PERGAMON

Automatica 38 (2002) 1719–1726

automatica

www.elsevier.com/locate/automatica

Brief Paper

Output attitude tracking for flexible spacecraft☆

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Received 6 October 1999; received in revised form 21 March 2002; accepted 3 April 2002

Abstract

In this work a class of nonlinear controllers has been derived for spacecraft with flexible appendages. The control aim is to track a given desired attitude. First, a static controller based on the measure of the whole state is determined. Then, a dynamic controller is designed; this controller does not use measures from the modal variables, and the variables measured are the parameters describing the attitude and the spacecraft angular velocity. Finally, it is shown that a relaxed version of the tracking problem can be solved when the only measured variable is the spacecraft angular velocity. Simulations show the performances of such control schemes.

Keywords: Nonlinear control; Output tracking; Dynamic control; Flexible spacecraft

1. Introduction

The attitude tracking problem, which basically consists of ensuring the tracking of a desired attitude while damping out the undesired vibrations, is an interesting test-bed for applying techniques developed for nonlinear systems. First applications of these techniques can be found in Dwyer (1984), Monaco and Stornelli (1985a, b), Monaco, Normand-Cyrot, and Stornelli (1986), Dwyer, Sira-Ramirez, Monaco, and Stornelli (1987), Wen and Kreutz-Delgado (1991), Crouch (1984), Di Gennaro, Monaco, Normand-Cyrot and Pignatelli (1997), and Di Gennaro, Monaco, and Normand-Cyrot (1999), which deal with the simple case of rigid spacecraft, and in Joshi (1989), Vadali (1990), Georgiou, Di Gennaro, Monaco, and Normand-Cyrot (1991), Joshi, Kelkar, and Wen (1995), Kelkar and Joshi (1996) and Di Gennaro (1998b), in the case of spacecraft with flexible appendages. These works require that the entire state of the system is available for feedback. Furthermore, they deal with the case of *point-to-point maneuvers* for flexible spacecraft or with the case of tracking maneuvers for *rigid* spacecraft; moreover, they do not consider terms in the controller which explicitly take into account the contribution to the attitude pointing due to the flexibility; this clearly diminishes the pointing precision.

To overcome these last limitations, in this work we propose a state-feedback controller which solves the attitude tracking problem for flexible spacecraft, and which contains also a direct compensation of the dynamic terms due to the flexibility. This controller is at the basis of the development of an output-feedback controller overcoming the first limitation mentioned, namely the state measurability. In fact, the measurability hypothesis is an important aspect (and limitation) in the implementation of sophisticated nonlinear control laws for flexible spacecraft. In particular, this means that not only the variables describing the attitude and the spacecraft angular velocity, but also the modal variables describing the deflection of the flexible elements have to be measured. The pointing precision of the payload carried by these flexible structures is a crucial issue and, therefore, the lack of modal measures may constitute a problem when applying fine attitude control strategies. Unfortunately, in some cases the availability of the overall system state is an unrealistic hypothesis, due to the impossibility or impracticability of using appropriate sensors. A solution to this problem are the proposed dynamic compensators.

Previous works on the design of dynamic controllers, but limited to rigid spacecraft, can be found in Aeyels (1985), where the stabilization problem of the angular velocity

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication by Associate Editor Hassan Khalil under the direction of Editor Henk Nijmeijer. This research was partially supported by the "Agenzia Spaziale Italiana" and by the University of L'Aquila, "Progetto di Ateneo—Il controllo attivo nella meccanica delle strutture".

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PII: S0005-1098(02)00082-1

equations is considered, and in Tsiotras (1995), and Lizarralde and Wen (1996), where the attitude maneuver problem is solved, exploiting the passivity approach, measuring only the spacecraft attitude. In the case of flexible spacecraft with feedback from the output and for point-to-point maneuvers, some recent results can be found in Di Gennaro (1996), where the attitude maneuver problem is solved exploiting the fact that the total angular momentum remains constant, and Di Gennaro (1998c), where a dynamic controller is derived, making use of the attitude parameters.

Exploiting the results contained in Di Gennaro (1998a), where a dynamic compensator for flexible spacecraft is derived to actively damp the vibrations with piezoelectric actuators during attitude tracking, in this work we determine an output-feedback controller solving the tracking problem for flexible spacecraft. Two versions of the output tracking problem are considered: in the first case, the modal variables are not measured. In the second case, generalizing the results of Di Gennaro (1998a), the same problem is solved when the only measured variable is the spacecraft angular velocity. Note that the lack of attitude measures is a more serious and interesting problem than the lack of the angular velocity. In this second case, it is obvious that the attitude tracking problem can be solved only in an approximated way, in the sense of ultimate boundedness of the system trajectories (Khalil, 1996). This can be also considered an attitude tracking problem with bounded attitude error, i.e. a milder version of the attitude tracking problem.

The rest of the paper is organized as follows. In Section 2 the mathematical model of a spacecraft with flexible appendages is recalled. In Section 3 the control problem is stated, while in Section 4 a nonlinear controller is derived, solving the posed problem under the hypothesis of measurability of the whole state of the system. This assumption is removed in Section 5, where a dynamic controller is presented either when the measured variables are the attitude and angular velocity parameters, or when we suppose that only angular velocity measures are available. This controller is tested in Section 6, where some simulation results are presented. Some comments conclude the paper.

