

This article was downloaded by:[Gennaro, S. Di]  
On: 10 October 2007  
Access Details: [subscription number 782928198]  
Publisher: Taylor & Francis  
Informa Ltd Registered in England and Wales Registered Number: 1072954  
Registered office: Mortimer House, 37-41 Mortimer Street, London W1T 3JH, UK



## International Journal of Control

Publication details, including instructions for authors and subscription information:  
<http://www.informaworld.com/smpp/title~content=t713393989>

### Non-linear control of electromagnetic valves for camless engines

S. Di Gennaro <sup>a</sup>, B. Castillo-Toledo <sup>b</sup>, M. D. Di Benedetto <sup>a</sup>

<sup>a</sup> Department of Electrical and Information Engineering and Center of Excellence DEWS, University of L'Aquila, Poggio di Roio, 67040 L'Aquila, Italy

<sup>b</sup> Centro de Investigación y de Estudios Avanzados-CINVESTAV del IPN Unidad Guadalajara Av., Zapopan, Jalisco, México

Online Publication Date: 01 November 2007

To cite this Article: Gennaro, S. Di, Castillo-Toledo, B. and Benedetto, M. D. Di (2007) 'Non-linear control of electromagnetic valves for camless engines', International Journal of Control, 80:11, 1796 - 1813

To link to this article: DOI: 10.1080/00207170701519748

URL: <http://dx.doi.org/10.1080/00207170701519748>

PLEASE SCROLL DOWN FOR ARTICLE

Full terms and conditions of use: <http://www.informaworld.com/terms-and-conditions-of-access.pdf>

This article maybe used for research, teaching and private study purposes. Any substantial or systematic reproduction, re-distribution, re-selling, loan or sub-licensing, systematic supply or distribution in any form to anyone is expressly forbidden.

The publisher does not give any warranty express or implied or make any representation that the contents will be complete or accurate or up to date. The accuracy of any instructions, formulae and drug doses should be independently verified with primary sources. The publisher shall not be liable for any loss, actions, claims, proceedings, demand or costs or damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of this material.

# Non-linear control of electromagnetic valves for camless engines

S. DI GENNARO\*†, B. CASTILLO-TOLEDO‡  
and M. D. DI BENEDETTO†

†Department of Electrical and Information Engineering and  
Center of Excellence DEWS, University of L'Aquila,  
Poggio di Roio, 67040 L'Aquila, Italy

‡Centro de Investigación y de Estudios Avanzados – CINVESTAV  
del IPN Unidad Guadalajara Av. Científica 1145,  
45010, Col. El Bajío, Zapopan, Jalisco, México

(Received in final form 18 June 2007)

Conventional internal combustion engines use mechanical camshafts to command the opening and closing phases of the intake and exhaust valves. The lift valve profile is designed in order to reach a good compromise among various requirements of the engine operating conditions. In principle, optimality in every engine condition can be attained by camless valvetrains. In this context, electromagnetic valves appear to be promising, although there are some relevant open problems. In fact, in order to eliminate acoustic noises and avoid damage to the mechanical components, the control specifications require sufficiently low impact velocities between the valve and the constraints (typically the valve seat), so that “soft-landing” is obtained. In this paper, the soft-landing problem is translated into a regulation problem for the lift valve profile, by imposing that the valve position tracks a desired reference, while the modelled disturbances are rejected. Both reference and disturbance are generated by an autonomous system. The submanifold characterized by the zeroing of the tracking error and the rejection of the disturbance, is determined. Finally, the stabilization problem of the system trajectory on such a manifold is solved.

## 1. Introduction

Conventional internal combustion engines use mechanical camshafts to command the opening and closing phases of the intake and exhaust valves. The lift valve profile, connected with the crankshaft angle and obtained with a proper cam profile, is designed in order to find a compromise among various requirements, such as the engine efficiency, pollution emissions, fuel economy, valvetrain noise and vibration, maximization of the output torque and power. In fact, the different engine operating conditions would need different lift valve profiles and valve timings, which

cannot be dynamically changed in a mechanically driven camshaft. To overcome these limitations and optimize the aforementioned requirements, a solution based on variable valve timing can be pursued.

