JOURNAL OF OPTIMIZATION THEORY AND APPLICATIONS: Vol. 98, No. 3, pp. 545-568, SEPTEMBER 1998

Adaptive Robust Tracking for Flexible Spacecraft in Presence of Disturbances[1]

S. DI GENNARO[2]

Communicated by L. Meirovitch

Abstract. The paper deals with trajectory tracking for a flexible spacecraft, subject to a gravity-gradient disturbance, under parameter uncertainties. The controls are gas jets and reaction wheels, and the measured variables describe the attitude and angular velocity of the rigid part. The flexible dynamics is treated as an additional disturbance acting on a rigid structure. First, an adaptive control is designed with only the gravity-gradient disturbance acting on the spacecraft; second, it is proved to be effective also in the presence of disturbance due to the flexibility, provided that appropriate robustness conditions on the controller gains are satisfied. These conditions use partial knowledge of the parameters describing the elastic dynamics. Simulations show the good performance of such control scheme and demonstrate its applicability even in the presence of input saturation.

Key Words. Flexible spacecraft, gravity-gradient disturbance, adaptive control, robustness, tracking.

1. Introduction

In this work, the problem of tracking a desired trajectory for flexible spacecraft, in the presence of environmental disturbances and parameter uncertainties, is considered. In fact, in practical situations, the structure is subject to disturbances of various nature and the spacecraft parameters are not known completely. These facts imply a deterioration of the control

[1]The author thanks Professor Salvatore Monaco for suggesting the topic. This research was supported by Ministero dell'Università e della Ricerca Scientifica e Tecnologica, National Program "Teoria dei Sistemi e del Controllo," and by Agenzia Spaziale Italiana under Contract 94 RS 17.

[2]Assistant Professor, Dipartimento di Ingegneria Elettrica, Università di L'Aquila, Poggio di Roio, L'Aquila, Italy.

0022-3239/98/0900-0545$15.00/0

behavior and must be taken into account in order to design a controller with high performance.

Among the environmental disturbances acting on spacecraft, the most important is due to gravity. In fact, the attitude dynamics of the spacecraft is influenced by the orbital elements through the gravity-gradient torque, which is at times treated as a disturbance torque in the design of spacecraft controllers, at times is used to aid the controller in stabilization problems. When dealing with tracking problems, it must be considered as a disturbance, which becomes important when the satellite is close enough to Earth and can be modeled by a nonlinear term multiplying the inverse of R^3, with R the distance from Earth. The nonlinear term is a function of the parameters used to describe the kinematics of the system (for instance, the unitary quaternions) and the inertia matrix J_T of the spacecraft. Both R and J_T are considered to be ill-known, with R varying in a fixed interval while J_T remains constant.

Another important role is played by disturbances due to flexible spacecraft structures. Their dynamics influence the attitude of the rigid part of the spacecraft, reducing the accuracy of the pointing; this influence can be treated as a disturbance acting on a rigid spacecraft. Such an approach is valid particularly when problems arise in measuring the modal variables, introduced for describing the flexible dynamics, and in the identification of the parameters involved in these equations.

In Ref. 1, conditions for the controllability of rigid bodies with gas jet or reaction wheel actuators are given. The first applications of the nonlinear technique for input-output linearization and decoupling can be found in Refs. 2 and 3. The controllability of rigid spacecraft in the presence of gravity-gradient torque is studied in great detail in Refs. 4 and 5 for systems actuated either by gas jets or reaction wheels. The tracking problem for rigid bodies is analyzed in Ref. 6, and different controllers are proposed in the cases of perfect or imprecise parameter knowledge. A dynamic adaptive controller for tracking a desired trajectory is also proposed in Refs. 7 and 8 for rigid spacecraft actuated by gas jets or gyrotorques and in the presence of uncertainties on parameters and disturbances acting on the structure.

The input-output linearization and decoupling techniques are extended to flexible spacecraft in case of large maneuvers in Refs. 9–12, where a servocompensator is introduced in the control system for augmenting the robustness of the control scheme in the presence of parameter uncertainties. In Refs. 13 and 14, the same technique is used along with an observer for estimating the modal variables. The adaptive approach for solving the tracking problem when parameters are unknown is used in Ref. 15; in Ref. 16, a robust controller for slew maneuvers is designed making use of collocated actuators and sensors in the presence of model uncertainties and disturbance torques. Following Ref. 7, in Ref. 17 an attitude controller for flexible spacecraft under bounded disturbance torques is proposed for asymptotically tracking a reference attitude and damping out the elastic oscillations. This dynamic compensator is derived assuming bounds on the elastic variables and imposing fast damping when the fixed desired attitude is reached.

In this work, the distance Earth–spacecraft is considered as an unknown time-varying parameter, taking its values in a compact set. Its variation, and therefore the corresponding gravity-gradient disturbance, can be considered reasonably slow with respect to the spacecraft dynamics. On the other hand, the disturbance due to the flexible elements of the spacecraft must be treated as fast-varying parameter. This difference suggests the use of an adaptive control scheme, based on a simplified model which considers only the rigid dynamics of the spacecraft, for eliminating the gravity-gradient action; the same technique can be used to face also uncertainties on the inertia matrices (Refs. 6, 8, 15, 16). Furthermore, the structure of the proposed adaptive controller can be maintained to achieve disturbance compensation with respect to flexibilities; in fact, from the estimated knowledge of the first significant modes, the controller parameters can be set to guarantee robustly a performing solution of the tracking problem.

The resulting controller presents the advantage, with respect to those proposed in the literature, to solve the tracking problem for flexible spacecraft taking into account either the gravity-gradient disturbance or parameter uncertainties in a unified framework. Moreover, if the tracked trajectory satisfies only some bounded conditions, the proposed control, defined everywhere, reduces automatically to an approximate controller which solves the tracking problem in the sense of the practical stabilization problem, while it ensures an exact tracking if additional conditions on the tracked trajectory are satisfied. Finally, this control scheme does not need measurements of the flexibility variables. Its robustness with respect to higher-order modes is shown by numerical simulations, also in the presence of input saturation.

The organization of the paper is the following. In Section 2, some notions on the mathematical model of a flexible spacecraft are given; in Section 3, an adaptive controller which solves asymptotically the control problem is derived, on the basis of a simplified model which neglects the flexible disturbance. This hypothesis is removed in Section 4, where a robust adaptive control law, which takes into account also the flexible dynamics, is designed. Simulation results are illustrated in Section 5.

2. Mathematical Model

A common nonminimal parametrization, usefully adopted for the kinematics of a spacecraft, is given by the four unitary quaternions $\theta_0 =$

$\cos(\Phi/2)$, $\theta = e\sin(\Phi/2)$, with Φ the rotation angle and e the unit vector along the rotation axis (Refs. 2, 3, 9, 18). These parameters are related by the norm constraint

$$\sum_{i=0}^{3} \theta_i^2 = 1.$$

This description of the orientation avoids the geometric singularities inherent with three parameter descriptions (Euler angles, etc.). Another advantage of this parametrization is that successive rotations result in simple successive multiplications of the quaternion matrices (Ref. 18). The quaternions can represent the spacecraft attitude with respect to a desired reference, moving with an angular velocity ω_r (namely, the attitude error with respect to the desired one). Denoting by

$$\omega = (\omega_1, \omega_2, \omega_3)^T = \omega_s - \omega_r$$

the difference between the spacecraft angular rate and the desired one (i.e., the error velocity), the quaternion dynamics are

$$\begin{bmatrix} \dot{\theta}_0 \\ \dot{\theta} \end{bmatrix} = (1/2) \begin{bmatrix} -\theta^T \\ \theta_0 I + \theta \times \end{bmatrix} \omega, \tag{1}$$

with $\times$ the usual vector product.