2. Mathematical model of flexible spacecraft

The mathematical model of a flexible spacecraft is here briefly recalled (for details see Monaco & Stornelli, 1985b; Monaco et al., 1986; Di Gennaro et al., 1999)

$$\begin{aligned}
\dot{e}_0 &= -\tfrac{1}{2}\, e^{\mathrm{T}}\omega_{\mathrm{e}},\\
\dot{e} &= \tfrac{1}{2}\,(e_0 I + \tilde{e})\omega_{\mathrm{e}},\\
\dot{\omega}_{\mathrm{e}} &= J_{\mathrm{mb}}^{-1}[-N(\omega_{\mathrm{e}},\psi,\omega_{\mathrm{r}})\\
&\quad + \delta^{\mathrm{T}}(C\psi + K\eta - C\delta\omega_{\mathrm{e}}) + u] - \dot{\omega}_{\mathrm{r}},\\
\begin{pmatrix}\dot{\eta}\\ \dot{\psi}\end{pmatrix} &= A\begin{pmatrix}\eta\\ \psi\end{pmatrix} - AB\delta\omega_{\mathrm{e}} - B\delta\dot{\omega}_{\mathrm{r}},
\end{aligned} \tag{1}$$

where e_0, e are the quaternions describing the attitude error (Ickes, 1970; Wertz, 1978) between the actual and the reference attitude, described by the quaternions q_0, q, and $q_{\mathrm{r}0}$, q_{r}, and given by Yuan (1988)

$$\begin{pmatrix} e_0\\ e\end{pmatrix} = \begin{pmatrix} q_0 & q^{\mathrm{T}}\\ -q & q_0 I + \tilde{q}\end{pmatrix}\begin{pmatrix} q_{\mathrm{r}0}\\ q_{\mathrm{r}}\end{pmatrix},\qquad \tilde{q} = \begin{pmatrix} 0 & -q_3 & q_2\\ q_3 & 0 & -q_1\\ -q_2 & q_1 & 0\end{pmatrix} \tag{2}$$

with I the identity matrix. Moreover, $\omega_{\mathrm{e}} = \omega - \omega_{\mathrm{r}}$ is the error between the spacecraft angular velocity ω and the reference ω_{r}. Note that ω_{r} and $\dot{\omega}_{\mathrm{r}}$, expressed in the spacecraft frame, depend on q_0, q

$$\omega_{\mathrm{r}} = 2\mathscr{R}(q_0, q; q_{\mathrm{r}0}, q_{\mathrm{r}})(-q_{\mathrm{r}} \quad q_{\mathrm{r}0}I - \tilde{q}_{\mathrm{r}})\begin{pmatrix}\dot{q}_{\mathrm{r}0}\\ \dot{q}_{\mathrm{r}}\end{pmatrix}, \tag{3}$$

$$\dot{\omega}_{\mathrm{r}} = 2\mathscr{R}(q_0, q; q_{\mathrm{r}0}, q_{\mathrm{r}})(-q_{\mathrm{r}} \quad q_{\mathrm{r}0}I - \tilde{q}_{\mathrm{r}})\begin{pmatrix}\ddot{q}_{\mathrm{r}0}\\ \ddot{q}_{\mathrm{r}}\end{pmatrix}, \tag{4}$$

where $\mathscr{R}(q_0, q; q_{\mathrm{r}0}, q_{\mathrm{r}})$ transforms vectors expressed in the reference frame into vectors in the spacecraft frame (Wertz, 1978). Furthermore, J_{mb} is the main body symmetric inertia matrix, $C = \mathrm{diag}\{2\zeta_i\Omega_{\mathrm{n}i},\ i = 1,\ldots,N_{\mathrm{e}}\}$, $K = \mathrm{diag}\{\Omega_{\mathrm{n}i}^2,\ i = 1,\ldots,N_{\mathrm{e}}\}$ are the damping and stiffness matrices (N_{e} elastic modes are considered, with $\Omega_{\mathrm{n}i}$ the natural frequencies and ζ_i the associated dampings), δ is the coupling matrix between elastic and rigid dynamics, η is the vector of the modal displacements and $\psi = \delta\omega_{\mathrm{e}} + \dot{\eta}$ is the difference between the total modal velocity $\delta\omega + \dot{\eta}$ and the reference modal velocity $\delta\omega_{\mathrm{r}}$. Finally, u is the external torque produced by gas jets

$$N(\omega_{\mathrm{e}}, \psi, \omega_{\mathrm{r}}) = (\tilde{\omega}_{\mathrm{e}} + \tilde{\omega}_{\mathrm{r}})(J_{\mathrm{mb}}\omega_{\mathrm{e}} + \delta^{\mathrm{T}}\psi + J\omega_{\mathrm{r}}) \tag{5}$$

is the gyroscopic term ($J = J_{\mathrm{mb}} + \delta^{\mathrm{T}}\delta$ is the symmetric inertia matrix of the undeformed structure) and

$$A = \begin{pmatrix} 0 & I\\ -K & -C\end{pmatrix},\qquad B = \begin{pmatrix} 0\\ I\end{pmatrix}. \tag{6}$$

In the following we will suppose that $\sigma(A) \in \mathbb{C}^-$, where $\sigma(\cdot)$ denotes the set of eigenvalues. This means that $K > 0$, $C > 0$, and that the spacecraft structure has a non-negligible internal damping.

The *attitude tracking problem* consists of determining a control law such that the actual spacecraft attitude follows a desired one, damping out the induced oscillations of the tips of the spacecraft flexible appendages, namely $\lim_{t\to\infty} e = 0$, $\lim_{t\to\infty} \mathscr{T}^{\mathrm{T}}\eta = 0$. Here, the constant matrix $\mathscr{T}$ transforms modal coordinates into real ones. Note that if $e \to 0$ then $e_0 \to 1$ as t tends to infinity, due to the constraint relation among the four quaternions (Wertz, 1978).

Moreover, we will deal with the *attitude tracking problem with bounded attitude error*, which represents a sort of "relaxed" attitude tracking problem. In this case, we want that the spacecraft tracks a fixed reference with bounded

bounded attitude error, and oscillation damping, namely $\lim_{t\to\infty}\|e\| = k_E$, $\lim_{t\to\infty}\mathscr{T}^{\mathrm{T}}\eta = 0$ with k_E a certain constant less than 1.

