a - B_a B_a' S$ is a stable matrix).

In the proof of this theorem it is shown that inequality (28) amounts to solving the min–max problem

$$\min_u \max_v (\| z \| - \gamma \| v \|).$$

Selecting the parameters of the auxiliary system as follows:

$$x_a = x, \quad A_a = A_0, \quad B_a = B_0, \quad P_a = (P \;\; H \;\; A_1 \;\; \cdots \;\; A_q \, B_1 \;\; \cdots \;\; B_r),$$

$$v = \begin{pmatrix} w \\ f(x) \\ f_1 x \\ \vdots \\ f_q x \\ g_1 u \\ \vdots \\ g_r u \end{pmatrix},$$

the state evolution of equation (21) becomes formally identical to that of equation (24). Note also that equations (18), (20), of the form (21), (22), are also of the form (24), (25) with $P_a = (P \;\; H \;\; A_1 \;\; A_2 \;\; A_3 \;\; B_1)$,

$$v = (\beta \;\; \alpha' \;\; f'(x) \;\; \alpha_1 x' \;\; \alpha_2 x' \;\; \alpha_3 x' \;\; \alpha_2 u')'.$$

Let now F_γ be any time-invariant controller yielding the auxiliary system (24), (25) finite attenuation γ over some interval $[0, t_f]$, i.e. F_γ is an asymptotic controller computed for the auxiliary system by means of equation (29). Denoting $\lambda_{\min}(M)$, $\lambda_{\max}(M)$ the minimum and maximum eigenvalue of $M'M$, being M a generic matrix, we can state the following result.

Theorem 2. Under Assumptions A1 and A2, the system (21), (22), with $u = F_\gamma x$ given by equation (29), has finite attenuation γ_0 over $[0, t_f]$ if the following condition holds:

$$k_3 = k_1 + \sum_{i=1}^{q} \bar{f}_i^2 + \lambda_{\max}(F_\gamma) \sum_{j=1}^{r} \bar{g}_j^2 < \frac{1 + \lambda_{\min}(F_\gamma)}{\gamma^2}. \tag{30}$$

Furthermore, the attenuation level satisfies:

$$\gamma_0^2 = \gamma^2 \frac{\lambda_{\max}(C) + \lambda_{\max}(F_\gamma)}{1 + \lambda_{\min}(F_\gamma) - \gamma^2 k_3}. \tag{31}$$

Proof. Hypothesis A2 ensures the applicability of Theorem 1, hence the state feedback $u = F_\gamma x$ can be computed from equation (29). Substituting this control into equation (21), the bound (28) is obtained, i.e., one has

$$\frac{\| z \|}{\| v \|^2} = \frac{\| x \|^2 + \| u \|^2}{\| v \|} = \left(1 + \frac{\| F_\gamma x \|^2}{\| x \|^2}\right) \frac{\| x \|^2}{\| v \|} \le \gamma^2, \quad \forall v \ne 0.$$

Therefore,

$$\frac{\| v \|^2}{\| x \|^2} \ge \frac{1 + (\| F_\gamma x \|^2 / \| x \|^2)}{\gamma^2} \ge \frac{1 + \lambda_{\min}(F_\gamma)}{\gamma^2}, \tag{32}$$

where the last inequality follows from:

$$\| F_\gamma x \|^2 = \lim_{t_f \to \infty} \frac{1}{t_f} \int_0^{t_f} \| x \|_2^2 \frac{\| F_\gamma x \|_2^2}{\| x \|_2^2} \, dt$$

$$\ge \lim_{t_f \to \infty} \frac{1}{t_f} \int_0^{t_f} \| x \|_2^2 \, \lambda_{\min}(F_\gamma) \, dt.$$

