JOURNAL OF OPTIMIZATION THEORY AND APPLICATIONS: Vol. 116, No. 1, pp. 41–60, January 2003 (© 2003)

Passive Attitude Control of Flexible Spacecraft from Quaternion Measurements[1,2]

S. Di Gennaro[3]

Communicated by M. A. Simaan

Abstract. In this work, we propose a dynamic controller for a spacecraft with flexible appendages and based on attitude measurements. This control ensures the asymptotic fulfillment of the objectives in the case of rest-to-rest maneuvers when a failure occurs on the accelerometer sensors, so that the angular velocity is not available for feedback. Also, it is assumed that the modal variables describing the flexible elements are not measured. This is a lower level controller and is to be selected at the higher level by a supervisor when an emergency situation is detected.

Key Words. Output control, passive control, flexible spacecraft.

1. Introduction

An important aspect in a spacecraft control scheme is its robustness versus sensor failures. In these cases, the controller should be capable of ensuring the continuation of the mission. From this point of view, the design of a controller based on only the available measurements is of interest. The total control system is then realized in a hierarchical manner: at the higher level, there is a supervisor deciding the type of control action to be applied, such as in the normal operation mode, in case of sensor failure, etc.; at the lower level, there are different controllers, one for each control action decided by the supervisor. The controller presented in this work is of the lower level type, to be used in the case of sensor failure. More precisely, in

[1]This research was partially supported by the Agenzia Spaziale Italiana and by the University of L'Aquila, Progetto di Ateneo, Controllo Attivo nella Meccanica delle Strutture.
[2]The author thanks Professor Salvatore Monaco for suggesting the research topic and for valuable advice concerning the proof of the main result.
[3]Assistant Professor, Department of Electrical Engineering, University of L'Aquila, Poggio di Roio, L'Aquila, Italy.

this work we design a controller based on only the attitude measurements. For instance, this is the case of a flexible spacecraft when there is a failure in the accelerometer sensors and when the variables describing the flexible elements of the spacecraft are not measured. In this situation, no information on the angular velocity and the modal variables is available.

In general, the spacecraft attitude can be determined either by attitude measurements with respect to some external reference (vector measurements), such as Sun, Earth, or other central body, magnetic field, stars, or by centrifugal acceleration measurements (Ref. 1). In the first case, the vector measures can be differentiated in time in order to obtain estimations of the spacecraft attitude and angular rate (Refs. 1–3). In the second case, one determines the change in orientation; this method goes under the name of inertial guidance, and entails the use of gyroscopes and accelerometers. Gyroscopes have high accuracy for limited time intervals. The main problem with inertial guidance is that it is necessary to integrate the attitude changes starting from an initial attitude; star sensors, due to their high accuracy, can provide a periodic update of the attitude, necessary to eliminate the accumulated errors in the integration and to update the gyroscopes. The scenario that we consider in this paper is that in which the information coming from the accelerometers is not available; hence, the proposed controller is not the one normally used during maneuvers, but rather the one applied in the case of emergency, according to the hierarchical control scheme previously outlined.

Previous works on the nonlinear control of spacecraft are based mostly on the knowledge of the entire state, and interesting examples are given in Refs. 4–10. More recently, some works on the design of controllers based on measured variables have been presented. In Ref. 11, a stabilization problem for flexible spacecraft is solved making use of a dynamic controller based on measurements of the spacecraft attitude; a limitation of this controller is that the total angular momentum is supposed constant, since the external torques are considered absent. In Ref. 12, the more general problem of tracking a desired attitude is solved when the attitude and angular rate are available for feedback, but not the modal variables. Inspired by the results given in Refs. 13–14 for rigid spacecraft and based on the passivity concept, a first example of dynamic controller, based on attitude measurements and for flexible spacecraft, is presented in Ref. 15. Continuing in this direction, a more sophisticated controller for flexible spacecraft and for large rest-to-rest maneuvers with a fixed axis is proposed in this paper. A comparison with the results in Ref. 11 shows that the resulting controller is simpler.

The paper is organized as follows. In Section 2, the mathematical model of a flexible spacecraft is presented; in Section 3, the control problem is stated and some basic results regarding dynamic controllers are reviewed. The main result is presented in Section 4, consisting of a dynamic controller based on only the attitude parameter measurements. Simulations results are presented in Section 5; in Section 6, final comments conclude the paper.

2. Equations of Motion of a Flexible Spacecraft

In this section, we recall briefly the mathematical model of a flexible spacecraft. The reader can find in Refs. 6–10 the details of the derivation (see also Refs. 9, 11, and references therein). It is well-known that the kinematics of a rigid body, representing the spacecraft main body, can be described efficiently by a nonminimal set of parameters, called unit quaternions or Euler parameters (Refs. 16, 17), given by

$$q_0 = \cos(\Phi/2), \qquad q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \epsilon \sin(\Phi/2), \tag{1}$$

and subject to the constraint

$$q_0^2 + q^T q = 1. \tag{2}$$

The rotation Φ is about the Euler axis, which is determined by the unit vector ϵ. The kinematic equations are therefore

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q} \end{bmatrix} = (1/2)\mathcal{Q}^T(q_0, q)\omega, \tag{3}$$

where

$$\mathcal{Q}(q_0, q) = [-q, q_0 I - \tilde{q}], \tag{4}$$

ω is the spacecraft angular velocity and

$$\tilde{q} = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix}$$

is a skew-symmetric matrix furnishing the diadic representation of the vector q. The unit quaternions are a nice nonsingular set of parameters with desirable computational properties (see Refs. 16, 18).