In this context, camless valvetrains are devices recently considered to decouple the camshaft and the valve lift dynamics (see Gray 1988, Ashhab *et al.* 1996, Schechter and Levine 1996, Pischinger *et al.* 2000). Their main advantages are fuel saving, increase of maximum and low speed torque, flattening of the torque characteristic and improvement of driveability, pollution and energy consumption reductions, increase of the burn rate, possible variability of the compression rate, improving of the combustion stability at low speed (Montanari *et al.* 2004). On the other hand, the electrohydraulic or electromechanical actuators of camless

\*Corresponding author. Email: digennar@ing.univaq.it

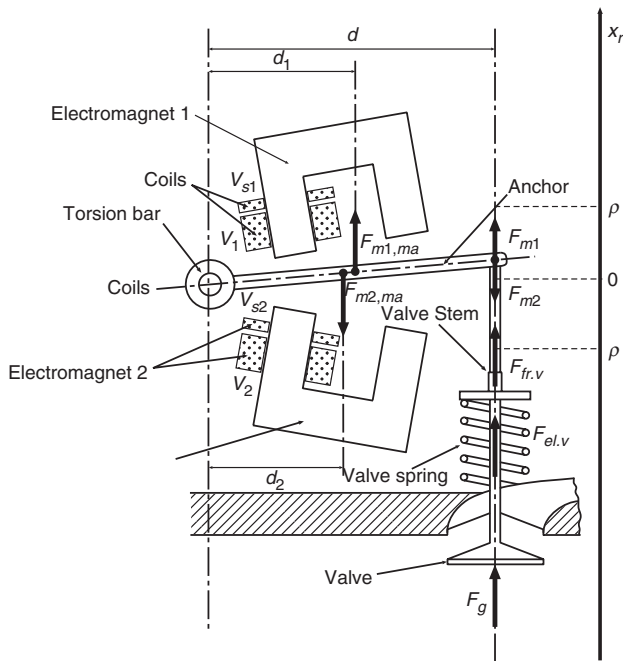


Figure 1. Scheme of an electromagnetic valve system (EMVS).

engines present considerable problems, which still remain open. In fact, these actuator dynamics are highly non-linear and unstable near the valve's terminal positions, while the control specifications require that impact velocities between the valve and the constraints (typically the valve seat) be sufficiently low in order to eliminate acoustic noises and avoid damage of the mechanical components. These problems are complicated by the short time (typically 3–5 ms) available at high engine speed to make a transition between the two valve's terminal positions, and the constraint in terms of actuator cost and space limitations. These last aspects imply that one typical request is the absence of the valve position sensor.

In Hoffmann and Stefanopoulou (2003) a precise control of the voltage applied to the coils has been proposed. A controller with a constant preset voltage, augmented by a voltage command based on a linear feedback, is determined and then modified by an iterative learning controller (ILC). In Tai *et al.* (2001) a control-oriented linear model for an EMCV has been considered, based on a gray-box approach which combines mathematical modelling and system identification. In Wang *et al.* (2002) a physics-based model for an EMCV is derived. Moreover, a sensitivity study has been conducted to characterize the ability of the control signal to affect the reduction on contact velocities. This model has been enriched by Peterson *et al.* (2002a) by introducing the impact dynamics, and a self-tuning non-linear controller has been designed. An observer

based output feedback controller has been proposed in Peterson *et al.* (2002b), while in Peterson *et al.* (2003) linear, non-linear and cycle-to-cycle self-tuning controllers have been considered.

Our work uses the model of an electromechanical actuator presented in Marchi *et al.* (2002) and Ronchi *et al.* (2002). The soft-landing problem is translated into a regulation problem for the lift valve profile, by imposing that the valve position tracks a desired reference, while the modeled disturbances are rejected. Both reference and disturbance are generated by an autonomous system, usually called exosystem. The sub-manifold characterized by the zeroing of the tracking error and the rejection of the disturbance, is determined. Finally, the stabilization problem of the system trajectory on such a manifold is solved. The resulting controller uses measurements of the whole state. Clearly, this is its main limitation. Nevertheless, its utility is represented by the fact that it can be used along with a state observer or as a first step in the development of a dynamic controller, which uses only output measurements.

