

## Discussion on ‘Stabilization for Continuum Models of Large Space Structures in Large Attitude Maneuvers’ by S. Di Gennaro and A. De Santis

### 1. Discussion by S. Battilotti<sup>1</sup>

The paper consider the problem of stabilizing an infinite dimensional model of a large space structure with flexible elements. The model describes a rigid spacecraft with a flexible element carrying a payload at the extremity. First, the case of full state information is considered. The state feedback law proposed consists of two parts: one cancels part of the nonlinearities of the model and the other is a linear term proportional to the boundary velocity deformations and angular velocities of the rigid body. The drawback of this feedback law is that it needs the knowledge of the deformations (and their velocities) of the structure, i.e. a distributed control law. The authors consider then the case of output feedback and assume that the state stays in some bounded and closed set of the state space. This can be guaranteed by a suitable choice of the Lyapunov function and a corresponding choice of the control law, which consists again of two parts: one cancels part of the nonlinearities of the model and the other is a linear term proportional to the boundary velocity deformations and angular velocities of the rigid body. The main advantage of using a bounded and closed set of the state space is that the part of the control which cancels some of the nonlinearities is no more distributed. The following comments can be done:

- The model is worked out without approximations in the frame of the Bernoulli’s theory;
- Previous works were limited to consider slow manoeuvres until the desired set point in such a way

to neglect the nonlinear effect of the elasticity during the manoeuvre;

- The approach followed by the authors is based on the state space representation and semigroup theory; with this setup so called weak solutions are obtained. The price to pay is non-exponential stability. This approach is opposed to partial differential equation theory with boundary control, which guarantee so called strong solutions and exponential stability;
- The main drawback of using a bounded and closed set of the state space is that the gains of the controller depend on the width of the region of attraction and may be large even for small initial conditions;
- When there is no flexibility (finite dimensional model) the control law reduces to classical passivity-based control;
- The result presented in this paper gives a general setup, in which not only stabilization problems can be studied. It is natural to study as well H-infinity and LQG problems.

### 2. Final Comments by the Authors S. Di Gennaro<sup>2</sup> and A. De Santis<sup>3</sup>

Following the discussion contribution given by Prof. S. Battilotti, we would like to further remark some key points of the paper.

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The suppression of the vibrations in distributed mechanical structures has been a widely investigated problem. Nevertheless the motion dynamics in rest-to-rest maneuvers has been always somewhat approximated by taking care of the flexibility only at the final rest position, after the slewing action was accomplished; this provides the advantage of dealing with a linear problem. Indeed, during the slewing action, the coupling between the flexible modes and the rigid body motion gives rise to nonlinear effects which we modelled in detail through a classical virtual works method, in a typical maneuver where only the bending deformation was considered. Extensions to a general maneuver with a complete deformations set are in progress.

Then a second order partial differential equations model is obtained (a linear nominal plant plus nonlinear terms), with suitable boundary conditions at given points along the structures where rigid bodies and other flexible appendages might be linked. This model is usually recast as an abstract first order differential equation on Hilbert space, but with an unbounded input operator from the control space  $U$  to the state space  $H$ ; the interested reader may check the detailed account [6] on this subject appeared on this Journal. The solutions are usually sought for smooth initial conditions and continuously differentiable control functions (strong solutions), see also [4]; actually in [6] is considered also the case of general  $L_2$  control functions. Along the same lines, even though not related to mechanical structures models, in [7] standard linear  $H_\infty$  problems are studied in presence of unbounded input and output operators: these are coped by mean of the usual embedding argument and, under suitable hypotheses, closed-loop exponential stability is obtained.

We used the approach first proposed in [1], the highlights being the inclusion of the boundary conditions in the state space definition and bounded input and output operators, obtaining a state space formulation more familiar to engineers, where non-smooth initial conditions and  $L_2$  control are considered, and weak solutions are dealt with. On the other hand with bounded input operators only strong stability can be achieved.

Besides the simple control problems solved in the paper, the state space formulation adopted allows for

more sophisticated control strategies. In the linear case, the LQR and LQG regulator problems were solved in [1,2] and an  $H_\infty$  disturbance attenuation problem was studied in [5]. By calling upon the semiglobal stabilization concept, we might think to consider feasible (i.e., boundary) controls for robust stabilization of the following class of continuum models of mechanical structures

$$dx(t) = (Ax(t) + B_1u(t) + B_2\phi(x, u)) dt + H(x, u) dw(t),$$

$$dy(t) = (C_1x(t) + C_2\psi(x, u)) dt + K(x, u)dw(t),$$

where the terms  $\phi(x, u)$  and  $\psi(x, u)$  collects the nonlinear dynamics terms as well as unmodelled dynamics terms, and both the state and output equations include stochastic disturbances properly described as Wiener processes  $w(t)$ .

An optimal control problem for systems belonging to this class, but in the finite dimensional setting, was solved in [3].

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