While the attitude tracking problem is a classical problem of rendering asymptotically stable the error dynamics (1), the attitude tracking problem with bounded attitude error is a problem of ultimate boundedness of the trajectories of these error dynamics (Khalil, 1996). To better understand the interest of this second problem, note that if we suppose to measure ω, but not q_0 and q, from (2) and (3) it is clear that the variables e_0, e and ω_{e} are not available for feedback, since either e_0, e or ω_{r} *in general* depend also on q_0, q. Moreover, also $\dot{\omega}_{\mathrm{r}}$ *in general* depends on q_0, q, and therefore is unknown (see (3)). Hence, in this case the attitude tracking problem cannot be solved; physically, it is not possible to rotate the spacecraft up to the desired attitude since we do not know the actual attitude from which it starts to slew. But one might be able to solve the relaxed version of this problem, ensuring bounded attitude error.

3. State-feedback control

In this section we suppose that the entire state is available for feedback. The following result shows that the attitude tracking problem is solvable in the case of measure of the whole state.

Theorem 1. *Let us suppose that the whole state is available for measure. For all $k_{\mathrm{p}} > 0$ and for $k_{\mathrm{d}} > 0$ large enough, the following static controller solves the attitude tracking problem for system* (1) *with a reference angular velocity $\omega_{\mathrm{r}} \in \mathscr{L}_\infty$ and derivative $\dot{\omega}_{\mathrm{r}} \in \mathscr{L}_2 \cap \mathscr{L}_\infty$*

$$\begin{aligned} u = & -k_{\mathrm{p}}e - k_{\mathrm{d}}\omega_{\mathrm{e}} - \tfrac{1}{2}J_{\mathrm{mb}}(e_0 I + \tilde{e})\omega_{\mathrm{e}} + N(\omega_{\mathrm{e}},\psi,\omega_{\mathrm{r}}) \\ & -\delta^{\mathrm{T}}(C\psi + K\eta - C\delta\omega_{\mathrm{e}}) + J_{\mathrm{mb}}\dot{\omega}_{\mathrm{r}} \end{aligned} \tag{7}$$

with $N(\omega_{\mathrm{e}},\psi,\omega_{\mathrm{r}})$ given by (5).

Proof. This result is proved by applying Barbalat theorem (Sastry & Bodson, 1989). To this aim, it is necessary to prove the boundedness and the square integrability of the state of the system. For, let us consider the following Lyapunov function (Khalil, 1996):

$$\begin{aligned} V(t,x) = & (k_{\mathrm{p}} + k_{\mathrm{d}})[(1-e_0)^2 + e^{\mathrm{T}}e] \\ & + \frac{1}{2}(e+\omega_{\mathrm{e}})^{\mathrm{T}}J_{\mathrm{mb}}(e+\omega_{\mathrm{e}}) \\ & + \frac{1}{2}(\eta^{\mathrm{T}} \quad \psi^{\mathrm{T}})P\begin{pmatrix}\eta\\ \psi\end{pmatrix}, \end{aligned} \tag{8}$$

where $V(t,x) \leqslant \alpha(\|x\|)$, $\alpha \in \mathscr{K}_\infty$, $P = P^{\mathrm{T}} > 0$ and $x = (e^{\mathrm{T}} \ \ \omega_{\mathrm{e}}^{\mathrm{T}} \ \ \eta^{\mathrm{T}} \ \ \psi^{\mathrm{T}})^{\mathrm{T}}$ is the state vector. Using Eqs. (1), the time derivative along the system trajectories is

$$\begin{aligned} \dot{V}(t,x) = & (k_{\mathrm{p}} + k_{\mathrm{d}})e^{\mathrm{T}}\omega_{\mathrm{e}} + (e+\omega_{\mathrm{e}})^{\mathrm{T}}\Bigg[\frac{1}{2}J_{\mathrm{mb}}(e_0 I + \tilde{e})\omega_{\mathrm{e}} \\ & - N(\omega_{\mathrm{e}},\psi,\omega_{\mathrm{r}}) + \delta^{\mathrm{T}}(C\psi + K\eta - C\delta\omega_{\mathrm{e}}) \\ & + u - J_{\mathrm{mb}}\dot{\omega}_{\mathrm{r}}\Bigg] + (\eta^{\mathrm{T}} \quad \psi^{\mathrm{T}})P \\ & \left[A\begin{pmatrix}\eta\\ \psi\end{pmatrix} - AB\delta\omega_{\mathrm{e}} - B\delta\dot{\omega}_{\mathrm{r}}\right]. \end{aligned} \tag{9}$$

Substituting (7) in (9), $\dot{V}(t,x)$ can be written as follows:

$$\begin{aligned} \dot{V}(t,x) = & -k_{\mathrm{p}}\|e\|^2 - k_{\mathrm{d}}\|\omega_{\mathrm{e}}\|^2 + (\eta^{\mathrm{T}} \quad \psi^{\mathrm{T}})PA\begin{pmatrix}\eta\\ \psi\end{pmatrix} \\ & -(\eta^{\mathrm{T}} \quad \psi^{\mathrm{T}})PAB\delta\omega_{\mathrm{e}} - (\eta^{\mathrm{T}} \quad \psi^{\mathrm{T}})PB\delta\dot{\omega}_{\mathrm{r}}. \end{aligned} \tag{10}$$