The dynamic equations can be written making use of the Euler theorem and under the hypothesis of small elastic deformations (Refs. 7–8),

$$J\dot{\omega} + \delta^T\ddot{\eta} = -\omega \times (J\omega + \delta^T\dot{\eta}) + u, \tag{5a}$$

$$\ddot{\eta} + C\dot{\eta} + K\eta = -\delta\dot{\omega}. \tag{5b}$$

Here, J is the total inertia (symmetric) matrix, u is the external torque acting on the main body of the structure, η is the modal coordinate vector, and

$$C = \mathrm{diag}\{2\zeta_i\omega_{ni}, i = 1, \ldots, N\}, \qquad K = \mathrm{diag}\{\omega_{ni}^2, i = 1, \ldots, N\}$$

are the damping and stiffness matrices. Finally, δ is the coupling matrix between flexible and rigid dynamics, namely the matrix which describes how the flexible dynamics influences the rigid dynamics, and vice versa. In the present model, N elastic modes are considered, with ω_{ni} the natural frequencies and ζ_i the associated dampings. From (5), it is possible to obtain the dynamics of the flexible spacecraft (Refs. 7, 8, 11),

$$\dot{\omega} = J_{mb}^{-1}[-\omega \times (J_{mb}\omega + \delta^T\psi) + \delta^T(C\psi + K\eta - C\delta\omega) + u], \tag{6a}$$

$$\dot{\eta} = \psi - \delta\omega, \tag{6b}$$

$$\dot{\psi} = -(C\psi + K\eta - C\delta\omega), \tag{6c}$$

with

$$J_{mb} = J - \delta^T\delta$$

the main body inertia matrix and

$$\psi = \dot{\eta} + \delta\omega$$

the total velocity of the flexible appendages. To sum up, the mathematical model of a spacecraft with flexible appendages is given by Eqs. (3) and (6).

3. Preliminaries and Some Basic Results

In this section, we state the control problem under study and we present some basic results about the design of a controller based on either the knowledge of the entire state (Refs. 4–10, 19) or the knowledge of the quaternions and the angular velocity (Refs. 11–15), which are instrumental for the presentation of the main results in Section 4.

3.1. Problem Statement. The control problem is to obtain a rest-to-rest maneuver about a fixed axis with elimination of the oscillations due to the flexible dynamics. In other words, we consider the problem of driving the body-fixed reference to a target reference, given by some desired value of the quaternions. Hence, these parameters express the spacecraft attitude error. At the same time, we want to damp out the induced flexible oscillations.

If $[q_{0r}, q_r] = [1, 0]$ is the desired attitude, the control problem is to find a control u, depending on the output measurements, such that

$$\lim_{t\to\infty} q = 0, \qquad \lim_{t\to\infty} \eta = 0, \qquad \lim_{t\to\infty} \psi = 0,$$

for any initial condition. Note that, if $q \to 0$, then $q_0 \to 1$ because of the constraint relation among the unit quaternions.

3.2. State-Feedback Controllers. In this section, we review briefly some results about state-feedback stabilization, for either rigid or flexible spacecraft. It is worth noting that, for the latter, the controller will need also the measurements of the modal variables. Clearly, this rarely occurs in practice; however, these results will be useful in view of the future developments and will render them clearer.

In the case of a rigid spacecraft, it can be shown that a simple proportional and derivative control suffices to globally and asymptotically stabilize the system (Ref. 19). To show this, first note that the rigid body motion is described by (3) and

$$\dot{\omega} = J_{mb}^{-1}(-\omega \times J_{mb}\,\omega + u). \tag{7}$$

Let us now consider the control law

$$u = -k_p q - k_d \omega, \tag{8}$$

with k_p, k_d positive scalars. Deriving along the system trajectories the following Lyapunov function candidate:

$$V = k_p[(q_0 - 1)^2 + q^T q] + (1/2)\omega^T J_{mb}\,\omega,$$

with $k_p > 0$, one obtains

$$\dot{V} = k_p q^T \omega + \omega^T(-\omega \times J_{mb}\,\omega + u) = -k_d \omega^T \omega \leq 0.$$

Since V is a continuously differentiable, radially unbounded and positive-definite function with negative semidefinite time derivative over the entire state, the global asymptotic stability can be stated by using the LaSalle theorem (Ref.20). In fact, the system trajectories converge to the largest invariant set $\mathscr{E}$ contained in

$$\begin{aligned} E &= \{x \in \mathbb{R}^n \,|\, \dot{V} = 0\} \\ &= \{x \in \mathbb{R}^n \,|\, \omega = 0\}. \end{aligned}$$

Since

$$J_{mb}\dot{\omega} = 0 = -k_p q \Rightarrow q = 0,$$

it is easy to see that

$$\mathscr{E} = \{x \in \mathbb{R}^n \,|\, q = 0,\ \omega = 0\}.$$

When dealing with a spacecraft with flexible appendages, a similar control can be designed. More precisely, a flexible spacecraft can be stabilized by a PD control plus a term which takes into account the flexible dynamics. If the modal variables are supposed measurable, the following static control is sufficient to ensure the global asymptotic stability:

$$u = -F\begin{bmatrix} q \\ \eta \\ \psi \end{bmatrix} - k_d\omega, \qquad F = \left[k_p I,\ \delta^T\left\{\begin{bmatrix} K \\ C \end{bmatrix} - P_1\begin{bmatrix} I \\ -C \end{bmatrix}\right\}^T\right], \tag{9}$$

where $P_1 = P_1^T > 0$. This can be shown easily by taking the Lyapunov function candidate

$$V = k_p[(q_0 - 1)^2 + q^T q] + (1/2)\omega^T J_{mb}\omega + (1/2)\,[\eta^T, \psi^T]\, P_1\begin{bmatrix} \eta \\ \psi \end{bmatrix}. \tag{10}$$

Its time derivative along the system trajectories (3), (6) is

$$\begin{aligned} \dot{V} &= \omega^T[k_p q - \omega \times (J_{mb}\omega + \delta^T\psi) + \delta^T(C\psi + K\eta - C\delta\omega) + u] \\ &\quad + [\eta^T, \psi^T]\, P_1\left[\begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix}\begin{bmatrix} \eta \\ \psi \end{bmatrix} - \begin{bmatrix} I \\ -C \end{bmatrix}\delta\omega\right] \\ &= -\omega^T(k_d I + \delta^T C\delta)\omega - [\eta^T, \psi^T]\, Q_1\begin{bmatrix} \eta \\ \psi \end{bmatrix} \leq 0, \end{aligned}$$

where the control (9) has been used and the matrix P_1 can be computed as solution of

$$P_1\begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix} + \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix}^T P_1 = -2Q_1, \tag{11}$$

for any fixed $Q_1 = Q_1^T > 0$. By the LaSalle theorem, we find that the largest invariant set $\mathscr{E}$ contained in

$$\begin{aligned} E &= \{x \in \mathbb{R}^n \,|\, \dot{V} = 0\} \\ &= \{x \in \mathbb{R}^n \,|\, \omega = 0,\ \eta = 0,\ \psi = 0\} \end{aligned}$$

is given by

$$\mathscr{E} = \{x \in \mathbb{R}^n | q = 0, \omega = 0, \eta = 0, \psi = 0\} \tag{12}$$

and the global asymptotic stability is proved. As previously noted, the need for modal measurements limits the use of this type of controller. This problem is solved in the following section.