## 2. Mathematical model of the electromagnetic valves and problem formulation

In this section we present the mathematical model of an electromagnetic valve system (EMVS), represented in figure 1. This valve is composed of an anchor moving between two electromagnets, connected with the stem of the valve. The valve is opened and closed by means of the attractive forces applied by the electromagnets on the anchor. In what follows we first present the dynamics of the electromagnets, then the mechanical dynamics.

### 2.1 Flux and eddy current dynamics

We first consider the equation describing the dynamics of the electric windings of the electromagnetic valve, which are (Marchi *et al.* 2002, Ronchi *et al.* 2002)

$$\dot{\phi}_{mj} = \frac{1}{N} (u_{mj} - R_j I_{mj}), \quad j = 1, 2, \quad (1)$$

where  $\phi_{mj}$  are the magnetic fluxes,  $R_j$  are the electrical resistances, and  $N$  is the number of turns of the windings, which is the same for both windings for each magnet. Moreover,  $u_{mj}$ ,  $I_{mj}$  are the voltages and currents of the electric circuits.

The currents  $I_{mj}$  can be given as a function of  $\phi_{mj}$  by expressing the magnetomotive force  $\hat{M}_{mj}$  as the sum of two terms (Ronchi *et al.* 2002)

$$\hat{M}_{mj}(\xi_{mj}, \phi_{mj}) = \mathcal{N}_j(\phi_{mj}) + \hat{\mathcal{R}}_j(\xi_{mj})\phi_{mj}, \quad j = 1, 2 \quad (2)$$

with  $\mathcal{N}_j(\phi_{mj}) \geq 0$  a non-linear function of the flux  $\phi_{mj}$ , describing the drop of the magnetomotive force in the magnetic flux path in the iron, while  $\hat{\mathcal{R}}_j(\xi_{mj})\phi_{mj}$  describes the drop in the air-gap, linear in the flux  $\phi_{mj}$  due to the absence of saturation in the air, and non-linear in the air-gap  $\xi_{mj}$  through the magnetic reluctance  $\hat{\mathcal{R}}_j(\xi_{mj}) \geq 0$ . In what follows, we assume that the forces closing the valve are positive. It is worth noting that we suppose that on the anchor no magnetic material (permanent magnets) is present; therefore, each electromagnet can only attract the anchor. We also suppose that only one electromagnet is supplied at each time instant.

The functions  $\mathcal{N}_j(\phi_{mj})$ ,  $j=1,2$ , can be described by some exponential functions obtained by a best fit procedure from experimental data

$$\mathcal{N}_j(\phi_{mj}) = \mathcal{N}_{0j} e^{n_j \phi_{mj}} \phi_{mj}, \quad j = 1, 2$$

$\mathcal{N}_{0j}$ ,  $n_j > 0$ . The magnetic reluctance  $\hat{\mathcal{R}}_j(\xi_{mj})$  is a non-linear function of the air-gap thickness  $\xi_{mj} \geq 0$ . Under the hypothesis that the anchor is connected with the valve stem tip, and that the arc can be approximated by the cord,  $\xi_{mj}$  can be expressed as functions of the anchor tip position  $x_a \in [-\rho, \rho]$

$$\xi_{mj} = \left( \rho + (-1)^j x_a \right) r_{l,mj}, \quad j = 1, 2, \quad (3)$$

where  $2\rho$  is the anchor tip displacement (coinciding with the valve vertical stroke),  $0 < r_{l,mj} = d_j/d < 1$ ,  $j=1,2$ , are the lever ratios, and  $x_a$  is positive in the same direction of  $x_v$ , see figure 1. Experimentally, one obtains the following expressions

$$\hat{\mathcal{R}}_j(\xi_{mj}) = p_{j1} \left( 1 - e^{-p_{j2} \xi_{mj}} \right) + p_{j3} \xi_{mj}, \quad j = 1, 2,$$

where  $p_{j1}$ ,  $p_{j2}$ ,  $p_{j3}$  are positive parameters obtained by identification. It is worth noting that  $\hat{\mathcal{R}}_j(\xi_{mj}) \geq 0$  for all  $\xi_{mj}$ .