Setting

$$Q = \begin{pmatrix} k_{\mathrm{p}}I & 0 & 0 \\ 0 & k_{\mathrm{d}}I & Q_{32}^{\mathrm{T}} \\ 0 & Q_{32} & Q_{33} \end{pmatrix}, \quad Q_{32} = PAB\delta/2,$$

$$Q_{33} = -(PA + A^{\mathrm{T}}P)/2 > 0 \tag{11}$$

with I the identity matrix, one has

$$\begin{aligned} \dot{V}(t,x) &= -x^{\mathrm{T}}Qx - (\eta^{\mathrm{T}} \quad \psi^{\mathrm{T}})PB\delta\dot{\omega}_{\mathrm{r}} \\ &\leqslant -\lambda_{\mathrm{m}}\|x\|^2 + \varDelta\|\dot{\omega}_{\mathrm{r}}\|\,\|x\|, \end{aligned} \tag{12}$$

where $\lambda_{\mathrm{m}} = \min\sigma(Q)$ and $\varDelta = \|PB\delta\|$. Once the matrix $Q_{33} > 0$ has been fixed and $P > 0$ is determined as solution of the Sylvester equation, the matrix Q is positive-definite for $k_{\mathrm{d}} > 0$ large enough. Therefore, $\lambda_{\mathrm{m}} > 0$. Since $\dot{\omega}_{\mathrm{r}} \in \mathscr{L}_\infty$, $\dot{\omega}_{\mathrm{r}}$ is bounded, say $\|\dot{\omega}_{\mathrm{r}}(t)\| \leqslant c$, $\forall t \geqslant t_0$ (this reflects the fact that $\ddot{q}_{\mathrm{r}0}$, $\ddot{q}_{\mathrm{r}}$ are bounded, since q, q_{r} are unitary vectors and $\dot{\omega}_{\mathrm{r}}$ is given by (4)). Hence, one has that $\dot{V}(t,x) \leqslant 0$ when $\|x(t)\| \geqslant c\varDelta/\lambda_{\mathrm{m}}$, namely $x(t)$ is bounded (see Khalil, 1996).

To prove the square integrability of $x(t)$, let us integrate both sides of (12)

$$\begin{aligned} & V(t,x) - V(t_0,x_0) \\ & \leqslant -\lambda_{\mathrm{m}}\int_{t_0}^{t}\|x(\tau)\|^2\,\mathrm{d}\tau + \varDelta\int_{t_0}^{t}\|\dot{\omega}_{\mathrm{r}}(\tau)\|\,\|x(\tau)\|\,\mathrm{d}\tau \\ & \leqslant -\lambda_{\mathrm{m}}\int_{t_0}^{t}\|x(\tau)\|^2\,\mathrm{d}\tau + \varDelta\left[\int_{t_0}^{t}\|\dot{\omega}_{\mathrm{r}}(\tau)\|^2\,\mathrm{d}\tau\right]^{1/2} \\ & \quad \times \left[\int_{t_0}^{t}\|x(\tau)\|^2\,\mathrm{d}\tau\right]^{1/2}, \end{aligned}$$

where the Schwarz inequality (Curtain & Pritchard, 1977) has been used and $x_0 = x(t_0)$. Considering the limit as t tends to infinity and denoting with $\|\cdot\|_2$ the $\mathscr{L}_2$-norm, one has

$$V(\infty,x) - V(t_0,x_0) \leqslant -\lambda_{\mathrm{m}}\|x\|_2^2 + \varDelta\|\dot{\omega}_{\mathrm{r}}\|_2\|x\|_2. \tag{13}$$

Moreover, since $V(\infty,x) \geqslant 0$,

$$\lambda_m \|x\|_2^2 - \Delta\|\dot{\omega}_r\|_2\|x\|_2 \leqslant V(t_0,x_0) - V(\infty,x) \leqslant V(t_0,x_0)$$

and this implies that $x \in \mathscr{L}_2$, since

$$\|x\|_2 \leqslant \frac{1}{\sqrt{\lambda_m}}\left[V(t_0,x_0) + \frac{\Delta^2}{4\lambda_m}\|\dot{\omega}_r\|_2^2\right]^{1/2} + \frac{\Delta}{2\lambda_m}\|\dot{\omega}_r\|_2, \tag{14}$$

$\dot{\omega}_r \in \mathscr{L}_2$ by hypothesis, and $V(t,x) \leqslant \alpha(\|x\|)$, $\alpha \in \mathscr{K}_\infty$, as previously observed. The application of Barbalat theorem allows one to conclude that $\lim_{t\to\infty} x = 0$, and $\lim_{t\to\infty} e = 0$, $\lim_{t\to\infty} \mathscr{T}^{\mathrm{T}}\eta = 0$, i.e. controller (7) fulfills the control objectives. □

4. Output-feedback control

In this section the hypothesis of measurability of the whole state is removed. First, we suppose that the modal variables are not measured. In what follows we show that, if the system parameters are perfectly known, it is possible to design a dynamic controller, based on the estimates $\hat{\eta}$, $\hat{\psi}$ of the modal variables, ensuring the control objectives for the attitude tracking problem.