The dynamics of a spacecraft with reaction wheels and flexible appendages (solar arrays, antennas, etc.) are (Refs. 3, 6, 7, 9, 11, 12, 16)

$$J_T\dot{\omega}_s + J_R\dot{\Omega} + \delta^T\ddot{\eta} = -\omega_s \times (J_T\omega_s + J_R\Omega + \delta^T\dot{\eta}) + T_e, \tag{2a}$$

$$J_R(\dot{\omega}_s + \dot{\Omega}) = u_2, \tag{2b}$$

$$\ddot{\eta} + C\dot{\eta} + K\eta = -\delta\dot{\omega}_s; \tag{2c}$$

here J_T, J_R are the symmetric inertia matrices of the whole system and the reaction wheels,

$$\Omega = (\Omega_1, \Omega_2, \Omega_3)^T$$

is the reaction wheel relative angular velocity (i.e., with respect to the main body),

$$T_e = u_1 + d$$

are the external torques acting on the structure, u_1, u_2 are the gas jet and reaction wheel controls, and d is the disturbance torque. Moreover, η are the relative modal displacements and δ, C, K are the coupling, damping, and stiffness matrices.

The structural parameters are supposed to be poorly known and are constant or can vary during spacecraft operations. In both cases, since their variation is assumed to be slow with respect to the spacecraft dynamics, their derivatives are or can be considered zero. Moreover, it is assumed that some perturbations are present. Among the disturbances acting on the structure, two are considered of interest here. The first is due to the gravity-gradient, while the second is due to flexible element dynamics. In fact, if flexible elements are present and if the displacements induced by maneuvers are not measurable, their dynamics can be considered as perturbations acting on the rigid main body.

The gravity-gradient disturbance torque is (Refs. 4, 5, 19)

$$d_g = 3\mu[\epsilon(q) \times J_T \epsilon(q)]/R^3,$$

where μ is the Earth gravitational constant, R the distance of the spacecraft center of mass from the center of the Earth, and $\epsilon(q)$ the unit vector of the direction Earth–spacecraft, expressed in the body-fixed frame, and hence dependent on the quaternions (q_0, q) describing the spacecraft attitude. These parameters are related to (θ_0, θ) and (θ_{0r}, θ_r), describing the position of the desired reference, by the relationship (Refs. 18, 20)

$$\begin{bmatrix} q_0 \\ q \end{bmatrix} = \begin{bmatrix} \theta_0 & \theta^T \\ -\theta & \theta_0 I - \theta \times \end{bmatrix} \begin{bmatrix} \theta_{0r} \\ \theta_r \end{bmatrix}.$$

If the distance R is an unknown time-varying parameter, taking its values in a compact set, and if one sets

$$p_T = [J_{T_1}, \ldots, J_{T_6}]^T, \qquad p_G = p_T/R^3,$$

$$\varphi_0^T(\theta, \theta_r) = 3\mu \begin{bmatrix} 0 & -\epsilon_2\epsilon_3 & \epsilon_2\epsilon_3 & -\epsilon_1\epsilon_3 & \epsilon_1\epsilon_2 & \epsilon_2^2 - \epsilon_3^2 \\ \epsilon_1\epsilon_3 & 0 & -\epsilon_1\epsilon_3 & \epsilon_2\epsilon_3 & \epsilon_3^2 - \epsilon_1^2 & -\epsilon_1\epsilon_2 \\ -\epsilon_1\epsilon_2 & \epsilon_1\epsilon_2 & 0 & \epsilon_1^2 - \epsilon_2^2 & -\epsilon_2\epsilon_3 & \epsilon_1\epsilon_3 \end{bmatrix},$$

the disturbance assumes the form

$$d_g = \varphi_0^T(\theta, \theta_r) p_G.$$

Therefore, in our case

$$T_e = u_1 + d_g = u_1 + \varphi_0^T(\theta, \theta_r) p_G.$$

The expression of the second disturbance action d_f can be obtained from (2) when solved for the variables of interest. Introducing the angular velocity

$$\psi = \omega + \Omega,$$

which is the reaction wheel error velocity, and the modal velocity

$$\varrho = \delta\omega + \dot{\eta},$$

representing the error velocity of the flexible elements, and setting

$$A = \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ I \end{bmatrix}, \qquad \omega_s = \omega + \omega_r,$$

(2) yields

$$\dot{\omega} = J_{MB}^{-1}[-N(\omega, \omega_r, \psi) - \delta^T C \delta\omega + u_1 - u_2 + \varphi_0^T(\theta, \dot{\theta}_r) p_G + r^T(\omega, \omega_r)\xi] - \dot{\omega}_r, \tag{3a}$$

$$\dot{\psi} = J_R^{-1} u_2 - \dot{\omega}_r, \tag{3b}$$

$$\dot{\xi} = A\xi - AB\delta\omega - B\delta\dot{\omega}_r, \tag{3c}$$

where

$$J_{MB} = J_T - J_R - \delta^T \delta$$

is the symmetric inertia matrix of the main body,

$$\xi = \begin{bmatrix} \eta \\ \varrho \end{bmatrix},$$

and

$$N(\omega, \omega_r, \psi) = (\omega + \omega_r) \times (J_{MB}\omega + J_R\psi + J_T\omega_r),$$

with the disturbance

$$d_f = r^T(\omega, \omega_r)\xi, \qquad r^T(\omega, \omega_r) = [\delta^T K, \delta^T C - (\omega + \omega_r) \times \delta^T].$$

The dynamics ξ, containing modal accelerations and velocities, must be treated as a fast-varying parameter, while p_T can be considered slowly varying. Hence, for eliminating the first disturbance due to the gravity-gradient, a simple adaptive control scheme can be proposed. Also, the poor knowledge of the parameters J_T, J_R is a problem which can be solved with an adaptive controller (Ref. 15). With this objective, we set

$$J_{MB} = \begin{bmatrix} J_{MB_1} & J_{MB_4} & J_{MB_5} \\ J_{MB_4} & J_{MB_2} & J_{MB_6} \\ J_{MB_5} & J_{MB_6} & J_{MB_3} \end{bmatrix}, \qquad J_R = \begin{bmatrix} J_{R_1} & J_{R_4} & J_{R_5} \\ J_{R_4} & J_{R_2} & J_{R_6} \\ J_{R_5} & J_{R_6} & J_{R_3} \end{bmatrix},$$

$$\varphi(\theta, \omega, \omega_r, \psi) = \begin{bmatrix} \varphi_0(\theta, \theta_r) \\ -\varphi_1(\omega, \omega_r) \\ -\varphi_{21}(\omega, \omega_r) \\ -\varphi_3(\omega, \omega_r, \psi) \\ -\varphi_4(\omega) \end{bmatrix}, \qquad p = \begin{bmatrix} p_G \\ p_T \\ p_{MB} \\ p_R \\ p_C \end{bmatrix},$$

$$p_{MB} = \begin{bmatrix} J_{MB_1} \\ \vdots \\ J_{MB_6} \end{bmatrix}, \qquad p_R = \begin{bmatrix} J_{R_1} \\ \vdots \\ J_{R_6} \end{bmatrix}, \qquad p_C = \begin{bmatrix} p_{C_1} \\ \vdots \\ p_{C_6} \end{bmatrix},$$

$$\varphi_1^T(\omega, \omega_r) = \begin{bmatrix} 0 & -\omega_{r2}\omega_{s3} & \omega_{r3}\omega_{s2} & -\omega_{r1}\omega_{s3} & \omega_{r1}\omega_{s2} & \omega_{r2}\omega_{s2} - \omega_{r3}\omega_{s3} \\ \omega_{r1}\omega_{s3} & 0 & -\omega_{r3}\omega_{s1} & \omega_{r2}\omega_{s3} & \omega_{r3}\omega_{s3} - \omega_{r1}\omega_{s1} & -\omega_{r2}\omega_{s1} \\ -\omega_{r1}\omega_{s2} & \omega_{r2}\omega_{s1} & 0 & \omega_{r1}\omega_{s1} - \omega_{r2}\omega_{s2} & -\omega_{r3}\omega_{s2} & \omega_{r3}\omega_{s1} \end{bmatrix},$$

$$\varphi_{21}^T(\omega, \omega_r) = \begin{bmatrix} 0 & -\omega_2\omega_{s3} & \omega_3\omega_{s2} & -\omega_1\omega_{s3} & \omega_1\omega_{s2} & \omega_2\omega_{s2} - \omega_3\omega_{s3} \\ \omega_1\omega_{s3} & 0 & -\omega_3\omega_{s1} & \omega_2\omega_{s3} & \omega_3\omega_{s3} - \omega_1\omega_{s1} & -\omega_2\omega_{s1} \\ -\omega_1\omega_{s2} & \omega_2\omega_{s1} & 0 & \omega_1\omega_{s1} - \omega_2\omega_{s2} & -\omega_3\omega_{s2} & \omega_3\omega_{s1} \end{bmatrix},$$