Denoting by  $I_{pj}$  the eddy currents in parasitic circuits with resistance  $R_{pj}$  and inductance  $L_{pj}$  coupled with the magnetic circuits, having dynamics (Marchi *et al.* 2002)

$$-R_{pj} I_{pj} = \dot{\phi}_{mj} + L_{pj} \dot{I}_{pj}, \quad j = 1, 2 \quad (4)$$

the expression of the magnetomotive force is (Marchi *et al.* 2002)

$$\begin{aligned} NI_{mj} + I_{pj} &= \hat{\mathcal{M}}_{mj}(\xi_{mj}, \phi_{mj}) \\ &= \mathcal{N}_j(\phi_{mj}) + \hat{\mathcal{R}}_j(\xi_{mj})\phi_{mj}, \quad j = 1, 2, \end{aligned} \quad (5)$$

where (2) has been used. Therefore, by substituting (5) into (1), one gets the flux dynamics

$$\dot{\phi}_{mj} = \frac{R_j}{N^2} \left( I_{pj} - \mathcal{M}_{mj}(x_a, \phi_{mj}) \right) + \frac{1}{N} u_{mj}, \quad j = 1, 2, \quad (6)$$

where

$$\begin{aligned} \mathcal{M}_{mj}(x_a, \phi_{mj}) &= \mathcal{N}_j(\phi_{mj}) + \mathcal{R}_j(x_a)\phi_{mj}, \quad j = 1, 2 \\ \mathcal{R}_j(x_a) &= \hat{\mathcal{R}}_j(\xi_{mj}) \Big|_{\xi_{mj} = (\rho + (-1)^j x_a) r_{l,mj}} \\ &= -\frac{a_j}{b_j} e^{-b_j x_a} + c_j x_a + d_j \\ a_j &= b_j p_{j1} e^{-p_{j2} r_{l,mj} \rho}, \quad b_j = (-1)^j r_{l,mj} p_{j2}, \\ c_j &= (-1)^j r_{l,mj} p_{j3}, \quad d_j = p_{j1} + \rho r_{l,mj} p_{j3}. \end{aligned}$$

Moreover, using (4), (6), one works out the eddy current dynamics

$$\begin{aligned} \dot{I}_{pj} &= -\frac{1}{L_{pj}} \left( R_{pj} + \frac{R_j}{N^2} \right) I_{pj} + \frac{R_j}{L_{pj} N^2} \mathcal{M}_{mj}(x_a, \phi_{mj}) \\ &\quad - \frac{1}{L_{pj} N} u_{mj}, \quad j = 1, 2. \end{aligned} \quad (7)$$

## 2.2 Mechanical dynamics

The mechanical dynamics of the actuator can be obtained by writing the balance of the forces acting on the anchor, of mass  $m_a$ , and on the valve, of mass  $m_v$ . In order to consider a specific situation, as anticipated, we suppose that the anchor remains always connected with the valve during its stroke. Moreover, the contribution due to the gravity is supposed to be negligible, and all the mechanical components of the valve are considered rigid bodies.

*Anchor dynamics.* One first considers the (attractive) forces developed by the electromagnets on the moving anchor. Choosing the magnetic axis as reference, these magnetic forces can be written as

$$F_{mj,ma}(\xi_{mj}, \phi_{mj}) = -(-1)^j \frac{1}{2} \frac{d\hat{\mathcal{R}}_j(\xi_{mj})}{d\xi_{mj}} \phi_{mj}^2, \quad j = 1, 2$$

with  $\phi_{mj}$  the flux and  $\xi_{mj}$  the air-gap thickness. Here we considered the force  $F_{m1,ma}$  closing the valve as positive force, and  $F_{m2,ma}$  opening the valve as a negative force (see figure 1). In order to write all the mechanical equations in the same reference, these forces can be replaced by the forces  $F_{mj}$ ,  $j=1,2$ , acting on the anchor's intersection point with the valve stem axis (see figure 1), made equivalent to the forces  $F_{mj,ma}$  by equating the generated momenta.