Theorem 2. *If the modal variables η, ψ are not measured, for all $k_p > 0$ and for $k_d > 0$ large enough the dynamic controller*

$$\begin{pmatrix} \dot{\hat{\eta}} \\ \dot{\hat{\psi}} \end{pmatrix} = A\begin{pmatrix} \hat{\eta} \\ \hat{\psi} \end{pmatrix} - AB\delta\omega_e - B\delta\dot{\omega}_r + \Gamma\begin{pmatrix} K\delta \\ \delta(\tilde{\omega}_e + \tilde{\omega}_r) + C\delta \end{pmatrix}(e + \omega_e), \tag{15}$$

$$u = -k_p e - k_d\omega_e - \tfrac{1}{2}J_{mb}(e_0 I + \tilde{e})\omega_e + N(\omega_e, \hat{\psi}, \omega_r) - \delta^{\mathrm{T}}(C\hat{\psi} + K\hat{\eta} - C\delta\omega_e) + J_{mb}\dot{\omega}_r \tag{16}$$

solves the attitude tracking problem for system (1) *with a reference angular velocity $\omega_r \in \mathscr{L}_\infty$ and derivative $\dot{\omega}_r \in \mathscr{L}_2 \cap \mathscr{L}_\infty$, where $N(\omega_e, \psi, \omega_r)|_{\psi=\hat{\psi}}$ is given by* (5), *A, B are given by* (6), *and $\Gamma = \Gamma^{\mathrm{T}} > 0$ is a gain matrix.*

Proof. The proof is similar to that of Theorem 1 and, therefore, is only sketched. In our case, we need to take into account the estimates on η and ψ; hence, an additional term is necessary in the Lyapunov function. Considering the function $V(t,x)$ given by (8), let us consider

$$V_1(t,x,e_\eta,e_\psi) = V(t,x) + \frac{1}{2}(e_\eta^{\mathrm{T}} \quad e_\psi^{\mathrm{T}})\Gamma^{-1}\begin{pmatrix} e_\eta \\ e_\psi \end{pmatrix}, \tag{17}$$

where $e_\eta = \eta - \hat{\eta}$, $e_\psi = \psi - \hat{\psi}$ are the estimate errors. Note that $\Gamma^{-1} > 0$. In the time derivative of V_1, along the trajectories of system (1), we have $\dot{V}(t,x)$ as in the proof of Theorem 1 and

$$\dot{V}_1(t,x,e_\eta,e_\psi) = \dot{V}(t,x) + (e_\eta^{\mathrm{T}} \quad e_\psi^{\mathrm{T}})\Gamma^{-1}\left[A\begin{pmatrix} \eta \\ \psi \end{pmatrix} - AB\delta\omega_e - B\delta\dot{\omega}_r - \begin{pmatrix} \dot{\hat{\eta}} \\ \dot{\hat{\psi}} \end{pmatrix}\right].$$

Therefore, substituting the control u given by (16) and the updating laws $\dot{\hat{\eta}}$, $\dot{\hat{\psi}}$ in (15), one has

$$\begin{aligned}\dot{V}_1(t,x,e_\eta,e_\psi) &= -k_p\|e\|^2 - k_d\|\omega_e\|^2 + (\eta^{\mathrm{T}} \quad \psi^{\mathrm{T}})PA\begin{pmatrix} \eta \\ \psi \end{pmatrix} \\ &\quad - (\eta^{\mathrm{T}} \quad \psi^{\mathrm{T}})PAB\delta\omega_e - (\eta^{\mathrm{T}} \quad \psi^{\mathrm{T}})PB\delta\dot{\omega}_r \\ &\quad + (e_\eta^{\mathrm{T}} \quad e_\psi^{\mathrm{T}})\Gamma^{-1}A\begin{pmatrix} e_\eta \\ e_\psi \end{pmatrix}\end{aligned} \tag{18}$$

similar to (10) except for the last term due to the estimate errors. Setting Q as in (11), and fixing the positive-definite matrix Q_e, so that Γ^{-1} is solution of the Sylvester equation $Q_e = -(\Gamma^{-1}A + A^{\mathrm{T}}\Gamma^{-1})/2$, one finally obtains

$$\dot{V}_1(t,x,e_\eta,e_\psi) \leqslant -\lambda_m\left\|\begin{matrix} x \\ e_\eta \\ e_\psi \end{matrix}\right\|^2 + \Delta\|\dot{\omega}_r\|\left\|\begin{matrix} x \\ e_\eta \\ e_\psi \end{matrix}\right\|, \tag{19}$$

where $\lambda_m = \min\{\sigma(Q), \sigma(Q_e)\}$ and $\Delta = \|PB\delta\|$. Arguing as in the proof of Theorem 1 one concludes that the control problem is solved, with $\lim_{t\to\infty} e_\eta = 0$, $\lim_{t\to\infty} e_\psi = 0$. □

In this second part of this section we suppose that not only η and ψ, but also q_0, q are not available. This happens, for instance, when an attitude sensor failure occurs. As pointed out before, when the quaternions q_0, q cannot be measured the spacecraft position is not known with precision; this means that also e_0, e (see (2)) and ω_r, $\dot{\omega}_r$ (see (3) and (4)) are unknown, and the attitude tracking problem does not have a solution. Therefore, we will try to solve the attitude tracking problem with bounded error.

We suppose to have an estimate $\hat{q}_0(0) \neq q_0(0)$, $\hat{q}(0) \neq q(0)$ of the spacecraft attitude at $t = 0$. By $\hat{q}_0$, $\hat{q}$ we denote the estimate of q_0, q, whose dynamics are chosen equal to

$$\dot{\hat{q}}_0 = -\tfrac{1}{2}\hat{q}^{\mathrm{T}}\omega,$$

$$\dot{\hat{q}} = \tfrac{1}{2}(\hat{q}_0 I + \tilde{\hat{q}})\omega. \tag{20}$$

Then, we will consider the attitude error $\hat{e}_0$, $\hat{e}$ between $\hat{q}_0$, $\hat{q}$ and the desired attitude q_{r0}, q_r, given by (2) with $\hat{q}_0$, $\hat{q}$ in the place of q_0, q. Furthermore, we will indicate by $\hat{\omega}_r$, $\dot{\hat{\omega}}_r$ the estimated angular velocity and acceleration, computed by (3) and (4), with q_0, q substituted by $\hat{q}_0$, $\hat{q}$.