$$\varphi_3^T(\omega, \omega_r, \psi) = \begin{bmatrix} 0 & -\psi_2\omega_{s3} & \psi_3\omega_{s2} & -\psi_1\omega_{s3} & \psi_1\omega_{s2} & \psi_2\omega_{s2} - \psi_3\omega_{s3} \\ \psi_1\omega_{s3} & 0 & -\psi_3\omega_{s1} & \psi_2\omega_{s3} & \psi_3\omega_{s3} - \psi_1\omega_{s1} & -\psi_2\omega_{s1} \\ -\psi_1\omega_{s2} & \psi_2\omega_{s1} & 0 & \psi_1\omega_{s1} - \psi_2\omega_{s2} & -\psi_3\omega_{s2} & \psi_3\omega_{s1} \end{bmatrix},$$

$$\varphi_4^T(\omega) = \begin{bmatrix} \omega_1 & 0 & 0 & \omega_2 & \omega_3 & 0 \\ 0 & \omega_2 & 0 & \omega_1 & 0 & \omega_3 \\ 0 & 0 & \omega_3 & 0 & \omega_1 & \omega_2 \end{bmatrix},$$

$$\delta^T C \delta = \begin{bmatrix} p_{C_1} & p_{C_4} & p_{C_5} \\ p_{C_4} & p_{C_2} & p_{C_6} \\ p_{C_5} & p_{C_6} & p_{C_3} \end{bmatrix}, \qquad \omega_{si} = \omega_i + \omega_{ri}, \qquad i = 1, 2, 3.$$

Then, from (3) we obtain

$$\dot{\omega}=J_{MB}^{-1}[\varphi^T(\theta,\omega,\omega_r,\psi)p+u_1-u_2+r^T(\omega,\omega_r)\xi]-\dot{\omega}_r, \tag{4a}$$

$$\dot{\psi}=J_R^{-1}u_2-\dot{\omega}_r, \tag{4b}$$

$$\dot{\xi}=A\xi-AB\delta\omega-B\delta\dot{\omega}_r, \tag{4c}$$

which along with (1) constitute the perturbed model.

The control objective is to track a desired reference attitude with disturbance rejection. Therefore, the outputs of interest are the quaternions θ; the control problem is to determine u_1, u_2 such that

$$\lim_{t\to\infty}\theta=0, \qquad \lim_{t\to\infty}\omega=0.$$

3. Adaptive Feedback Stabilization

As discussed in Section 2, the action due to ξ is considered an additional disturbance acting on the spacecraft. Therefore, the control law solving the control problem is designed herein on the basis of an unperturbed model, that is, a model without the disturbance d_f, constituted by (1) and

$$\dot{\omega}=J_{MB}^{-1}[\varphi^T(\theta,\omega,\omega_r,\psi)p+u_1-u_2]-\dot{\omega}_r, \tag{5a}$$

$$\dot{\psi}=J_R^{-1}u_2-\dot{\omega}_r. \tag{5b}$$

This is basically the model of a rigid spacecraft subject to the gravity-gradient disturbance. When applied to the real system [the perturbed model (1), (4)], this control does not ensure solving the control problem anymore; but, as shown in Section 4, it holds also for the perturbed system provided that appropriate conditions, depending on the structural properties of the flexible elements, are verified.

Let us therefore design an adaptive control which stabilizes asymptotically the unperturbed system (1), (5). The control derived is similar to that proposed in Ref. 6, extended to the case of spacecraft with reaction wheels. Note that, in our case, with gravity-gradient disturbance, the direct application of the control proposed in Ref. 6 does not ensure the convergence of θ to zero, and an appropriate modification is necessary in order to take into account this disturbance.

Let us consider the following candidate Lyapunov function:

$$V_{UN}=(k_p+k_d)[(\theta_0-1)^2+\theta^T\theta]+(1/2)(\theta+\omega)^TJ_{MB}(\theta+\omega)$$
$$+(1/2)\psi^TJ_R\psi+(1/2)(p-\hat{p})^T\Gamma^{-1}(p-\hat{p}),$$

with $k_p, k_d > 0$, $\Gamma = \Gamma^T > 0$, and $\hat{p}$ the estimate of p. Since

$$\begin{bmatrix} 2(k_p+k_d)I + J_{MB} & J_{MB} \\ J_{MB} & J_{MB} \end{bmatrix} > 0,$$

and since $J_R > 0$, V_{UN} is positive definite. Its time derivative along the trajectories of the unperturbed system (1), (5) is

$$\begin{aligned} \dot{V}_{UN} = {} & (k_p+k_d)\theta^T\omega + (\theta+\omega)^T \\ & \times [(1/2)J_{MB}(\theta_0 I + \theta\times)\omega + \varphi^T(\theta, \theta_r, \omega, \omega_r, \psi)p + u_1 - u_2 - J_{MB}\dot{\omega}_r] \\ & + \psi^T(v_2 - J_R\dot{\omega}_r) - (p-\hat{p})^T\Gamma^{-1}\dot{\hat{p}}. \end{aligned} \tag{6}$$

Now setting

$$J_{MB}(\theta_0 I + \theta\times)\omega/2 = \varphi_{22}^T(\theta, \omega)p_{MB}, \tag{7a}$$

$$J_{MB}\dot{\omega}_r = \varphi_{23}^T(\dot{\omega}_r)p_{MB}, \qquad J_R\dot{\omega}_r = \varphi_{23}^T(\dot{\omega}_r)p_R, \tag{7b}$$

$$\begin{aligned} \phi_1(\theta, \theta_r, \omega, \omega_r, \dot{\omega}_r, \psi) = {} & \varphi(\theta, \theta_r, \omega, \omega_r, \psi) \\ & + [0, 0, \varphi_{22}(\theta, \omega) - \varphi_{23}(\dot{\omega}_r), 0, 0]^T, \end{aligned} \tag{7c}$$

$$\phi_2(\dot{\omega}_r) = [0, 0, 0, \varphi_{23}(\dot{\omega}_r), 0]^T, \tag{7d}$$

with

$$\varphi_{22}^T(\theta, \omega) = (1/2)\begin{bmatrix} \bar{\omega}_1 & 0 & 0 & \bar{\omega}_2 & \bar{\omega}_3 & 0 \\ 0 & \bar{\omega}_2 & 0 & \bar{\omega}_1 & 0 & \bar{\omega}_3 \\ 0 & 0 & \bar{\omega}_3 & 0 & \bar{\omega}_1 & \bar{\omega}_2 \end{bmatrix},$$

$$\varphi_{23}^T(\dot{\omega}_r) = \begin{bmatrix} \dot{\omega}_{r1} & 0 & 0 & \dot{\omega}_{r2} & \dot{\omega}_{r3} & 0 \\ 0 & \dot{\omega}_{r2} & 0 & \dot{\omega}_{r1} & 0 & \dot{\omega}_{r3} \\ 0 & 0 & \dot{\omega}_{r3} & 0 & \dot{\omega}_{r1} & \dot{\omega}_{r2} \end{bmatrix},$$

$$\bar{\omega}_1 = \theta_0\omega_1 - \theta_3\omega_2 + \theta_2\omega_3, \qquad \bar{\omega}_2 = \theta_3\omega_1 + \theta_0\omega_2 - \theta_1\omega_3,$$

$$\bar{\omega}_3 = -\theta_2\omega_1 + \theta_1\omega_2 + \theta_0\omega_3,$$

(6) can be rewritten as

$$\begin{aligned} \dot{V}_{UN} = {} & (k_p+k_d)\theta^T\omega + (\theta+\omega)^T[\phi_1^T(\theta, \theta_r, \omega, \omega_r, \psi)p + u_1 - u_2] \\ & + \psi^T[u_2 + \phi_2^T(\dot{\omega}_r)p] - (p-\hat{p})^T\Gamma^{-1}\dot{\hat{p}}. \end{aligned}$$

Therefore, with the control

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -k_p\theta - k_d\omega - k_r\psi - [\phi_1^T(\theta, \theta_r, \omega, \omega_r, \dot{\omega}_r, \psi) + \phi_2^T(\dot{\omega}_r)]\hat{p} \\ -k_r\psi - \phi_2^T(\dot{\omega}_r)\hat{p} \end{bmatrix}, \tag{8}$$