Since inequality (32) holds for any $v \neq 0$, it holds in particular for $v|_{u=F_\gamma x} = \bar{v}$. As already noticed, with this choice of v the state evolution of the auxiliary system is identical to that of equation (21) under control $u = F_\gamma x$ and disturbance w. Using hypothesis A1 we have

$$\|\bar{v}\|^2 \leq \|w\|^2 + \|f(x)\|^2 + \sum_{i=1}^{q} \|f_i x\|^2 + \sum_{j=1}^{r} \|g_i F_\gamma x^2\|$$

$$\leq \|w\|^2 + (k_1 + \sigma_f + \lambda_{\max}(F_\gamma)\sigma_g)\|x\|^2$$

$$= \left(\frac{\|w\|^2}{\|x\|^2} + k_3\right)\|x\|^2,$$

where

$$\sigma_f = \sum_{i=1}^{q} \bar{f}_i^2, \quad \sigma_g = \sum_{j=1}^{r} \bar{g}_j^2, \quad k_3 = k_1 + \sigma_f + \lambda_{\max}(F_\gamma)\sigma_g. \quad (33)$$

Thus

$$\frac{1}{(\|w\|^2/\|x\|^2) + k_3} \leq \frac{\gamma^2}{1 + \lambda_{\min}(F_\gamma)},$$

hence,

$$\frac{\|x\|^2}{\|w\|^2} \leq \frac{\gamma^2}{1 + \lambda_{\min}(F_\gamma) - \gamma^2 k_3}.$$

The boundedness of this ratio is ensured by condition (30) of the statement. Finally, from equation (22):

$$\frac{\|y\|^2}{\|w\|^2} \leq \frac{\|Cx\|^2 + \|F_\gamma x\|^2}{\|w\|^2} \leq (\lambda_{\max}(C) + \lambda_{\max}(F_\gamma))\frac{\|x\|^2}{\|w\|^2}$$

$$\leq \gamma^2 \frac{\lambda_{\max}(C) + \lambda_{\max}(F_\gamma)}{1 + \lambda_{\min}(F_\gamma) - \gamma^2 k_3}$$

which is the bound (31).

Remark 2. When the number of controls is less than the number of state components, matrix $F'_\gamma F_\gamma$ is not full-rank hence $\lambda_{\min}(F_\gamma) = 0$ in condition (31).

Remark 3. In order to verify condition (31) it is necessary to compute first the controller F_γ and assess its attenuation γ via equation (27). Notice that the presence of P_a in this equation makes F_γ dependent upon the structure of the uncertainty. Once F_γ is obtained, one computes the constants $k_1, \bar{f}_i, \bar{g}_j$ and therefore determines the allowed tolerance on the parameter uncertainties. Some flexibility in this determination is allowed by the fact that the set of constants satisfying equation (31) is, in general, non-unique. For $t_f \to \infty$ Theorem 2 retains its validity and controller F_γ guarantees finite attenuation γ_0 and uniform global asymptotic stability of the origin for all systems in the class (21) satisfying equation (31).

It is easily verified that for synchronous motors condition (30) becomes

$$\bar{\alpha}_2^2 \bar{x}_3^2 + \bar{\alpha}_1^2 + \bar{\alpha}_2^2 + \bar{\alpha}_3^2 + \bar{\alpha}_2^2 \lambda_{\max}(F_\gamma) < \frac{1}{\gamma^2}. \quad (34)$$

In the following section it is shown that, based on realistic numerical values, this condition can be satisfied with reasonably large bounds on the uncertain parameters.

4. *Simulation results*

A real motor has been simulated, characterized by the following nominal parameter values:

$$R_0 = 0.6\ \Omega, \qquad L_0 = 1.2\ 10^{-3}\ \mathrm{H}, \qquad F_0 = 1.4\ 10^{-3}\ \mathrm{N\ m\ s},$$

and with

$$J = 2.5\ 10^{-3}\ \mathrm{kg\ m^2}, \qquad \Phi = 0.12\ \mathrm{Wb}, \qquad p = 4.$$

One obtains:

$$i_{d_r} = 0\ \mathrm{A}, \qquad i_{q_r} = 24.54\ \mathrm{A}, \qquad \tilde{v}_{d_r} = 0\ \mathrm{V}, \qquad \tilde{v}_{q_r} = 104.5\ \mathrm{V},$$

corresponding to a reference angular velocity $\Omega_r = 187$ rad/s and a reference load torque $c_r = 11.52$ N m.