3.3. Output-Feedback Controllers. It is quite easy to extend the controller (9) to the case of output control, i.e., when one measures only the attitude and the angular velocity. It is sufficient to define the estimates $\hat{\eta}$, $\hat{\psi}$ of the modal variables and to introduce the errors (Ref. 12)

$$e_\eta = \eta - \hat{\eta}, \quad e_\psi = \psi - \hat{\psi}.$$

Hence, the Lyapunov function candidate V is given by (10) plus the term

$$(1/2)\,[e_\eta^T, e_\psi^T]\, P_2 \begin{bmatrix} e_\eta \\ e_\psi \end{bmatrix}, \tag{13}$$

with $P_2 = P_2^T$ a positive-definite matrix. By deriving this term taking into account Eqs. (6), one obtains in $\dot{V}$ the additional term

$$[e_\eta^T, e_\psi^T]\, P_2 \left[\begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix} \begin{bmatrix} \eta \\ \psi \end{bmatrix} - \begin{bmatrix} I \\ -C \end{bmatrix} \delta\omega - \begin{bmatrix} \dot{\hat{\eta}} \\ \dot{\hat{\psi}} \end{bmatrix} \right].$$

Fixing

$$Q_i = Q_i^T > 0, \qquad i = 1, 2,$$

computing P_1 as in (11), P_2 as in (14) below,

$$P_2 \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix} + \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix}^T P_2 = -2Q_2, \tag{14}$$

and using the dynamic controller

$$\begin{bmatrix} \dot{\hat{\eta}} \\ \dot{\hat{\psi}} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix} \begin{bmatrix} \hat{\eta} \\ \hat{\psi} \end{bmatrix} - \begin{bmatrix} I \\ -C \end{bmatrix} \delta\omega + P_2^{-1} \left[\begin{bmatrix} K \\ C \end{bmatrix} - P_1 \begin{bmatrix} I \\ -C \end{bmatrix} \right] \delta\omega, \tag{15}$$

$$u = -F \begin{bmatrix} q \\ \hat{\eta} \\ \hat{\psi} \end{bmatrix} - k_d \omega, \tag{16}$$

with F as in (9), finally one obtains

$$\dot{V} = -\omega^T (k_d I + \delta^T C \delta)\omega - [\eta^T, \psi^T]\, Q_1 \begin{bmatrix} \eta \\ \psi \end{bmatrix} - [e_\eta^T, e_\psi^T]\, Q_2 \begin{bmatrix} e_\eta \\ e_\psi \end{bmatrix} \leq 0.$$

The application of the LaSalle theorem shows that the control problem is globally solved.

4. Passive Controller for Flexible Spacecraft

In this section, we present a dynamic controller for a flexible spacecraft, based on passivity concepts (Ref.21), which will need only attitude measurements. In the derivation of this controller, we follow the works of Refs.13-14 for the stabilization of a rigid spacecraft. Using the notation of Section 3.3, we consider in place of (16) the control

$$u = -F \begin{bmatrix} q \\ \hat{\eta} \\ \hat{\psi} \end{bmatrix} + v,$$

where F is given in (9). We note that the map from the new input v to ω is passive; i.e., there exists some constant $\gamma_0 = \gamma_0(x_0)$ such that

$$\int_0^T \omega^T v \, dt \geq -\gamma_0^2, \qquad \forall T \geq 0. \tag{17}$$

In fact, considering the function V as in (10), (13) along the trajectories of the system (3), (6), (15), one gets

$$\dot{V} = \omega^T v - \omega^T \delta^T C \delta \omega - [\eta^T, \psi^T] \, Q_1 \begin{bmatrix} \eta \\ \psi \end{bmatrix} - [e_\eta^T, e_\psi^T] \, Q_2 \begin{bmatrix} e_\eta \\ e_\psi \end{bmatrix} \leq \omega^T v.$$

Therefore, it follows that (17) holds true, because

$$\int_0^T \omega^T v \, dt \geq V(x(T)) - V(x_0) \geq -V(x_0), \qquad \forall T \geq 0;$$

namely, the system is passive.

Therefore, analogously to what was done in Refs. 13–14, we consider first a velocity feedback, namely the controller (15)–(16) presented in Section 3.3, which we have already seen to solve the control problem. The first step is to rewrite this controller as follows:

$$\begin{bmatrix} \dot{\hat{\eta}} \\ \dot{\hat{\psi}} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix} \begin{bmatrix} \hat{\eta} \\ \hat{\psi} \end{bmatrix} + 2 P_2^{-1} M \delta \mathcal{Q}(q_0, q) \begin{bmatrix} \dot{q}_0 \\ \dot{q} \end{bmatrix}, \tag{18}$$

$$u = -F \begin{bmatrix} q \\ \hat{\eta} \\ \hat{\psi} \end{bmatrix} - 2 k_d \mathcal{Q}(q_0, q) \begin{bmatrix} \dot{q}_0 \\ \dot{q} \end{bmatrix}, \tag{19}$$

where F is given in (9), $\mathcal{Q}(q_0, q)$ is as in (4),

$$M = \begin{bmatrix} K \\ C \end{bmatrix} - (P_1 + P_2)\begin{bmatrix} I \\ -C \end{bmatrix}, \tag{20}$$

and P_1, P_2 are positive-definite symmetric matrices, which are solutions of (11), (14), respectively, for fixed symmetric matrices $Q_1, Q_2 > 0$. Here, we have used the fact that, from the kinematic equations (3) and from (4), one gets

$$\omega = 2\mathcal{Q}(q_0, q)\begin{bmatrix} \dot{q}_0 \\ \dot{q} \end{bmatrix},$$

since the matrix $\mathcal{Q}^T(q_0, q)$ is left invertible.