It is now clear that the attitude tracking problem with bounded error can be solved as an attitude tracking problem where the spacecraft error attitude is $\hat{e}_0$, $\hat{e}$ instead of e_0, e. This also leads to consider the estimates $\hat{\omega}_r$, $\hat{\omega}_e = \omega - \hat{\omega}_r$, $\dot{\hat{\omega}}_r$ in the place of ω_r, ω_e, $\dot{\omega}_r$. Therefore, the posed problem can be solved by applying the controller of Theorem 2, in which the estimation dynamics $\hat{\eta}$, $\hat{\psi}$ and the control u are appropriately modified. This intuitive discussion is formalized in the following theorem, which solves the attitude tracking problem with bounded error.

Theorem 3. *Let us suppose that only the spacecraft angular velocity ω is available for measure. Moreover, let*

$$\begin{pmatrix} E_0 \\ E \end{pmatrix} = \begin{pmatrix} q_0 & q^T \\ -q & q_0 I + \tilde{q} \end{pmatrix} \begin{pmatrix} \hat{q}_0 \\ \hat{q} \end{pmatrix} \tag{21}$$

be the error between $\hat{q}_0$, $\hat{q}$ and q_0, q, and let us indicate $\|E(0)\|$ by k_E. For all $k_p > 0$ and for $k_d > 0$ large enough, the dynamic controller (15), (16) *with e_0, e, ω_e, ω_r, $\dot{\omega}_r$ substituted by $\hat{e}_0$, $\hat{e}$, $\hat{\omega}_e = \omega - \hat{\omega}_r$, $\hat{\omega}_r$, $\dot{\hat{\omega}}_r$, and with $\hat{q}_0$, $\hat{q}$ given by* (20), *solves the attitude tracking problem with bounded error for system* (1), *with an estimated reference angular velocity $\hat{\omega}_r \in \mathscr{L}_\infty$ and derivative $\dot{\hat{\omega}}_r \in \mathscr{L}_2 \cap \mathscr{L}_\infty$. Moreover, the estimate error E_0, E remain constant with $\lim_{t\to\infty} \|e\| = k_E$.*

Proof. The mathematical model in the new coordinates $\hat{e}_0$, $\hat{e}$, $\hat{\omega}_e = \omega - \hat{\omega}_r$, η, $\psi = \delta\hat{\omega}_e + \dot{\eta}$, can be derived as in Section 2 and is given by

$$\dot{\hat{e}}_0 = -\tfrac{1}{2}\,\hat{e}^T\hat{\omega}_e,$$

$$\dot{\hat{e}} = \tfrac{1}{2}\,(\hat{e}_0 I + \tilde{\hat{e}})\hat{\omega}_e,$$

$$\dot{\hat{\omega}}_e = J_{mb}^{-1}[-N(\hat{\omega}_e, \hat{\psi}, \hat{\omega}_r) + \delta^T(C\psi + K\eta - C\delta\hat{\omega}_e) + u] - \dot{\hat{\omega}}_r,$$

$$\begin{pmatrix} \dot{\eta} \\ \dot{\psi} \end{pmatrix} = A\begin{pmatrix} \eta \\ \psi \end{pmatrix} - AB\delta\hat{\omega}_e - B\delta\dot{\hat{\omega}}_r \tag{22}$$

with $N(\omega_e, \psi, \omega_r)|_{\omega_e=\hat{\omega}_e,\ \psi=\hat{\psi},\ \omega_r=\hat{\omega}_r}$ as in (5). The proof of the theorem follows from arguments analogous to those used in the proofs of Theorems 1 and 2. The Lyapunov function is again the function (17), with $\hat{e}_0$, $\hat{e}$, $\hat{\omega}_e$ in the place of e_0, e, ω_e, and $x = (\hat{e}\ \ \hat{\omega}_e^T\ \ \eta^T\ \ \psi^T\ \ e_\eta^T\ \ e_\psi^T)^T$, $e_\eta = \eta - \hat{\eta}$, $e_\psi = \psi - \hat{\psi}$. Making use of dynamics (22), and the dynamic controller, one works out (see (18) and (19))

$$\dot{V}(t,x) \leqslant -\lambda_m\|x\|^2 + \Delta\|\dot{\hat{\omega}}_r\|\,\|x\|$$

with Δ, Q_{33}, Q_e, $\lambda_m = \min\{\sigma(Q), \sigma(Q_e)\}$, Q as in the proof of Theorem 2. The same arguments used in the proofs of Theorems 1 and 2, with the bound (14) substituted by

$$\|x\|_2 \leqslant \frac{1}{\sqrt{\lambda_m}}\left[V(t_0,x_0) + \frac{\Delta^2}{4\lambda_m}\|\dot{\hat{\omega}}_r\|_2^2\right]^{1/2} + \frac{\Delta}{2\lambda_m}\|\dot{\hat{\omega}}_r\|_2$$

yield $\lim_{t\to\infty} x = 0$. In particular, $\lim_{t\to\infty} \hat{e} = 0$. It is easy to see that $(\hat{e}_0, \hat{e}) = (1, 0)$ if and only if the attitude estimation error E is equal to the tracking error e. Then, $\lim_{t\to\infty}\|e\| = \|E\| = k_E$. Moreover, $\lim_{t\to\infty} \mathscr{T}^T\eta = 0$. Finally, E_0, E remain constant since

$$\dot{E}_0 = -\tfrac{1}{2}\,E^T(\omega - \omega) = 0,$$

$$\dot{E} = \tfrac{1}{2}\,(E_0 I + \tilde{E})(\omega - \omega) = 0. \qquad \square$$

This result states that during the tracking the attitude error does not increase with respect to the initial error; roughly speaking, the controller does not worsen the initial attitude error. Obviously, the presence of noises and measurement and integration errors worsen the estimate integration (20) in practice, since (20) do not contain terms forcing $\hat{q}$ to converge to q. The discussion of this case goes beyond the aim of this paper. Nevertheless, this is a valid method in practical situations for solving an approximated attitude tracking problem, at least for limited time intervals.