Choosing $i_0 = i_{q_r}$, $\Omega_0 = \Omega_r$, $v_0 = 104$ V, $c_0 = 11.52$ N m as base values, from the Riccati equation (27) we find $u = F_\gamma x$ with

$$F_\gamma = \begin{pmatrix} -1.3219 & 0.3686 & -0.0117 \\ 0.3686 & -2.7814 & -2.9716 \end{pmatrix},$$

$\gamma = 1.44$ and $\lambda_{\max}(F_\gamma) = 8.32$.

Assuming deviations e_i for each parameter

$R = R_0(1 + e_1)$ for resistance,

$L = L_0(1 + e_2)$ for inductance,

$F = F_0(1 + e_3)$ for friction coefficient,

$\bar{x}_3 = e_4$ for angular velocity,

we find equation (34) is satisfied by the maximum individual tolerances

$$e_1 \simeq 69\%, \qquad e_2 \simeq 27\%, \qquad e_3 \simeq 69\%,$$

whereas for $e_1 = e_2 = e_3 = 0$ the term $f(x)$ in equation (19) vanishes, hence attenuation γ holds for every value of e_4.

In practice, we often are faced with simultaneous parameter variations. Assuming, for instance, a base percent deviation e with

$R = R_0(1 + 10\,e)$ for resistance,

$L = L_0(1 + 6\,e)$ for inductance,

$F = F_0(1 + 5\,e)$ for friction coefficient,

$\bar{x}_3 = 24\,e$ for angular velocity

and solving for e in equation (34), our controller would tolerate in this example a variation of nearly 42% for R, 25% for L, 21% for F from R_0, L_0, F_0, and 100% for Ω from Ω_r, with a guaranteed attenuation γ_0 given by equation (31).

As a basis for comparison we also computed a controller by the traditional *LQ* technique. In particular we solved the problem $\min\|y\|^2$, where y is given by equation (20), subject to equation (18) with $D(t) = 0$. Since the output y is the same considered with the H^∞ approach, this establishes a fair comparison between the two approaches. This resulted in the feedback gain matrix:

$$F_{LQ} = \begin{pmatrix} -0.8689 & 0 & 0 \\ 0 & -0.8720 & -0.4390 \end{pmatrix}.$$

Figures 1–4 show the frequency responses $\Omega(\omega)/\beta(\omega)$, $\Omega(\omega)/\alpha_i(\omega)$, $i = 1, 2, 3$, of the system controlled by the H^∞ controller (solid curves) and by the LQ controller (dotted curves). Our design technique seems quite advantageous in attenuating the effect of uncertainties in the electric parameters, especially in the low-frequency range. Similar results, although less pronounced, hold for the output i_d.

Next, we show the controller performance during transient operation. The values of the parameters R, L and F and the load c and the angular velocity Ω_r are supposed to vary according to the following laws:

$$R(t) = \mathrm{R}_0(1 - 0.4\,\mathrm{e}^{-t/0.1}\,t)$$

$$L(t) = L_0\left(1 + 0.25 \sin\frac{\Omega}{p\pi}\,t\right)$$

$$F(t) = F_0(1 + 0.2 \sin 50\Omega\,t)$$

$$c(t) = \begin{cases} c_r & \text{if } t < t_1, \\ 127\%\,c_r(1 - \mathrm{e}^{-100(t-t_1)}) & \text{if } t \in [t_1, t_2], \\ 90\%\,c_r(1 - \mathrm{e}^{-100(t-t_2)}) & \text{if } t > t_2, \end{cases}$$

$$\Omega_r(t) = \begin{cases} \Omega_0 & \text{if } t < t_3, \\ 60\%\,\Omega_0 & \text{if } t \in [t_3, t_4], \\ \Omega_0 & \text{if } t > t_4, \end{cases}$$

with $t_1 = 0.2$, $t_2 = 0.35$ s and $t_3 = 0.4$, $t_4 = 0.5$ s.