Let us now eliminate the use of $\dot{q}_0, \dot{q}$. Setting

$$x = \begin{bmatrix} q_0 \\ q \\ \omega \\ \eta \\ \psi \\ e_\eta \\ e_\psi \end{bmatrix},$$

$$f(x) = \begin{bmatrix} (1/2)\mathcal{Q}^T(q_0, q)\omega \\ J_{mb}^{-1}\left[-\omega \times (J_{mb}\omega + \delta^T\psi) + \delta^T\begin{bmatrix} K \\ C \end{bmatrix}^T\begin{bmatrix} \eta \\ \psi \end{bmatrix} - \delta^T C\delta\omega - F\begin{bmatrix} q \\ \hat{\eta} \\ \hat{\psi} \end{bmatrix}\right] \\ \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix}\begin{bmatrix} \eta \\ \psi \end{bmatrix} - \begin{bmatrix} I \\ -C \end{bmatrix}\delta\omega \\ \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix}\begin{bmatrix} e_\eta \\ e_\psi \end{bmatrix} - \begin{bmatrix} I \\ -C \end{bmatrix}\delta\omega \end{bmatrix},$$

$$g(x) = \begin{bmatrix} 0 \\ -2k_d J_{mb}^{-1}\mathcal{Q}(q_0, q) \\ 0 \\ -2P_2^{-1} M\delta\mathcal{Q}(q_0, q) \end{bmatrix},$$

$$v = k(x) = \begin{bmatrix} \dot{q}_0 \\ \dot{q} \end{bmatrix} = (1/2)\mathcal{Q}^T(q_0, q)\omega,$$

the feedback system (3), (6), (18), (19) is in the form

$$\dot{x} = f(x) + g(x)k(x). \tag{21}$$

It is easy to show (see for instance Ref. 22, page 193) that, if the system (21) is asymptotically stable in the first approximation, then also the system

$$\dot{x} = f(x) + g(x)\begin{bmatrix}\xi_0 \\ \xi\end{bmatrix},$$

$$\begin{bmatrix}\dot{\xi}_0 \\ \dot{\xi}\end{bmatrix} = (1/\epsilon)\left[-\begin{bmatrix}\xi_0 \\ \xi\end{bmatrix} + k(x)\right]$$

is asymptotically stable in the first approximation for $\epsilon > 0$ sufficiently small. The only thing to check is that the system (21) is asymptotically stable in the first approximation. To this aim, let us consider first the state variable

$$\rho_0 = 1 - q_0$$

in the place of q_0, so that the origin is the equilibrium point. Clearly, the kinematics equations (3) are rewritten as

$$\begin{bmatrix}\dot{\rho}_0 \\ \dot{q}\end{bmatrix} = (1/2)\begin{bmatrix}q^T \\ I + (-\rho_0 I + \tilde{q})\end{bmatrix}\omega.$$

Second, we consider the following system, which is the linearization of the system (21) at the equilibrium:

$$\begin{bmatrix}\dot{\rho}_0 \\ \dot{q} \\ \dot{\omega} \\ \dot{\eta} \\ \dot{\psi} \\ \dot{e}_\eta \\ \dot{e}_\psi\end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I/2 & 0 & 0 \\ 0 & -k_p J_{mb}^{-1} & -J_{mb}^{-1}(k_d I + \delta^T C \delta) & J_{mb}^{-1}\delta^T \begin{bmatrix} I \\ -C \end{bmatrix}^T P_1 & J_{mb}^{-1}\delta^T M_1^T \\ 0 & 0 & -\begin{bmatrix} I \\ -C \end{bmatrix}\delta & \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix} & 0 \\ 0 & 0 & -P_2^{-1} M_1 \delta & 0 & \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix} \end{bmatrix} \begin{bmatrix}\rho_0 \\ q \\ \omega \\ \eta \\ \psi \\ e_\eta \\ e_\psi\end{bmatrix},$$

with

$$M_1 = \begin{bmatrix}K \\ C\end{bmatrix} - P_1\begin{bmatrix}I \\ -C\end{bmatrix},$$

and where it has been considered that

$$\delta^T \begin{bmatrix} K \\ C \end{bmatrix}^T \begin{bmatrix} \eta \\ \psi \end{bmatrix} - F \begin{bmatrix} q \\ \hat{\eta} \\ \hat{\psi} \end{bmatrix}$$

$$= -k_p q + \delta^T \begin{bmatrix} I \\ -C \end{bmatrix}^T P_1 \begin{bmatrix} \eta \\ \psi \end{bmatrix} + \delta^T M_1^T \begin{bmatrix} e_\eta \\ e_\psi \end{bmatrix},$$

$$\begin{bmatrix} \hat{\eta} \\ \hat{\psi} \end{bmatrix} = \begin{bmatrix} \eta \\ \psi \end{bmatrix} - \begin{bmatrix} e_\eta \\ e_\psi \end{bmatrix}.$$

Considering the Lyapunov function candidate

$$\mathscr{V} = k_p(\rho_0^2 + q^T q) + (1/2)\omega^T J_{mb}\,\omega$$
$$+ (1/2)[\eta^T, \psi^T]\, P_1 \begin{bmatrix} \eta \\ \psi \end{bmatrix} + (1/2)[e_\eta^T, e_\psi^T]\, P_2 \begin{bmatrix} e_\eta \\ e_\psi \end{bmatrix},$$

one obtains easily

$$\dot{\mathscr{V}} = -\omega^T(k_d I + \delta^T C \delta)\omega - [\eta^T \;\; \psi^T] Q_1 \begin{bmatrix} \eta \\ \psi \end{bmatrix} - [e_\eta^T \;\; e_\psi^T] Q_2 \begin{bmatrix} e_\eta \\ e_\psi \end{bmatrix}.$$

The largest invariant set $\mathscr{E}$ contained in

$$E = \{x \in \mathbb{R}^n \mid \dot{\mathscr{V}} = 0\}$$
$$= \{x \in \mathbb{R}^n \mid \omega = 0, \eta = 0, \psi = 0, e_\eta = 0, e_\psi = 0\}$$

reduces to the origin, which hence is asymptotically stable. This is equivalent to the fact that the dynamic matrix of the linear system is Hurwitz, and this guarantees the asymptotic stability in the first approximation of the feedback system (21).