$k_r > 0$, and the parameter updating

$$\dot{\hat{p}} = \Gamma[\phi_1(\theta, \theta_r, \omega, \omega_r, \dot{\omega}_r, \psi)(\theta + \omega) + \phi_2(\dot{\omega}_r)\psi], \tag{9}$$

we obtain

$$\dot{V}_{UN} = -k_p\|\theta\|^2 - k_d\|\omega\|^2 - k_r\|\psi\|^2 \leq 0.$$

Therefore, V_{UN} is not increasing along the unperturbed system trajectories and $\theta, \omega, \psi \in L_\infty[0, \infty)$. From (1), (5), (7)-(9), under the hypothesis that $\omega_r, \dot{\omega}_r \in L_\infty[0, \infty)$, we have also that $\dot{\theta}, \dot{\omega}, \dot{\psi} \in L_\infty[0, \infty)$. Furthermore, we deduce that

$$\lim_{t\to\infty} V_{UN} = V_{UN\infty}$$

exists and is finite. Finally,

$$V_{UN_0} - V_{UN_\infty} = k_p \int_0^\infty \|\theta(\tau)\|^2 \, d\tau + k_d \int_0^\infty \|\omega(\tau)\|^2 \, d\tau + k_r \int_0^\infty \|\psi(\tau)\|^2 \, d\tau,$$

or

$$\|\theta(\tau)\|_2^2 + \|\omega(\tau)\|_2^2 + \|\psi(\tau)\|_2^2 \leq (V_{NU_0} - V_{UN_\infty})/k_m \leq \infty,$$

$$k_m = \min\{k_p, k_d, k_r\}$$

$\|\cdot\|_2$ denoting the L_2 norm. Hence, $\theta, \omega, \psi \in L_2[0, \infty)$. Since $\theta, \omega, \psi \in L_2[0, \infty) \cap L_\infty[0, \infty)$ and $\dot{\theta}, \dot{\omega}, \dot{\psi} \in L_\infty[0, \infty)$, from the Barbalat lemma (Ref. 21) we deduce that

$$\lim_{t\to\infty} \theta = 0, \qquad \lim_{t\to\infty} \omega = 0, \qquad \lim_{t\to\infty} \psi = 0,$$

i.e., the control problem is solved, while

$$\lim_{t\to\infty} \hat{p} = \text{const.}$$

Moreover, since

$$\Omega = \psi - \omega,$$

also the reaction wheel angular velocity goes to zero.

Remark 3.1. If the parameters J_{MB}, J_R are known, since the signal $\dot{\omega}_r$ is matched by the input, it can be compensated directly. Hence, defining

$$u_1 = v_1 + (J_{MB} + J_R)\dot{\omega}_r, \qquad u_2 = v_2 + J_R\dot{\omega}_r,$$

practically implementable if $\dot{\omega}_r \in L_\infty[0, \infty)$, since the actuators work under finite energy, we obtain an autonomous system. In this case, with the new controls and the updating law

$$v_1 = -k_p\theta - k_d\omega - k_r\psi - \phi^T(\theta, \theta_r, \omega, \omega_r, \psi)\hat{p},$$

$$v_2 = -k_r\psi,$$

$$\dot{\hat{p}} = \Gamma\phi(\theta, \theta_r, \omega, \omega_r, \psi)(\theta + \omega),$$

where

$$\begin{aligned}&\phi^T(\theta, \theta_r, \omega, \omega_r, \psi)\\ &= \varphi^T(\theta, \theta_r, \omega, \omega_r, \psi) + [0, 0, \varphi_{22}^T(\theta, \omega), 0, 0]^T,\end{aligned}$$

and with $\omega_r \in L_\infty[0, \infty)$, and making use of the La Salle theorem (Ref. 22), it can be easily shown that the control problem is solved.

4. Robust Adaptive Control Law

Let us turn now to our main problem, concerning the presence of gravity-gradient and flexibility disturbances, with a poor or imprecise knowledge of structure information. In this case, it is not evident if the control designed in Section 3, applied to the perturbed system (1), (4), can still ensure asymptotic stability or at least bounded state evolution. In fact, to take into account the disturbance due to the flexibility, the control law must be designed to be robust, in the sense that the control gain magnitudes must be constrained to satisfy relations depending on the parameters of the flexible dynamics. These parameters (namely, the matrices δ, C, K) are supposed to be known to a certain extent. In particular, their minimum and maximum values are assumed to be known.

In order to see if the control (8)–(9) can still solve the control problem, let us choose the following Lyapunov function:

$$V_P = V_{UN} + (1/2)\xi^T P\xi,$$

i.e.,

$$\begin{aligned}V_P &= (k_p + k_d)[(\theta_0 - 1)^2 + \theta^T\theta] + (1/2)(\theta + \omega)^T J_{MB}(\theta + \omega)\\ &\quad + (1/2)\psi^T J_R\psi + (1/2)(p - \hat{p})^T\Gamma^{-1}(p - \hat{p}) + (1/2)\xi^T P\xi,\end{aligned}$$

where

$$P = P^T > 0.$$

Clearly, V_P is positive definite. Taking the time derivative along the perturbed dynamics (1), (4), and considering the positions (7), one has

$$\begin{aligned}\dot{V}_P = &(k_p + k_d)\theta^T\omega + (\theta + \omega)^T \\ &\times [\phi_1^T(\theta, \theta_r, \omega, \omega_r, \psi)p + u_1 - u_2 + r^T(\omega, \omega_r)\xi] \\ &+ \psi^T[u_2 + \phi_2^T(\dot{\omega}_r)p] - (p - \hat{p})^T\Gamma^{-1}\dot{\hat{p}} \\ &+ \xi^T P(A\xi - AB\delta\omega - B\delta\dot{\omega}_r).\end{aligned}$$

Since A has all its eigenvalues in the left-hand plane, for every fixed $\lambda > 0$ there exist a symmetric and positive-definite solution P of the Sylvester equation

$$(PA + A^TP)/2 = -\lambda I,$$

given by

$$P = \lambda\tilde{P}, \qquad \tilde{P} = \begin{bmatrix} C^{-1} + K^{-1}C + KC^{-1} & K^{-1} \\ K^{-1} & C^{-1} + K^{-1}C^{-1} \end{bmatrix}.$$

Therefore, with (8)–(9), we have

$$\begin{aligned}\dot{V}_P = &-k_p\|\theta\|^2 - k_d\|\omega\|^2 - k_r\|\psi\|^2 - \lambda\|\xi\|^2 \\ &+ \theta^T r^T(\omega, \omega_r)\xi + \xi^T[r(\omega, \omega_r) - \lambda\tilde{P}AB\delta]\omega - \lambda\xi^T\tilde{P}B\delta\dot{\omega}_r.\end{aligned}$$

Since the structural characteristic of the spacecraft are known up to a certain extent, only the upper bounds on the norms are supposed known. In this case, if $\omega_r \in L_\infty[0, \infty)$ and if one sets

$$\max_{\omega_r}\|[\delta^TK, \delta^TC - \omega_r \times \delta^T]\| \le 2\alpha, \qquad \|\delta\| \le 2\beta, \tag{10a}$$

$$\left\|\left[\begin{bmatrix} K \\ C \end{bmatrix} - \lambda\tilde{P}AB\right]\delta\right\| + \max_{\omega_r}\|\omega_r\|\,\|\delta\| \le 2\gamma(\lambda), \qquad \|\tilde{P}B\delta\| \le \upsilon, \tag{10b}$$

we have

$$\begin{aligned}&\theta^T r^T(\omega, \omega_r)\xi \le 2\alpha\|\theta\|\,\|\xi\| + 2\beta\|\omega\|\,\|\xi\|, \\ &\xi^T(r(\omega, \omega_r) - \lambda\tilde{P}AB\delta)\omega \le 2\gamma(\lambda)\|\omega\|\,\|\xi\|, \\ &\xi^T\tilde{P}B\delta\dot{\omega}_r \le \nu\|\dot{\omega}_r\|\,\|\xi\|,\end{aligned}$$

where we recall that

$$\|\theta\| \leq 1, \qquad \omega^T[0, \omega \times \delta^T]\xi = 0.$$

Hence,

$$\dot{V}_P \leq -k_r \|\psi\|^2 - [\|\theta\|, \|\omega\|, \|\xi\|] Q \begin{pmatrix} \|\theta\| \\ \|\omega\| \\ \|\xi\| \end{pmatrix} + \lambda v \|\dot{\omega}_r\| \, \|\xi\|,$$

$$Q = \begin{bmatrix} k_p & 0 & -\alpha \\ 0 & k_d & -(\beta + \gamma(\lambda)) \\ -\alpha & -(\beta + \gamma(\lambda)) & \lambda \end{bmatrix}.$$