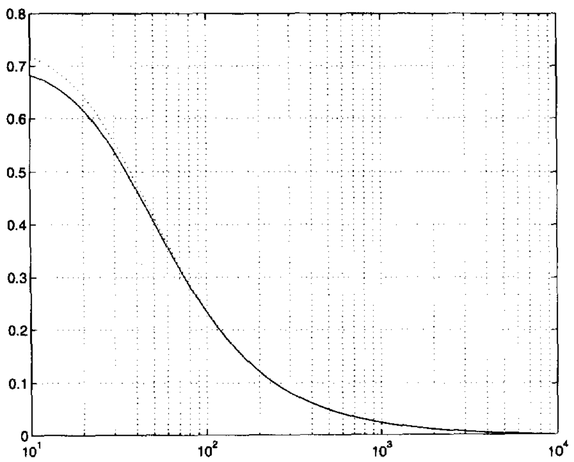

Fig. 1. $\Omega(\omega)/\beta(\omega)$.

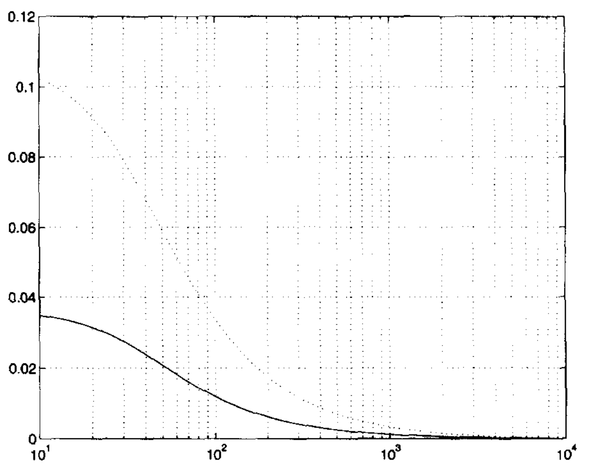

Fig. 2. $\Omega(\omega)/\alpha_1(\omega)$.

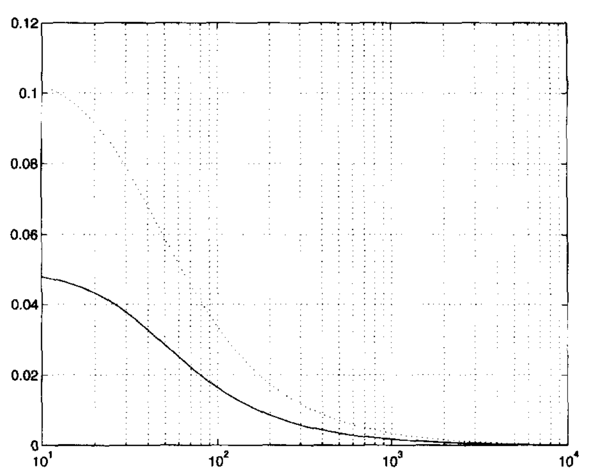

Fig. 3. $\Omega(\omega)/\alpha_2(\omega)$.

The initial values considered for the motor states are:

$$x_1(0) = 0, \qquad x_2(0) = 0, \qquad x_3(0) = 0.5,$$

where $x_3(0)$ corresponds to an initial angular velocity 93 rad/s, i.e. to 50% of Ω_r.

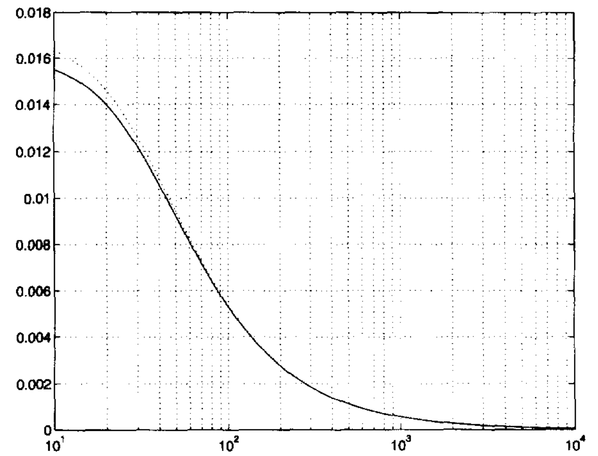

Fig. 4. $\Omega(\omega)/\alpha_3(\omega)$.