We conclude by noting that it is not necessary to have $\dot{q}_0, \dot{q}$ available to realize the feedback

$$\begin{bmatrix} \dot{\xi}_0 \\ \dot{\xi} \end{bmatrix} = (1/\epsilon)\left\{ -\begin{bmatrix} \xi_0 \\ \xi \end{bmatrix} + \begin{bmatrix} \dot{q}_0 \\ \dot{q} \end{bmatrix} \right\},$$

$$\bar{u} = \begin{bmatrix} \xi_0 \\ \xi \end{bmatrix}.$$

In fact, setting

$$\begin{bmatrix} \xi_0 \\ \xi \end{bmatrix} = \begin{bmatrix} \dot{\chi}_0 \\ \dot{\chi} \end{bmatrix},$$

one can substitute the dynamics of ξ_0, ξ with

$$\begin{bmatrix} \dot{\chi}_0 \\ \dot{\chi} \end{bmatrix} = (1/\epsilon)\left\{ -\begin{bmatrix} \chi_0 \\ \chi \end{bmatrix} + \begin{bmatrix} q_0 \\ q \end{bmatrix} \right\},$$

$$\bar{u} = \begin{bmatrix} \dot{\chi}_0 \\ \dot{\chi} \end{bmatrix} = (1/\epsilon)\left\{ -\begin{bmatrix} \chi_0 \\ \chi \end{bmatrix} + \begin{bmatrix} q_0 \\ q \end{bmatrix} \right\}.$$

Therefore, the dynamic controller is finally

$$\begin{bmatrix} \dot{\eta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix} \begin{bmatrix} \hat{\eta} \\ \hat{\psi} \end{bmatrix} + (2/\epsilon)\, P_2^{-1} M \delta \mathcal{Q}(q_0, q) \left\{ -\begin{bmatrix} \chi_0 \\ \chi \end{bmatrix} + \begin{bmatrix} q_0 \\ q \end{bmatrix} \right\}, \tag{22a}$$

$$\begin{bmatrix} \dot{\chi}_0 \\ \dot{\chi} \end{bmatrix} = (1/\epsilon)\left\{ -\begin{bmatrix} \chi_0 \\ \chi \end{bmatrix} + \begin{bmatrix} q_0 \\ q \end{bmatrix} \right\}, \tag{22b}$$

$$u = -k_p q + (2/\epsilon)\, k_d \mathcal{Q}(q_0, q) \begin{bmatrix} \chi_0 \\ \chi \end{bmatrix} - \delta^T M_1^T \begin{bmatrix} \hat{\eta} \\ \hat{\psi} \end{bmatrix}, \tag{22c}$$

where it was noted that

$$\mathcal{Q}(q_0, q) \begin{bmatrix} q_0 \\ q \end{bmatrix} = 0.$$

Therefore, we arrive at the following theorem.

Theorem 4.1. The controller (22) solves the control problem with attitude measurements.

As a final comment, we compare the controller (22) with the one obtained in Refs. 13-14 for a rigid spacecraft,

$$\dot{\chi} = \mathcal{A}\chi + \mathcal{B}q, \tag{23a}$$

$$u = -k_p q - k_d (q_0 I - \tilde{q}) \mathcal{C}(\mathcal{A}\chi + \mathcal{B}q), \tag{23b}$$

where $\mathcal{A}, \mathcal{B}, \mathcal{C}$, with $(\mathcal{A}, \mathcal{B})$ controllable and $(\mathcal{A}, \mathcal{C})$ observable, constitute the realization of a strictly-positive real transfer function

$$C(s) = \mathcal{C}(sI - \mathcal{A})^{-1}\mathcal{B}$$

which satisfies the Kalman–Yakubovich–Popov lemma (Ref. 20); namely,

$$\Lambda\mathscr{A}+\mathscr{A}^T\Lambda=-Q_3, \qquad \Lambda\mathscr{B}=\mathscr{C}^T,$$

for appropriate matrices Λ, Q_3 positive-definite and symmetric. One notes that (22) has additional dynamics, similar to (15), in order to estimate the modal variables, and has an additional term in the control law. The fact that (22) contains also the dynamics of χ_0 is due only to the decision of using all the four quaternions (but a similar controller can be derived also when using only q).

A further argument of comparison is given by the following observation. We have proved that the estimation errors e_η, e_ψ go to zero asymptotically. But, at the end of the maneuver, q, ω also tend to zero; since the flexible dynamics is asymptotically stable this means that η, ψ go to zero. Therefore, the information $\hat{\eta}$, $\hat{\psi}$ about η, ψ is useful but not indispensable, in the sense that its use can improve the performance of the transient; in any case, the stability can be achieved even if $\hat{\eta}$, $\hat{\psi}$ are not computed. In fact, it is possible to stabilize the spacecraft without this information. This happens not only in the present case, but also when dealing with the state-feedback controller of Section 3.2. In fact, if we use the PD controller (8), the derivative of (10) becomes

$$\dot{V}=-[\omega^T,\eta^T,\psi^T]\begin{bmatrix}k_dI+\delta^TC\delta & \delta^TM_1^T/2\\ M_1\delta/2 & Q_1\end{bmatrix}\begin{bmatrix}\omega\\ \eta\\ \psi\end{bmatrix},$$

and it is clear that the condition of positive definitiveness,

$$k_dI+\delta^TC\delta-\delta^TM_1^TQ_1^{-1}M_1\delta/4>0,$$

is verified for k_d large enough. A similar result is valid for the controller (23). In this sense, the controller (23) reveals to be robust with respect to unmodeled dynamics, as stated by the following statement.

Proposition 4.1. The controller (23) is robust with respect to the unmodeled dynamics describing the flexibility.