5. Simulation results

The spacecraft implemented on a digital computer is characterized by the inertia matrix

$$J_{mb} = \begin{pmatrix} 400 & 3 & 10 \\ 3 & 300 & 12 \\ 10 & 12 & 200 \end{pmatrix}\ \text{kg m}^2.$$

The flexible appendage considered in these simulations has a length of 20 m, is constituted by aluminum, with a density of 2.76×10^3 kg/m^3, a Young modulus of 6.8×10^{10} N/m^2 and a shear modulus of 2.5×10^{10} N/m^2. The point of attachment of the appendage to the main body is given by the vector components $r_x = 1.5$, $r_y = 2.3$, $r_z = -0.8$ m. Three elastic modes result from the modal analysis of the structure, with natural frequencies $\Omega_{n1} = 19.38$, $\Omega_{n2} = 77.98$, $\Omega_{n3} = 157.22$ rad/s and dampings $\zeta_1 = 0.0001$, $\zeta_2 = 0.00005$, $\zeta_3 = 0.00001$. Only the first two modes have been considered in the controller design, so taking into account possible spillover effects. The coupling matrix δ is given by

$$\delta = \begin{pmatrix} 14.3961 & 8.37634 & -5.29354 \\ -20.4871 & 7.59188 & -6.08014 \\ 4.50401 & 11.5222 & -12.6033 \end{pmatrix}\ \text{kg}^{1/2}\ \text{m}.$$

Finally, a payload of 30 kg is present at the tip of the appendage.

The reference trajectory to be tracked is described as follows:

$$q_{r0} = \cos\frac{\phi_r}{2}, \quad q_r = \begin{pmatrix} \cos 0.5t \\ \sin 0.5t \\ 0 \end{pmatrix}\sin\frac{\phi_r}{2},$$

$$\phi_r = \sin\gamma t, \quad \gamma = 0.035\ \text{rad/s}.$$

These quaternions correspond to a spiral maneuver which, starting from the initial spacecraft attitude, diverges when ϕ_r increases and converges when ϕ_r decreases. Note that

$$\dot{q}_{r0}|_{t=0} = -\frac{\dot{\phi}_r}{2}\sin\frac{\phi_r}{2}\bigg|_{t=0} = 0,$$

$$\dot{q}_r|_{t=0} = \left[\dot{\varepsilon}_r \sin\frac{\phi_r}{2} + \varepsilon_r \frac{\dot{\phi}_r}{2}\cos\frac{\phi_r}{2}\right]|_{t=0} = \begin{pmatrix}\gamma/2\\0\\0\end{pmatrix}$$

and therefore, from (3), $\omega_r(0) = (\gamma\ \ 0\ \ 0)^T$ when $q_0(0) = q_{r0}(0)$, $q(0) = q_r(0)$ (see later). In the following simulations it has been supposed that the initial error angular velocity is $\omega_e(0) = -\omega_r(0)$, i.e. at the initial time the spacecraft is idle. Moreover, the initial modal variables $\eta(0)$, $\psi(0)$ values are supposed given by $\eta(0)=0$, $\psi(0)=\delta\omega_e(0)+\dot{\eta}(0)=-\delta\omega_r(0)$, i.e. the flexible appendage is undeformed.

As far as the initial error between actual attitude and reference attitude is concerned, for the first two cases, in which the spacecraft attitude is measured (controllers of Theorems 1 and 2), it has been supposed that $e(0) = 1$ and $e(0) = 0$, i.e. that the body fixed frame and the reference frame coincide at $t = 0$. In the third case, regarding Theorem 3 where the quaternions q_0, q are not measured, the initial values for the estimated quaternions $\hat{q}_0$, $\hat{q}$ (see dynamics (20)) have been set equal to $\hat{q}_0(0) = 0.866$, $\hat{q}(0) = (0.05\ \ -0.17\ \ -0.467)^T$. These values are equivalent to an error w.r.t. the actual attitude corresponding to a rotation of about 60° about the axis $(0.1\ \ -0.34\ \ -0.935)^T$.

For all the controllers we have considered $k_p = 10^5$, $k_d = 3\times 10^5$. For the dynamic controllers (Theorems 2 and 3) the gain matrix Γ has been set equal to the identity matrix, while the initial conditions for the estimated modal variables are $\hat{\eta}(0) = 0$, $\hat{\psi}(0) = 0$. As previously stressed, in the controller design only the first two modes were taken into account.

The simulations are rendered more realistic by respecting the fact that the gas jets work in a bang-bang manner, with saturation values at 60 Nm. This renders harder the control task. It is worth noting that k_p and k_d are large enough to ensure the trajectory tracking, at least at low velocity, but small enough to avoid the bad behavior determined by the gas jet saturation.

The first simulation regards controller (7) (Theorem 1, "controller a"). Figs. 1(a) and 3(a) show a good behavior in the quaternion tracking. Fig. 2(a) shows the flexible dynamics. Analogous comments hold true for controllers (15), (16) (Theorem 2, "controller b"), for which the behavior of the quaternions is practically the one seen for the controller (7) (Fig. 1(b); see also Fig. 3(b)). In this case, however, the modal variables are estimated (their behavior in Fig. 2(b) is almost the same of the variables to be estimated). Finally, the third simulation regards the controller of Theorem 3 ("controller c"). Obviously, in this case the performance cannot be compared with those previously obtained. The quaternion tracking is clearly imprecise (Fig. 1(c)) and the control action is more active, so that the modal dynamics are always excited and their estimations are imprecise. However, it is interesting to note that the maneuver remains qualitatively similar to the previous ones (Fig. 3(c)), so that, even in presence of big errors in the attitude determination, the controller is capable to alleviate the error given by a possible attitude sensor failure.