The matrix Q is positive definite if there exists a triplet of positive parameters (k_p, k_d, λ) such that the following conditions are satisfied:

$$k_d > ((\beta + \gamma(\lambda))^2/\lambda, \qquad |Q| = k_p[k_d\lambda - (\beta + \gamma(\lambda))^2] - \alpha^2 k_d > 0. \tag{11}$$

With this objective, given a spacecraft with certain structural properties, and therefore with certain values for the bounds (10), it is always possible to fix a value for λ, and accordingly determine the values for α, β, $\gamma(\lambda)$ such that conditions (11) can be satisfied by appropriate gains k_p, k_d. While k_p, k_d appear in the control (8)–(9), λ is simply a positive parameter which can be fixed arbitrarily and does not influence the controller performance.

Defining

$$\lambda_m = \min\{k_r, \sigma(Q)\},$$

$\sigma(Q)$ the set of eigenvalues of Q, eventually we have

$$\dot{V}_P \leq -\lambda_m \|x\|^2 + \lambda v \|\dot{\omega}_r\| \, \|\xi\|, \tag{12a}$$

$$x = [\theta^T, \omega^T, \psi^T, \xi^T]^T. \tag{12b}$$

It is now possible to distinguish two operative situations. In the first, we suppose that $\dot{\omega}_r \in L_2[0, \infty) \cap L_\infty[0, \infty)$, which is a rather restrictive hypothesis; in the second, $\|\dot{\omega}_r\|$ is assumed to be simply bounded.

In the first case, from (12) we have also

$$\dot{V}_P \leq -\lambda_m \|x\|^2 + \lambda v \|\dot{\omega}_r\| \, \|x\|,$$

and the integration of both sides leads to

$$V_P(t) - V_{P0} \leq -\lambda_m \int_0^t \|x(\tau)\|^2 \, d\tau + \lambda v \int_0^t \|\dot{\omega}_r(\tau)\| \, \|x(\tau)\| \, d\tau$$

$$\leq -\lambda_m \int_0^t \|x(\tau)\|^2 \, d\tau$$

$$+ \lambda v \sqrt{\int_0^t \|\dot{\omega}_r(\tau)\| \, d\tau} \sqrt{\int_0^t \|x(\tau)\| \, d\tau}. \tag{13}$$

This can be rewritten as

$$\lambda_m \int_0^t \|x(\tau)\|^2 \, d\tau - \lambda v \int_0^t \|\dot{\omega}_r(\tau)\| \, \|x(\tau)\| \, d\tau \leq V_{P0} - V_P(t) \leq V_{P0},$$

since V_P is positive definite; therefore,

$$\lambda_m \|x\|_2^2 - \lambda v \|\dot{\omega}_r\|_2 \|x\|_2 \leq V_{P0}. \tag{14}$$

Hence, we obtain the bound

$$\|x\|_2^2 \leq [(V_{P0} + c_1^2/(4\lambda_m))/\lambda_m]^{1/2} + c_1/(2\lambda_m),$$

with $c_1 = \lambda v \|\dot{\omega}_r\|_2$, i.e., $x \in L_2[0, \infty)$. Moreover, from (13),

$$V_P(t) \leq V_{P0} - \lambda_m \int_0^t \|x(\tau)\|^2 \, d\tau + \lambda v \int_0^t \|\dot{\omega}_r(\tau)\| \, \|x(\tau)\| \, d\tau,$$

that is, V_P is uniformly bounded along the trajectories of the perturbed system and $x \in L_\infty[0, \infty)$. From (1), (4), we deduce that $\dot{x}$ is uniformly bounded, and therefore x is uniformly continuous. By the Barbalat lemma applied to x (Ref. 21), we infer that

$$\lim_{t \to \infty} x = 0,$$

i.e., the control problem is solved,

$$\lim_{t \to \infty} \theta = 0, \qquad \lim_{t \to \infty} \omega = 0.$$

Moreover, since also

$$\lim_{t \to \infty} \psi = 0, \qquad \lim_{t \to \infty} \xi = 0,$$

we deduce that

$$\lim_{t\to\infty} \Omega = 0, \qquad \lim_{t\to\infty} \eta = 0, \qquad \lim_{t\to\infty} \dot{\eta} = 0.$$

On the other hand in the second situation, in which we suppose only that $\dot{\omega}_r \in L_\infty[0, \infty)$, (13) is not applicable any more, since $\|\dot{\omega}_r\|_2$ could be infinite in (14). Actually, it turns out that, in this case, we cannot solve the control problem, since it is not possible to ensure asymptotic stability, but only that the state evolution remains bounded. In fact, denoting

$$\epsilon = \max_{\|\dot{\omega}_r\|} \lambda v \|\dot{\omega}_r\| / \lambda_m,$$

if $\|\xi\| < \epsilon$ for $t = t_0$, from (12) we get the following bound:

$$\dot{V}_P(t) \leq -\lambda_m \|x\|^2 + \lambda v \|\dot{\omega}_r\| \, \|\xi\| < \lambda_m \epsilon \|\xi\| < \lambda_m \epsilon^2,$$

and integrating over the interval $[t_0, t_1)$ we obtain

$$V_P(t_1) - V_{P0} < \lambda_m \epsilon^2 (t_1 - t_0) \Rightarrow V_P(t_1) < V_{P0} + \lambda_m \epsilon^2 (t_1 - t_0).$$

Here, t_1 represents the instant when $\|\xi\|$ becomes larger than ϵ.

Conversely, if $\|\xi\| \geq \epsilon$ for $t \geq t_1$, again from (12) we have

$$\begin{aligned} \dot{V}_P(t) &\leq -\lambda_m \|x\|^2 + \lambda v \|\dot{\omega}_r\| \, \|\xi\| \\ &\leq -\lambda_m \|x\|^2 + \lambda_m \epsilon^2 \|\xi\| / \epsilon \\ &\leq -\lambda_m \left\| \begin{bmatrix} \theta \\ \omega \\ \psi \end{bmatrix} \right\|^2 \leq 0, \end{aligned}$$

and over the interval $[t_1, t)$ we have that

$$V_P(t) \leq V_P(t_1).$$

Moreover, since V_P is positive definite,

$$\lambda_1 \left\| \begin{bmatrix} x \\ p - \hat{p} \end{bmatrix} \right\|^2 \leq V_P \leq \lambda_2 \left\| \begin{bmatrix} x \\ p - \hat{p} \end{bmatrix} \right\|^2,$$

and using the previous result we have

$$\lambda_1 \left\| \begin{bmatrix} x \\ p - \hat{p} \end{bmatrix} \right\|^2 \leq V_P(t) \leq V_P(t_1) < V_{P0} + \lambda_m \epsilon^2 (t_1 - t_0),$$

i.e.,

$$\left\|\begin{bmatrix} x \\ p-\hat{p} \end{bmatrix}\right\| < \sqrt{[V_{P0}+\lambda_m \epsilon^2(t_1-t_0)]/\lambda_1}.$$

Obviously, for t sufficiently large, we could have again $\|\xi\|<\epsilon$. Indicating by Δt_m the maximum time interval in which $\|\xi\|<\epsilon$, we have

$$\left\|\begin{bmatrix} x \\ p-\hat{p} \end{bmatrix}\right\| < c_2 := \sqrt{\left[\lambda_2 \max_{\|\xi\|<\epsilon} \left\|\begin{bmatrix} x_0 \\ p-\hat{p}_0 \end{bmatrix}\right\| + \lambda_m \epsilon^2 \Delta t_m\right] \Big/ \lambda_1};$$

that is, the control problem is not solved, but x and $p-\hat{p}$ remain bounded. Note that, in this second situation, the Barbalat lemma cannot be applied, since $\dot{V}_P$ is seminegative definite only for $\|\xi\|>\epsilon$, and can be positive definite otherwise. Moreover, if the flexible elements are not present, we have

$$\dot{V}_P = -k_p\|\theta\|^2 - k_d\|\omega\|^2 - k_r\|\psi\|^2 \leq 0,$$

and the same results of Section 3 are obtained. Therefore, in this second case, the solution given by the control (8)–(9) can be considered to be only an approximate solution of the control problem, and the approximation becomes more accurate as ϵ is decreased, by choosing appropriate gains according to (11).