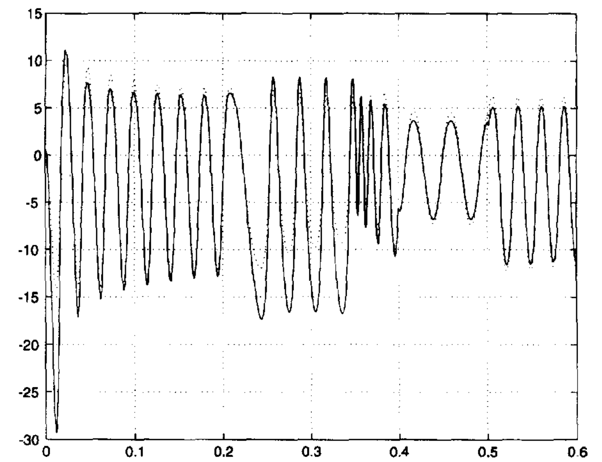

Fig. 5. Current i_d.

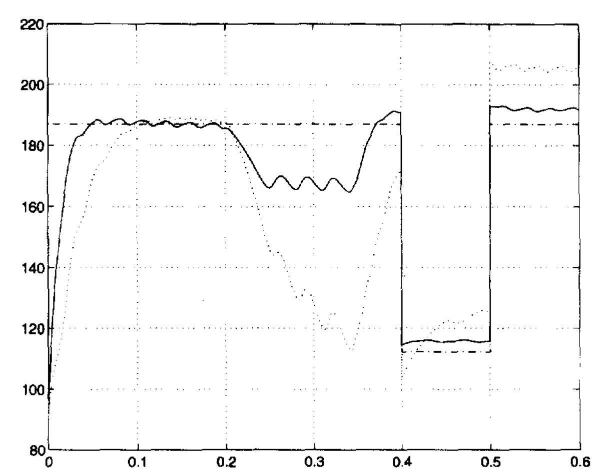

Fig. 6. Angular velocity Ω.

In the simulations shown below, all variables are expressed in MKSA units. Figures 5–8 show the superiority of our controller (solid curves) over the LQ controller (dotted curves), especially in the attenuation of the peak values of the state variables corresponding to changes of the load torque c. Notice that the steady-state error in Fig. 6 is due to the persistent nature of the

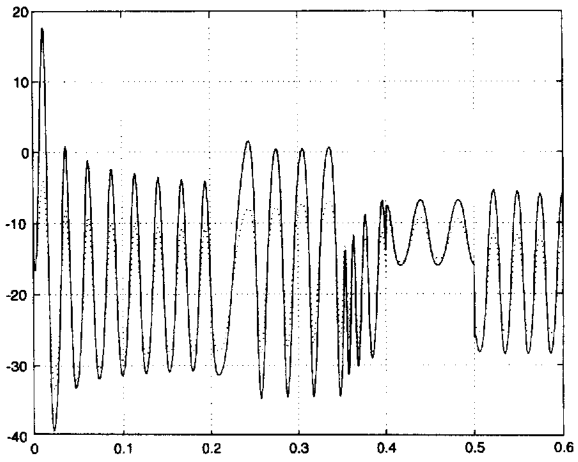

Fig. 7. Voltage v_d.

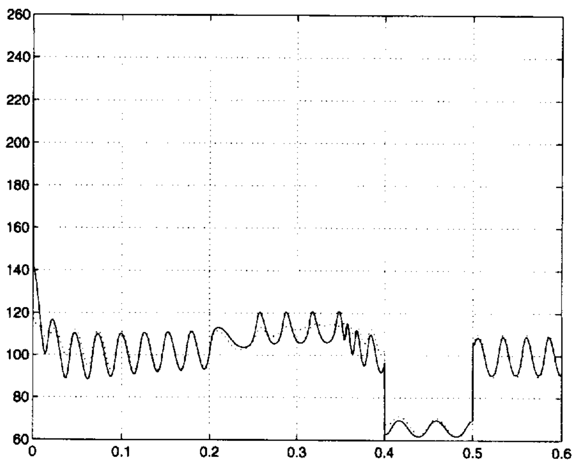

Fig. 8. Voltage v_q.

variations in L, F and c with respect to their nominal values. The optimality nature of our controller ensures the best guaranteed attenuation of these disturbances. Naturally, full integral action, with zero steady-state error, is recovered when disturbances vanish.