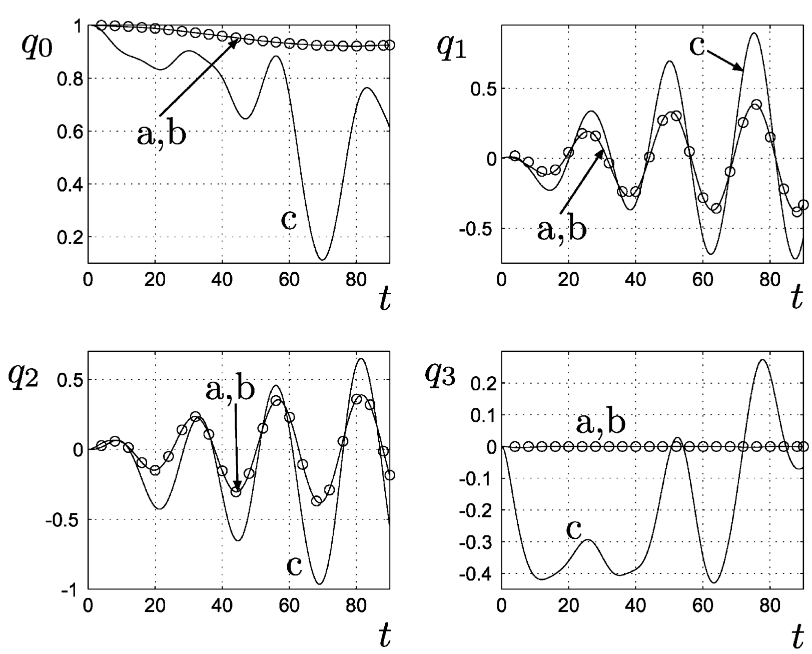

Fig. 1. Reference quaternions q_{r0}, q_r (circles) and actual quaternions q_0, q.

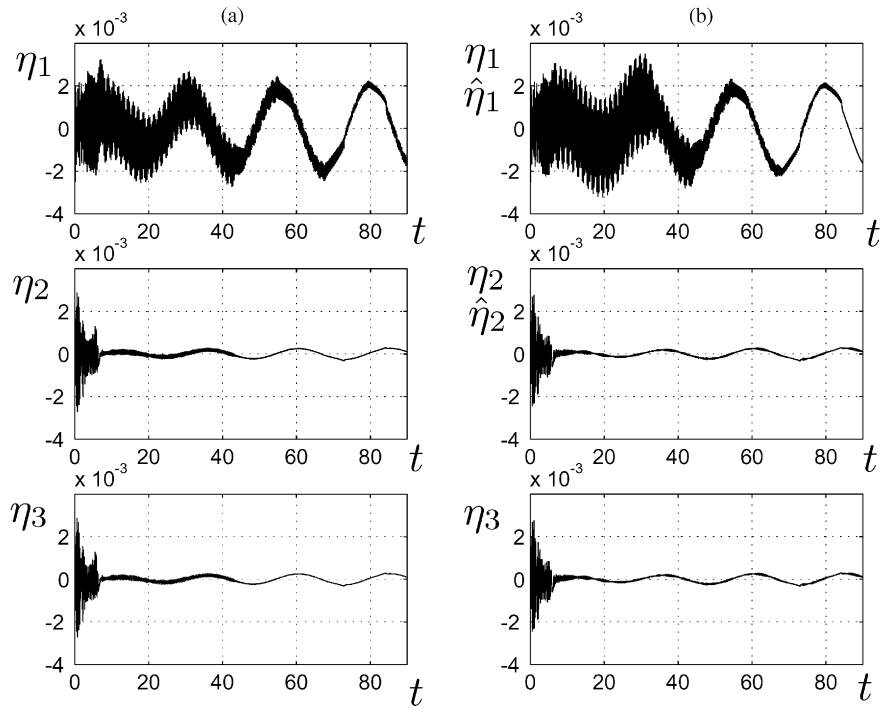

Fig. 2. Modal displacements η_1, η_2, η_3 and estimates $\hat{\eta}_1, \hat{\eta}_2$.

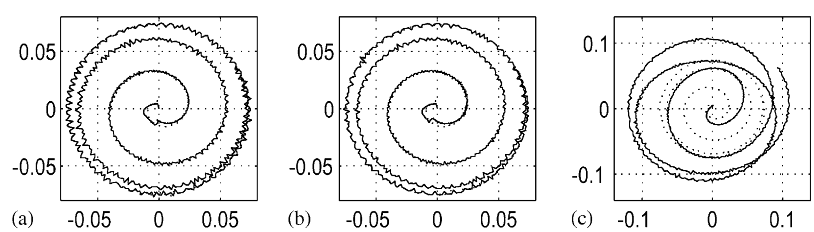

Fig. 3. Projections on a plane of the reference (dotted lines) and actual (solid lines) trajectories.

6. Conclusions

In this paper we have proposed a class of controllers which solve the tracking problem for a flexible spacecraft. Some of these controllers do not need the measure of the modal and the attitude variables, and this represents a clear advantage for practical implementations. On the other hand, they relay on the perfect knowledge of the system parameters, in particular those describing the elastic motion (natural frequencies and damping ratios). This is an obvious limitation, since they are not usually known accurately. Future work will regard the design of structurally stable controllers which avoid this drawback.

Acknowledgements

The author thanks the anonymous referees for the helpful suggestions which greatly improved the presentation of the results, in particular, the one who outlined the right presentation of the proofs. The author is also indebted with Salvatore Monaco for his precious ideas and technical suggestions.

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