We stress the fact that the fulfillment of conditions (11) renders the control (8)–(9) robust with respect to parameter and dynamic uncertainties, and that the only structural knowledge used here is given by the loose upper bound values α, β, $\gamma(\lambda)$, ν.

We can summarize these results with the following theorem.

Theorem 4.1. The control (8)–(9), with $k_r>0$, Γ positive-definite, and k_p, k_d satisfying (11), solves the tracking problem of a given trajectory with $\omega_r \in L_\infty[0, \infty)$ for the system (1), (4) ensuring bounded state evolution if $\dot{\omega}_r \in L_2[0, \infty) \cap L_\infty[0, \infty)$ and an approximate solution of the tracking problem with bounded state evolution if $\dot{\omega}_r \in L_\infty[0, \infty)$.

5. Simulation Results

The application of the control (8)–(9) to a flexible spacecraft has been simulated and the results are reported herein. The nominal inertia and coupling matrices are (Ref. 9).

$$J_{MB}=\begin{bmatrix}800 & 12 & 5\\ 12 & 400 & 1.5\\ 5 & 1.5 & 600\end{bmatrix}, \qquad J_R=\begin{bmatrix}10 & 0 & 0\\ 0 & 10 & 0\\ 0 & 0 & 10\end{bmatrix}\mathrm{kg\,m^2},$$

$$\delta=\begin{bmatrix}10 & 0.5 & 0.2\\ 0.5 & 2 & 0\\ 0.1 & 10.9 & 0.8\\ 1 & 0.5 & 0.5\end{bmatrix}\mathrm{kg^{1/2}\,m},$$

while the nominal altitude is 2000 km, and the Earth gravitational constant is $\mu=3.986\times 10^{14}\ \mathrm{m^3/s^2}$. Two different trajectories have been considered, one for each situation discussed in Section 4; in the first,

$$\omega_r\in L_2[0,\infty)\cap L_\infty[0,\infty);$$

in the second,

$$\omega_r\in L_\infty[0,\infty).$$

In both cases, the tracked trajectory is specified by the quaternions $[\theta_{r_0},\theta_r^T]^T$, describing the desired attitude for the spacecraft, from which it is possible to derive the expression of ω_r (Ref. 20). In the first case, we have chosen

$$\theta_{r0}=\cos 10\pi(1-e^{-t/18.6})/2,$$

$$\theta_r=[1\sqrt{18},\ 1/\sqrt{18},\ \sqrt{8}/3]^T\sin 10\pi(1-e^{-t/18.6})/2,$$

which corresponds to tracking a large maneuver of almost 10π rad; in the second case,

$$\theta_{r0}=\cos 3\pi\sin(0.02t)/2,$$

$$\theta_r=[\cos 0.34t,\ \sin 0.34t,\ 0]^T\sin 3\pi\sin(0.02t)/2.$$

The initial conditions have been set at

$$\theta_{r0}(0)=1,\qquad \theta_{r1}(0)=0,\qquad \theta_{r2}(0)=0,\qquad \theta_{r3}(0)=0,$$

i.e., for $t=0$ the spacecraft attitude coincide with the desired one, and

$$\omega_i(0)=0,\quad \Omega_i(0)=0\ \mathrm{rad/s},\qquad i=1,2,3.$$

Four elastic modes have been taken into account in the model used for simulating the spacecraft at

$$\omega_{n1}=1.9,\qquad \omega_{n2}=4.1,\qquad \omega_{n3}=5.8,\qquad \omega_{n4}=6\ \mathrm{rad/s},$$

with damping

$$\zeta_1=0.08, \qquad \zeta_2=0.30, \qquad \zeta_3=0.60, \qquad \zeta_4=0.75,$$

with modal variable initial values

$$\eta_i(0)=0, \quad z_i(0)=(0), \qquad i=1,\ldots,4.$$

For the design of the control law (8), we have used a reduced model, in which only the first two modes, corresponding to ω_{n1} and ω_{n2}, have been considered. The matrices K, C, δ appearing in this approximate model can be used as estimates of the real ones. On the basis of these estimates, the parameters appearing in (10) and the control gains have been computed. In this way, it has been possible to determine α, β, γ, ν in (10), by considering a worst-case analysis. Then, the control (8) so determined has been applied to the simulation model with four modes. The simulation results thus obtained can be regarded as a further test of robustness and evidence of the absence of spillover phenomena.

The parameters $\alpha, \beta, \gamma, \nu$ have been determined by increasing the values computed from (10) by 5%. For $\lambda=4.41$, we have obtained for the first case

$$\alpha_1=195.25, \qquad \beta_1=5.2674, \qquad \gamma_1(\lambda)=369.16, \qquad \nu_1=705.03,$$

and for the second case

$$\alpha_2=336.95, \qquad \beta_2=5.2674, \qquad \gamma_2(\lambda)=584.44, \qquad \nu_2=705.03,$$

with α_i, $i=1,2$, determined considering the components ω_{ri}, $i=1,2,3$, of the reference angular rate, ranging respectively in the intervals $(-42, 42)$, $(-41, 41)$, $(-0.8, 0.8)$ rad/s, as in the second tracking. On the basis of these values and (11), the gains chosen in (8)–(9) have been

$$k_{p1}=5.4030\times 10^4, \qquad k_{d1}=3.9739\times 10^4, \qquad k_{r1}=1.0\times 10^3,$$

$$k_{p2}=1.6090\times 10^5, \qquad k_{d2}=9.8569\times 10^4, \qquad k_{r2}=1.0\times 10^3,$$

in the two cases; correspondingly, $\max_{\omega_r}\|\omega_r\|$ is 1.5168 and 42.386 rad/s, respectively. These gains are 25% larger than the minimum values necessary to satisfy (11). In this way, we have taken into account further lack of structural information. Saturation at 100 N m and 5 N m on gas jet and reaction wheel inputs was considered, in order to have a more realistic situation. Finally, the initial values of the 30 adapted parameters have been chosen equal to

$$\hat{p}_G(0)=\begin{bmatrix}1.064\times 10^{-18}\\ 8.360\times 10^{-19}\\ 2.874\times 10^{-19}\\ 0\\ 0\\ 0\end{bmatrix},\qquad \hat{p}_T(0)=\begin{bmatrix}770\\ 605\\ 208\\ 0\\ 0\\ 0\end{bmatrix},$$

$$\hat{p}_{MB}(0)=\begin{bmatrix}750\\ 600\\ 200\\ 0\\ 0\\ 0\end{bmatrix},\qquad \hat{p}_R(0)=\begin{bmatrix}20\\ 5\\ 8\\ 0\\ 0\\ 0\end{bmatrix},\qquad \hat{p}_C(0)=\begin{bmatrix}0\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix},$$

with errors of about $+30\%$ for the spacecraft altitude, $\pm 100\%$ for the entries of J_{MB}, J_R, δ with respect to the real values, and adaption gains

$$\Gamma=\operatorname{diag}\{10^{-29}I,\ 1I,\ 10I\},$$

I being the 6×6 identity matrix. In this way, we included in the simulation tests also uncertainties in the knowledge of the rigid model.

Referring to the first trajectory, the solid lines in Figs. 1–6 show that the control problem is solved, with θ_0 converging to 1 (Fig. 1) and ω converging to zero (Fig. 2). Due to the norm constraint, we have that θ converges to zero; this also implies the almost-perfect convergence of the spacecraft attitude to the desired one. The controls are nonzero even when the maneuver, performed with trajectory tracking, is concluded due to the persistence of the gravity-gradient disturbance. This is shown in Fig. 3, which describes the gas jets; the reaction wheels have a similar behavior. The modal displacements and velocities of the first and third mode are given in Figs. 4–5; note that the third mode is the first of the modes not considered in the design of the controller and its behavior must be examined in order to check the absence of spillover effects. Finally, two of the thirty adapted parameters (namely $\hat{J}_{MB_1}$, $\hat{J}_{R_1}$), are illustrated in Fig. 6. Note the convergence of the adapted parameters to constant values.