Proof. For the proof of this statement, let us consider the Lyapunov function candidate

$$V=k_p[(q_0-1)^2+q^Tq]+(1/2)\omega^TJ_{mb}\omega$$
$$+(1/2)\,[\eta^T,\psi^T]\,P_1\begin{bmatrix}\eta\\ \psi\end{bmatrix}+k_d\xi^T\Lambda\xi,$$

with $P_1 = P_1^T > 0$ to be determined. Along the system dynamics, with u given by (see Refs. 13–14)

$$\dot{\xi} = \mathscr{A}\xi + \mathscr{B}\dot{q},$$

$$u = -k_p q - k_d(q_0 I - \tilde{q})\mathscr{C}\xi,$$

one has

$$\begin{aligned}\dot{V} &= \omega^T[k_p q + \delta^T(C\psi + K\eta - C\delta\omega) + u] \\ &\quad + [\eta^T, \psi^T]\, P_1\left\{\begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix}\begin{bmatrix} \eta \\ \psi \end{bmatrix} - \begin{bmatrix} I \\ -C \end{bmatrix}\delta\omega\right\} \\ &\quad + 2k_d\xi^T\Lambda[\mathscr{A}\xi + \mathscr{B}\dot{q}] \\ &= -\omega^T\delta^T C\delta\omega - [\eta^T, \psi^T]\, Q_1\begin{bmatrix} \eta \\ \psi \end{bmatrix} - k_d\xi^T Q_3\xi - [\eta^T, \psi^T]\, M_1\delta\omega \\ &= -(\omega^T\delta^T, \eta^T, \psi^T)\begin{bmatrix} C & M_1^T/2 \\ M_1/2 & Q_1 \end{bmatrix}\begin{bmatrix} \delta\omega \\ \eta \\ \psi \end{bmatrix} - k_d\xi^T Q_3\xi.\end{aligned}$$

The matrix P_1 can be determined such that

$$R = \begin{bmatrix} C & M_1^T/2 \\ M_1/2 & Q_1 \end{bmatrix}$$

is semipositive definite. In fact, let

$$P_1 = \begin{bmatrix} P_a & P_b \\ P_b^T & P_c \end{bmatrix}$$

be a block partition of P_1. Hence,

$$R = (1/2)\begin{bmatrix} 2C & (P_a - P_b C - K)^T & (P_b^T - P_c C - C)^T \\ P_a - P_b C - K & P_b K + K P_b^T & -P_a + P_b C + K P_c \\ P_b^T - P_c C - C & (-P_a + P_b C + K P_c)^T & P_c C + C P_c - P_b - P_b^T \end{bmatrix}.$$

It is clear that setting

$$P_a = K, \qquad P_b = 0, \qquad P_c = I,$$

one obtains

$$\dot{V} = -[\omega^T \delta^T, \eta^T, \psi^T] \begin{bmatrix} C & 0 & -C \\ 0 & 0 & 0 \\ -C & 0 & C \end{bmatrix} \begin{bmatrix} \delta\omega \\ \eta \\ \psi \end{bmatrix} - k_d \xi^T Q_3 \xi$$

$$= -\dot{\eta}^T C \dot{\eta} - k_d \xi^T Q_3 \xi,$$

where we recall that

$$\dot{\eta} = \psi - \delta\omega$$

is the modal velocity. Hence,

$$E = \{x \in \mathbb{R}^n \,|\, \dot{\eta} = \psi - \delta\omega = 0, \xi = 0\},$$

and from $\dot{\xi} = 0$, one finally obtains $\omega = 0$ and therefore $q = 0$; moreover, from $\omega = 0$, one deduces $\psi = 0$ and $\dot{\psi} = K\eta = 0$; i.e., $\mathscr{E}$ is given by (12). □

5. Simulation Results

As an example of a flexible spacecraft to which apply the designed controller let us consider the thermoelectric outer planet spacecraft (TOPS), which is provided with a great parabolic communication antenna (see Ref. 23). The main parameters characterizing TOPS are the following:

$$J_{mb} = \begin{bmatrix} 1543.9 & -2.3 & -2.8 \\ -2.3 & 471.6 & -35 \\ -2.8 & -35 & 1713.3 \end{bmatrix},$$

$$\delta = \begin{bmatrix} -9.4733 & -15.5877 & 0.0052 \\ -0.5331 & 0.4855 & 18.0140 \\ 0.5519 & 4.5503 & 16.9974 \\ -12.1530 & 11.7138 & -0.0002 \\ -0.0289 & 0.0199 & 6.2378 \\ 0.2268 & 0.8289 & -35.7298 \\ -0.8935 & 5.4516 & 1.5005 \\ 1.1628 & 2.6350 & -0.0989 \\ -0.1688 & 0.3131 & 3.6231 \\ -1.4910 & 2.0020 & -0.2893 \end{bmatrix},$$

with J_{mb} in Kg m^2 and δ in $Kg^{1/2}$ m, with the natural frequencies (in rad/s)

$$\omega_{n1} = 0.7400, \quad \omega_{n2} = 0.7500, \quad \omega_{n3} = 0.7600, \quad \omega_{n4} = 0.7600, \quad \omega_{n5} = 1.1600,$$

$$\omega_{n6} = 3.8500, \quad \omega_{n7} = 5.0200, \quad \omega_{n8} = 5.6600, \quad \omega_{n9} = 5.6600, \quad \omega_{n10} = 5.6900,$$

and the dampings

$$\zeta_1 = 0.004, \quad \zeta_2 = 0.005, \quad \zeta_3 = 0.0064, \quad \zeta_4 = 0.008, \quad \zeta_5 = 0.0085,$$

$$\zeta_6 = 0.0092, \quad \zeta_7 = 0.0105, \quad \zeta_8 = 0.012, \quad \zeta_9 = 0.015, \quad \zeta_{10} = 0.017,$$

associated to the first 10 natural modes.

The compensator (22) has been implemented by considering the gains

$$k_p = 300, \qquad k_d = 800,$$

$\epsilon = 0.1$, and choosing $Q_1 = I$, $Q_2 = 10\,I$ in (11), (14) for determining P_1, P_2.