It is important to remark that, while mathematically plausible, the parameter variation selected in the simulations is definitely not the worst case. Although physically unrealistic, a worst case simulation would have made the comparison even more favorable to the H^∞ controller.

5. *Conclusions*

In this paper we have shown the application of a robust control technique to a synchronous motor, based on the H^∞ approach. The advantages of this technique lie on the possibility of stabilizing the motor dynamics under various uncertainty sources, for which no statistical information are available. These sources may be either external disturbances or intrinsic parameter changes. Moreover, the approach permits to handle either non-linearities or disturbances arising from unmodeled dynamics in the system equations. In all these cases the performance of the controlled system can be optimized in terms of the output–noise ratio, thus preserving and in fact enhancing the benefits of H^∞ control, well-known for the simpler case of certain systems. The application of the theory to the control of a synchronous motor with unknown parameters seems to offer a valid design alternative to more conventional techniques.

Acknowledgements—Work supported by Ministero dell'Università e della Ricerca Scientifica e Tecnologica, National Program "Teoria dei Sistemi e del Controllo", and CNR–PFT2. The authors appreciated the discussions with Franco Parasiliti and his comments. The second author wishes also to thank Salvatore Monaco and Dorothée Normand-Cyrot for the valuable suggestions received during the preparation of this paper.

References

Basar, T. and P. Bernhard (1991). *H^∞ Optimal Control and Related Minimax Design Problems. A Dynamic Game Approach.* Birkhauser, Boston, MA.

Caravani, P. and S. Di Gennaro (1995). H^∞ Control of a Nonlinear Synchronous Motor with Uncertaint Parameters, *Proc. 3rd European Control Conf.—ECC* 95, Rome, Italy, pp. 242–247.

Cerruto, E., A. Consoli, A. Raciti and A. Testa (1995). A robust adaptive controller for PM motor drives in robotic applications. *IEEE Trans. Power Electron.*, **10**, 62–71.

Chelouah, A., S. Di Gennaro and M. Tursini (1993). Nonlinear Digital Control of a Synchronous Motor: Comparative Simulation Results, *Proc. IEEE Int. Conf. on Systems, Man and Cybernetics*, Vol. 5, Le Touquet, France, pp. 96–101.

Di Gennaro, S. and M. Tursini (1994). Control techniques for synchronous motor with flexible shaft. *Proc. IEEE Conf. on Control Appl.*, Glasgow, Scotland, U.K., pp. 471–476.

Doyle, J. C., B. A. Francis and A. R. Tannenbaum (1992). *Feedback Control Theory.* Macmillan, New York.

Georgiou, G. and B. Le Pioufle (1991). Nonlinear speed control of synchronous servomotor with robustness. *Proc. EPE'*91, Firenze, Italy.

Georgiou, G., A. Chelouah, S. Monaco and D. Normand-Cyrot (1992). Nonlinear multirate adaptive control of a synchronous motor. *Proc.* 31*st IEEE Conf. on Decision and Control*, Tucson, Arizona, U.S.A., pp. 3523–3528.

Ho, E. and P. C. Sen (1995). High-performance decoupling control techniques for various rotating field machines. *IEEE Trans. Industrial Electron.*, **42**, 40–49.

Isidori, A. (1995). *Nonlinear Control System: An Introduction*, 3rd edn, Springer Verlag, London.

Leonard, W. (1985). *Control of Electrical Drives.* Springer-Verlag, Berlin.

Morimoto, S., Y. Takeda, K. Hatanaka, Y. Tong and T. Hirasa (1993). Design and control system of inverter-driven permanent magnet synchronous motors for high torque operation. *IEEE Trans. Industry Appl.*, **29**, 1150–1155.

Rossi, C. and A. Tonielli (1994). Robust control of permanent magnet motors: VSS techniques lead to simple hardware implementations. *IEEE Trans. Industrial Electron.*, **41**, 451–460.