Analogously, for the second tracked trajectory, the dotted lines in Figs. 1–6 show the performance of the proposed controller. In this case, as expected, the control problem is solved only in an approximate way, as explained in Section 4. In fact, θ and ω remain bounded (Figs. 1, 2); this implies a very good tracking of the desired attitude. Obviously, the control

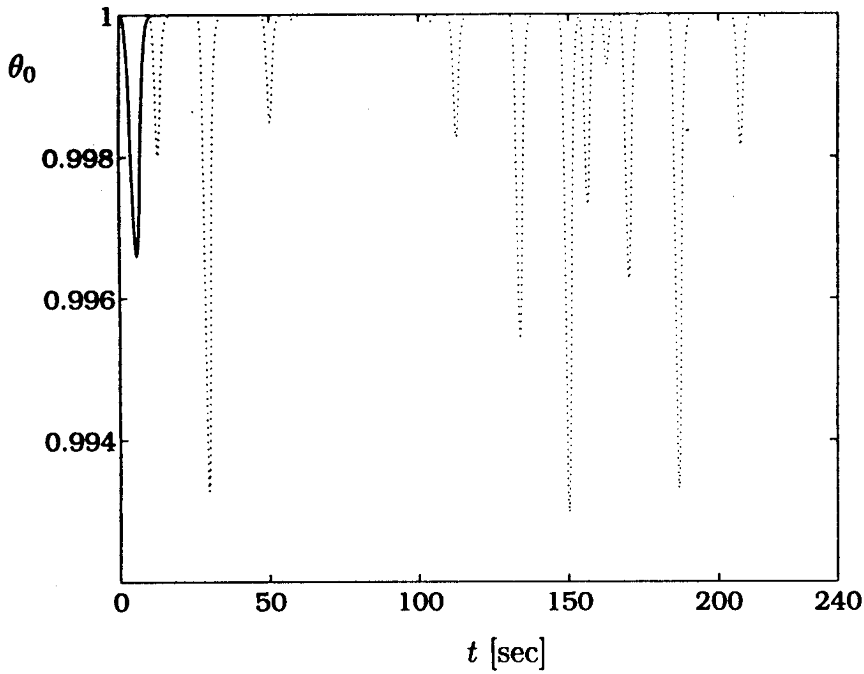

Fig. 1. Error quaternion θ_0.

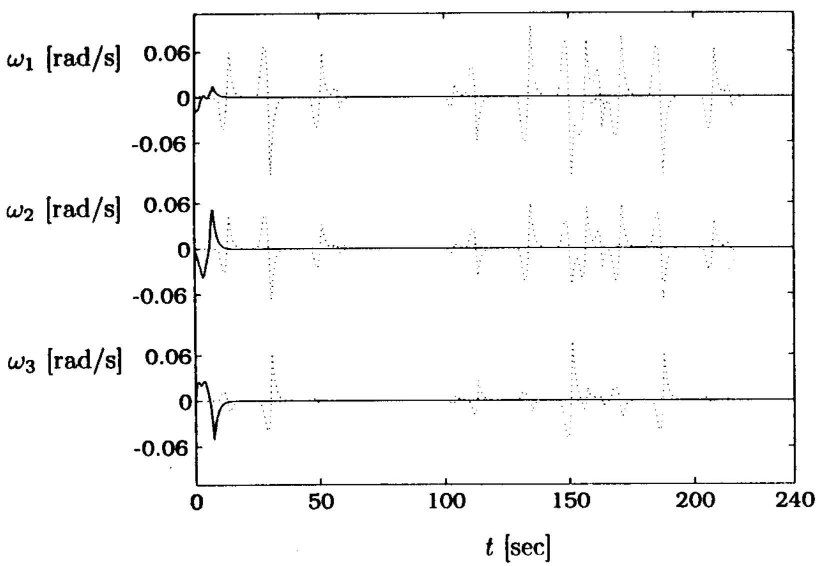

Fig. 2. Error angular velocity components ω_1, ω_2, ω_3.

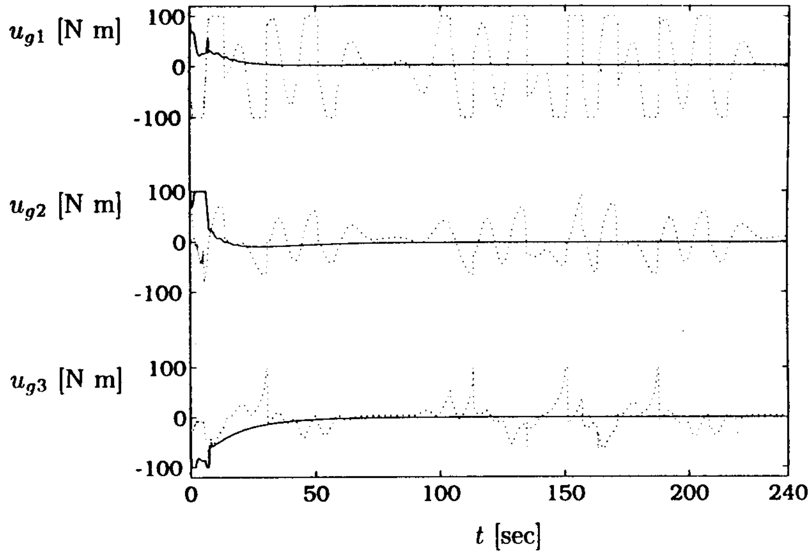

Fig. 3. Gas jet control components u_{g1}, u_{g2}, u_{g3}.

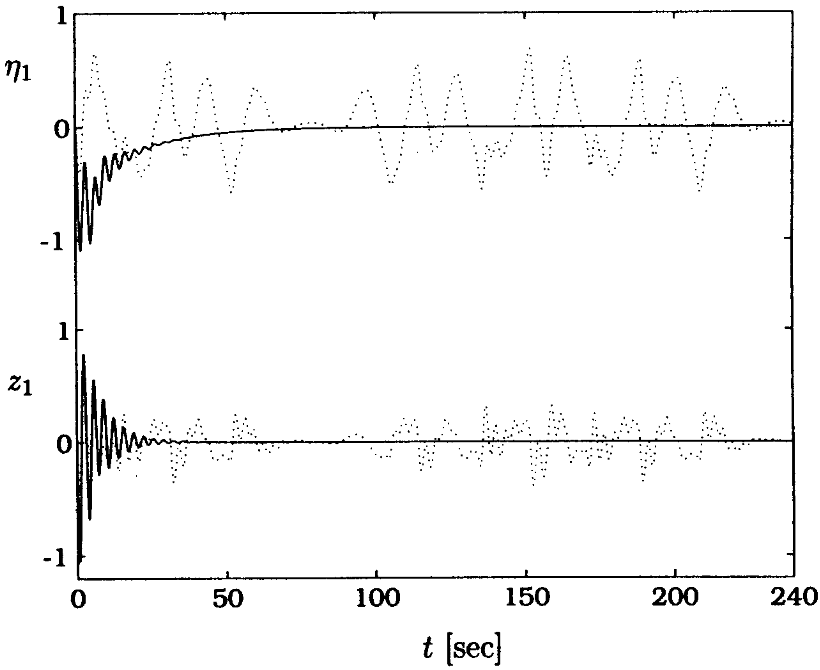

Fig. 4. First modal displacement and velocity η_1, z_1.