The initial attitude is described by the quaternions

$$q_0(0) = 0.1736, \qquad q(0) = \begin{bmatrix} q_1(0) \\ q_2(0) \\ q_3(0) \end{bmatrix} = \begin{bmatrix} -0.5264 \\ -0.2632 \\ 0.7896 \end{bmatrix},$$

and the rest-to-rest maneuver brings the quaternions to $q_0 = 1, q = 0$. This maneuver corresponds to a rotation of 160° with an Euler axis which is

$$\epsilon = [-2/\sqrt{14}, 1/\sqrt{14}, 3/\sqrt{14}]^T$$

at the initial time. Moreover, initially the spacecraft is still, so that

$$\omega(0) = 0, \qquad \eta(0) = 0, \qquad \psi(0) = 0.$$

The comparison of the controller (22) with the controller (23) has been conducted by choosing

$$A = \mathcal{C} = I, \qquad \mathcal{B} = 2.5\,I,$$

where I is the 3×3 identity matrix and the gains

$$k_p = 150, \qquad k_d = 450.$$

These last values, lower than the ones used for the controller (22), are justified by the fact that the control input results to be of the same amplitude of the control determined by (22), at least in the initial transient, so establishing a fair comparison.

The simulation results are summarized in Figs. 1–4. The behavior of the quaternions q_0, q_1, q_2, q_3 is given in Fig. 1, while the control components u_1, u_2, u_3 are shown in Fig. 2. The modal displacements η_1, η_2 and their estimates $\hat{\eta}_1, \hat{\eta}_2$ are shown in Fig. 3. Finally, in Fig. 4, the quaternions

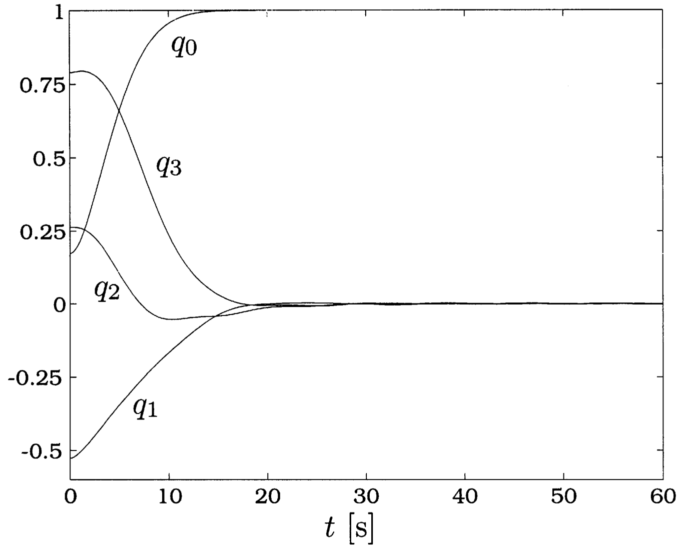

Fig. 1. Controller (22): Quaternion q_0, q_1, q_2, q_3 vs. time.

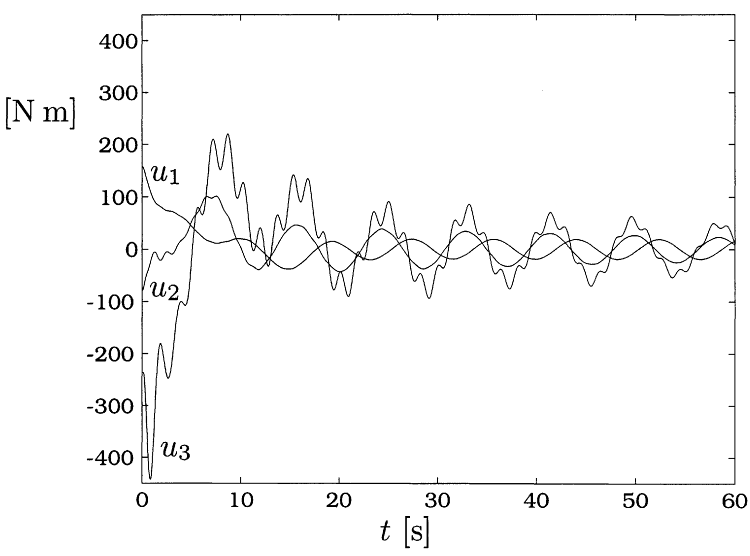

Fig. 2. Controller (22): Input components u_1, u_2, u_3 vs. time.

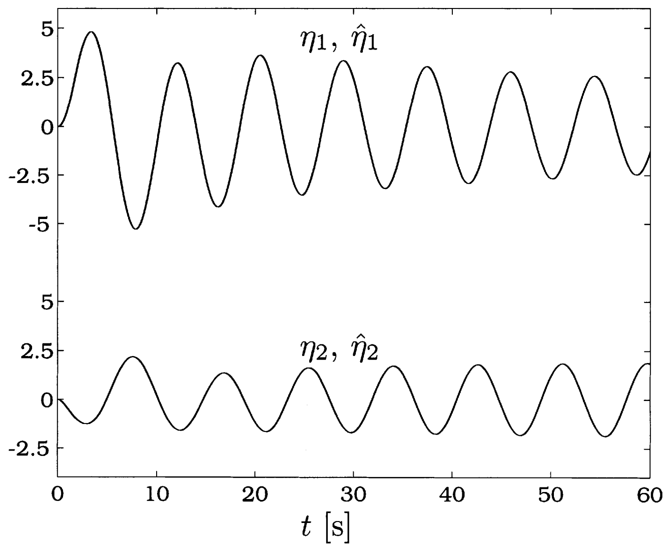

Fig. 3. Controller (22): Modal displacements and estimates η_i, $\hat{\eta}_i$, $i = 1, 2$, vs. time.

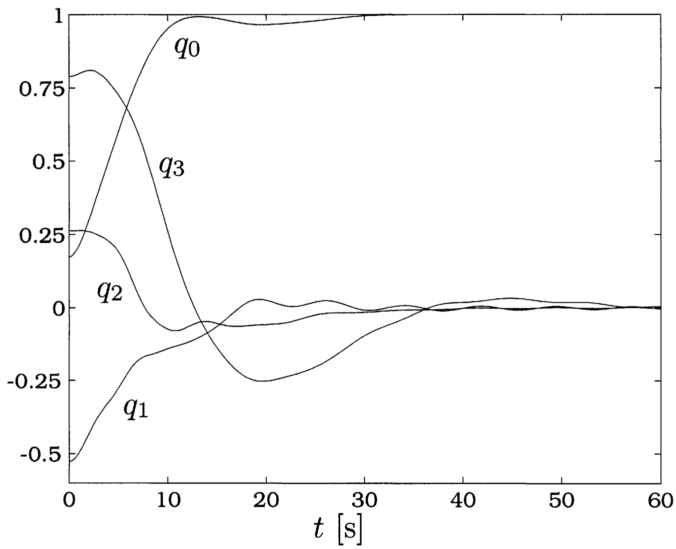

Fig. 4. Controller (23): Quaternion q_0, q_1, q_2, q_3 vs. time.

for the controller (23) are reported. Comparing Figs. 1 and 4, the improvements obtained via the controller (22) are clear; it reaches the desired attitude in about 15 s, while the controller (23) needs more than 30 s.