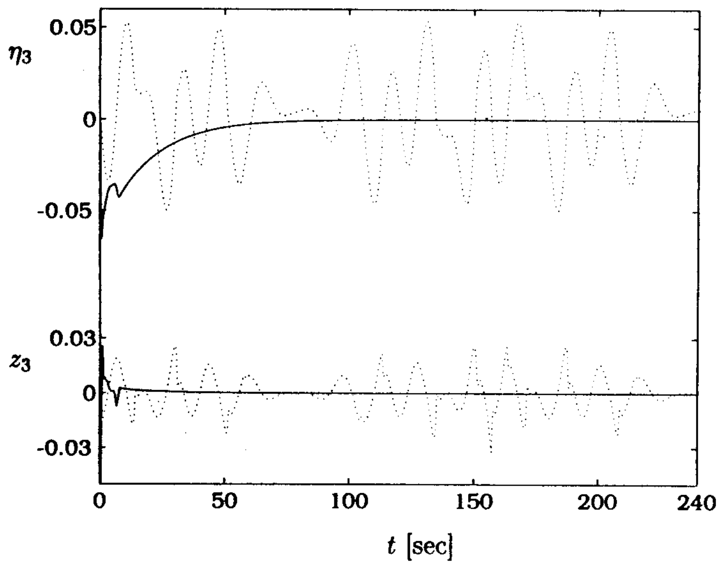

Fig. 5. Third modal displacement and velocity η_3, z_3.

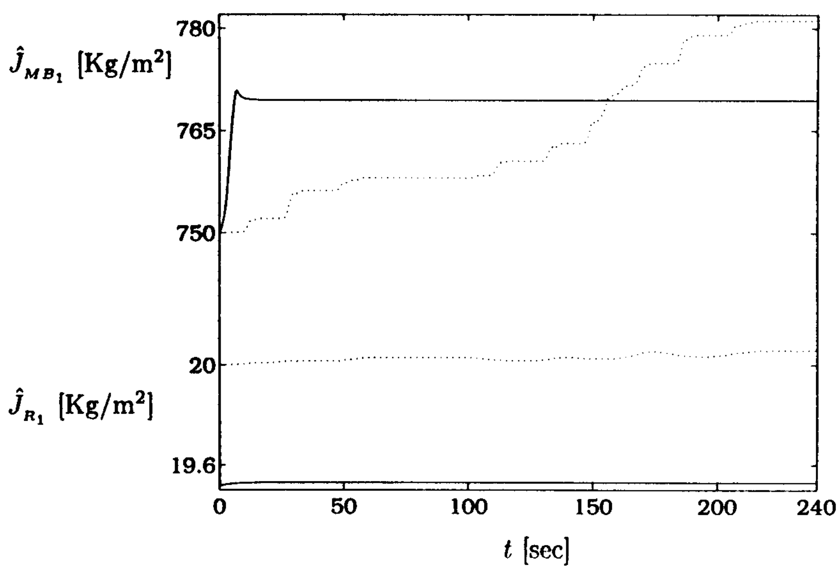

Fig. 6. Adapted parameters $\hat{J}_{MB_1}, \hat{J}_{R_1}$.

is more active (Fig. 3), and during the tracking the modal displacements and velocities remain bounded (see Fig. 4 for the first mode and Fig. 5 for the third). The adapted parameters do not converge to constant values, but tend to be piecewise constant (see Fig. 6 for $\hat{J}_{MB_1}$ and $\hat{J}_{R_1}$).

6. Conclusions

An adaptive robust control was designed for a rigid body subjected to gravity-gradient disturbances and applied to a flexible spacecraft; it was pointed out that the robustness conditions on the control gains guarantee the satisfaction, exact or approximate, of the control requirements, under appropriate conditions on the tracked trajectory. The satisfaction of the robustness conditions is largely sufficient for ensuring good performance of the control scheme, and as shown by simulation results, no difficulties are encountered, even in the presence of saturation on the inputs and higher-order modes. Future work is planned to study the digital implementation of such control scheme on hardware platforms for attitude control experimentation.

References

1. CROUCH, P. E., *Spacecraft Attitude Control and Stabilization: Applications of Geometric Control Theory to Rigid Body Models*, IEEE Transactions on Automatic Control, Vol. 29, pp. 321–331, 1984.
2. DWYER, T. A. W., *Exact Nonlinear Control of Large-Angle Rotational Maneuvers*, IEEE Transactions on Automatic Control, Vol. 29, pp. 769–774, 1984.
3. MONACO, S., and STORNELLI, S., *A Nonlinear Feedback Control Law for Attitude Control*, Algebraic and Geometric Methods in Nonlinear Control Theory, Edited by M. Hazewinkel and M. Fliess, Reidel, Dordrecht, Holland, pp. 573–595, 1986.
4. LIAN, K. Y., WANG, L. S., and FU, L. C., *Controllability of Spacecraft Systems in a Central Gravitational Field*, Proceedings of the 32nd Conference on Decision and Control, pp. 3105–3110, 1993.
5. LIAN, K. Y., WANG, L. S., and FU, L. C., *Controllability of Spacecraft in a Central Gravitational Field*, IEEE Transactions on Automatic Control, Vol. 39, pp. 2426–2441, 1994.
6. WEN, J. T., and KREUTZ-DELGADO, K., *The Attitude Control Problem*, IEEE Transactions on Automatic Control, Vol. 36, pp. 1148–1162, 1991.
7. SINGH, S. N., *Nonlinear Adaptive Attitude Control of Spacecraft*, IEEE Transactions on Aerospace and Electronic Systems, Vol. 23, pp. 371–379, 1987.
8. SINGH, S. N., *Nonlinear Adaptive Attitude Control of Satellite Using Gyrotorquers*, Proceedings of the 29th Conference on Decision and Control, pp. 3357–3361, 1990.

9. DWYER, T. A., SIRA-RAMIREZ, H., MONACO, S., and STORNELLI, S., *Variable-Structure Control of Globally Feedback Decoupled Deformable Vehicle Maneuvers*, Proceedings of the 27th Conference on Decision and Control, pp. 1281–1287, 1987.
10. MONACO, S., and STORNELLI, S., *A Nonlinear Attitude Control Law for a Satellite with Flexible Appendages*, Proceedings of the 24th Conference on Decision and Control, pp. 1654–1659, 1985.
11. MONACO, S., NORMAND-CYROT, D., and STORNELLI, S., *Sampled Nonlinear Control for Large-Angle Maneuvers of Flexible Spacecraft*, ESA SP-255, pp. 31–38, 1986.
12. SINGH, S. N., *Robust Nonlinear Attitude Control of Flexible Spacecraft*, IEEE Transactions on Aerospace and Electronic Systems, Vol. 23, pp. 380–387, 1987.
13. AZAM, M., SINGH, S. N., IYER, A., and KAKAD, Y. P., *Detumbling and Reorientation Maneuvers and Stabilization of NASA SCOLE System*, IEEE Transactions on Aerospace and Electronic Systems, Vol. 28, pp. 80–91, 1992.
14. AZAM, M., SINGH, S. N., IYER, A., and KAKAD, Y. P., *Nonlinear Rotational Maneuver and Vibration Damping of NASA SCOLE System*, Acta Astronautica, Vol. 32, pp. 211–220, 1994.
15. GEORGIOU, G., DI GENNARO, S., MONACO, S., and NORMAND-CYROT, D., *On the Nonlinear Adaptive Control of a Flexible Spacecraft*, ESA SP-323, pp. 509–514, 1991.
16. SINGH, S. N., *Rotational Maneuver of Nonlinear Uncertain Elastic Spacecraft*, IEEE Transactions on Aerospace and Electronic Systems, Vol. 24, pp. 114–123, 1988.
17. SINGH, S. N., *Nonlinear Attitude Control of Flexible Spacecraft under Disturbance Torque*, Acta Astronautica, Vol. 13, pp. 507–514, 1986.
18. ICKES, B. P., *A New Method for Performing Digital Control System Attitude Computations Using Quaternions*, AIAA Journal, Vol. 8, pp. 13–17, 1970.
19. WERTZ, J., Editor, *Spacecraft Attitude Determination and Control*, Kluwer Academic Publishers, Dordrecht, Holland, 1978.
20. YUAN, J. S. C., *Closed-Loop Manipulator Control Using Quaternion Feedback*, IEEE Journal of Robotics and Automation, Vol. 4, pp. 434–440, 1988.
21. SASTRY, S. S., and BODSON, M., *Adaptive Control: Stability, Convergence, and Robustness*, Prentice-Hall, Englewood Cliffs, New Jersey, 1989.
22. KHALIL, H. K., *Nonlinear Systems*, Macmillan Publishing Company, New York, New York, 1992.