6. Conclusions

The application of the proposed dynamic control, based on quaternion measurements only, ensures the asymptotic convergence to zero of the attitude errors and damps out the flexible motion induced in the maneuver, with an enhancement of the transient performance with respect to that of the controllers previously presented. This controller constitutes a first step toward the design of a multilevel control scheme, composed of a high-level supervisor plus various lower-level controllers performing different tasks, among which those regarding the continuation of a mission in the case of sensor failures.

References

1. Azor, R., Bar-Itzhack, I. Y., and Harman, R. R., *Satellite Angular Rate Estimation from Vector Measurements*, Journal of Guidance, Control, and Dynamics, Vol. 21, No. 3, pp. 450–457, 1998.
2. Natanson, G., *A Deterministic Method for Estimating Attitude from Magnetometer Data Only*, Paper IAF-92-0036, 43rd Congress of the International Astronautical Federation, Washington, DC, 1992.
3. Challa, M., Natanson, G., Deutschmann, J., and Galal, K., *A PC-Based Magnetometer-Only Attitude and Rate Determination System for Gyroless Spacecraft*, Flight Mechanics/Estimation Theory Symposium, NASA Goddard Space Flight Center, pp. 83–96, 1995.
4. Dwyer, T. A. W., *Exact Nonlinear Control of Large-Angle Rotational Maneuvers*, IEEE Transactions on Automatic Control, Vol. 29, No. 9, pp. 769–774, 1984.
5. Dwyer, T. A. W., Sira-Ramirez, H., Monaco, S., and Stornelli, S., *Variable Structure Control of Globally Feedback Decoupled Deformable Vehicle Maneuvers*, 27th Conference on Decision and Control, Los Angeles, California, pp. 1281–1287, 1987.
6. Monaco, S., and Stornelli, S., *A Nonlinear Feedback Control Law for Attitude Control*, Algebraic and Geometric Methods in Nonlinear Control Theory, Edited by M. Hazewinkel and M. Fliess, Reidel, Dordrecht, Holland, pp. 573–595, 1985.
7. Monaco, S., and Stornelli, S., *A Nonlinear Attitude Control Law for a Satellite with Flexible Appendages*, 24th Conference on Decision and Control, Ft. Lauderdale, Florida, pp. 1654–1659, 1985.

8. Monaco, S., Normand-Cyrot, D., and Stornelli, S., *Sampled Nonlinear Control for Large-Angle Maneuvers of Flexible Spacecraft*, Paper ESA SP-255, 2nd International Symposium on Spacecraft Flight Dynamics, Darmstadt, Germany, pp. 31–38, 1986.
9. Di Gennaro, S., Monaco, S., and Normand-Cyrot, D., *Nonlinear Digital Scheme for Attitude Tracking*, AIAA Journal of Guidance, Control, and Dynamics, Vol. 22, No. 3, pp. 467–477, 1999.
10. Di Gennaro, S., Monaco, S., Normand-Cyrot, D., and Pignatelli, A., *Digital Controllers for Attitude Maneuvering: Experimental Results*, Paper ESA SP-381, 2nd International Symposium on Spacecraft Flight Dynamics, Noordwijk, Netherlands, pp. 439–446, 1997.
11. Di Gennaro, S., *Output Feedback Stabilization of Flexible Spacecraft*, 35th Conference on Decision and Control, Kobe, Japan, pp. 497–502, 1996.
12. Di Gennaro, S., *Active Vibration Suppression in Flexible Spacecraft Attitude Tracking*, Journal of Guidance, Control, and Dynamics, Vol. 21, No. 3, pp. 400–408, 1998.
13. Lizarralde, F., and Wen, J. T., *Attitude Control without Angular Velocity Measurement: A Passivity Approach*, IEEE Transactions on Automatic Control, Vol. 41, No. 3, pp. 468–472, 1996.
14. Tsiotras, P., *A Passivity Approach to Attitude Stabilization Using Nonredundant Kinematic Parametrizations*, 34th Conference on Decison and Control, New Orleans, Louisiana, pp. 515–520, 1995.
15. Di Gennaro, S., *Output Attitude Control of Flexible Spacecraft from Quaternion Measures: A Passivity Approach*, 37th IEEE Conference on Decision and Control, Tampa, Florida, pp. 4549–4550, 1998.
16. Ickes, B. P., *A New Method for Performing Digital Control System Attitude Computations Using Quaternions*, AIAA Journal, Vol. 8, No. 1, pp. 13–17, 1970.
17. Wertz, J., Editor, *Spacecraft Attitude Determination and Control*, Kluwer Academic Publishers, Dordrecht, Holland, 1978.
18. Friedland, B., *Analysis of Strapdown Navigation Using Quaternions*, IEEE Transactions on Aerospace and Electronic Systems, Vol. 14, No. 5, pp. 764–769, 1978.
19. Wie, B., and Barba, P. M., *Quaternion Feedback for Spacecraft Large-Angle Maneuvers*, AIAA Journal of Guidance, Control, and Dynamics, Vol. 8, No. 3, pp. 360–365, 1985.
20. Khalil, H. K., *Nonlinear Systems*, Prentice-Hall, Upper Saddle River, New Jersey, 1996.
21. Desoer, C. A., and Vidyasagar, M., *Feedback Systems: Input–Output Properties*, Academic Press, New York, NY, 1975.
22. Isidori, A., *Nonlinear Control Systems*, Springer Verlag, Berlin, Germany, 1995.
23. Likins, P. W., and Fleischer, G. E., *Results of Flexible Spacecraft Attitude Control Studies Utilizing Hybrid Coordinates*, Journal of Spacecraft and Rockets, Vol. 8, No. 3, pp. 264–273, 1971.