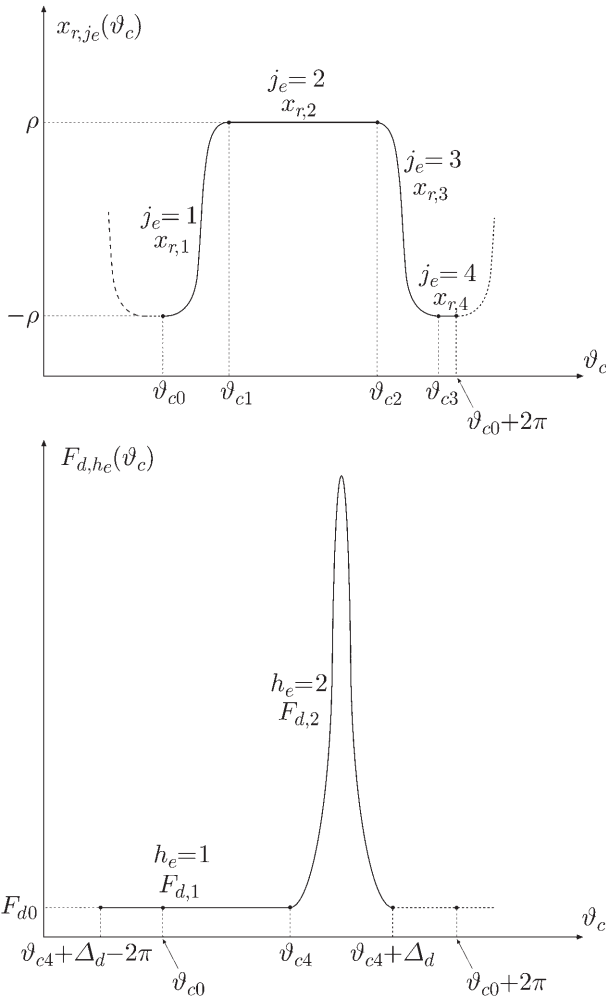


Figure 2. Reference trajectory  $x_r$  and disturbance  $F_d$ .

Considering relations (3), in this way one gets the following expressions:

$$\begin{aligned}
 F_{mj} &= r_{l,mj} F_{mj,ma}(\xi_{mj}, \phi_{mj}) \Big|_{\xi_{mj}=r_{l,mj}(\rho+(-1)^j x_a)} \\
 &= -\frac{1}{2} \mathcal{R}'_j(x_a) \phi_{mj}^2, \quad j = 1, 2
 \end{aligned} \tag{8}$$

with

$$\mathcal{R}'_j(x_a) = \frac{d\mathcal{R}_j(x_a)}{dx_a} = a_j e^{-b_j x_a} + c_j, \quad j = 1, 2.$$

Clearly  $\mathcal{R}'_1(x_a) < 0$  and  $\mathcal{R}'_2(x_a) > 0$ , so that  $F_{m1} > 0$  (closing the valve) and  $F_{m2} < 0$  (opening the valve), accordingly to the convention on the force signs. It is clear that both electromagnets may be supplied at the same time in order to impose some specific behaviour on the anchor. For the sake of the simplicity and

without loss of generality, we suppose that only the electromagnet 1 (see figure 1) is supplied during the closing phase of the valve, while during the opening phase that only the electromagnet 2 is supplied.

A torsional spring keeps the anchor in an intermediate position. This force can be rendered equivalent (by equating the momenta) to a force  $F_{el,a} = -k_a(x_a - x_a^\circ)$  acting on the tip of the anchor, with  $x_a^\circ$  the intermediate position, and  $k_a$  the elastic coefficient for the torsion bar. Hence, considering the (equivalent) spring elastic force  $F_{el,a}$ , the viscous friction force  $F_{fr,a} = -b_a \dot{x}_a$ , the net magnetic force  $F_m = F_{m1} + F_{m2}$ , and the constraint force  $F_{c,a}$  due to the electromagnet surface, each considered with its sign, the anchor motion equation is given by

$$\begin{aligned}
 m_a \ddot{x}_a &= F_{el,a} + F_{fr,a} + F_m + F_{c,a} \\
 &= -k_a(x_a - x_a^\circ) - b_a \dot{x}_a + F_m + F_{c,a}
 \end{aligned} \tag{9}$$

where  $b_a$  is the viscous friction for anchor. Note that  $F_{c,a}$  is always zero except for  $x_a = \pm \rho$ .

*Valve dynamics.* The valve has two springs, one closing it and another opening it, which can be modelled equivalently by a single linear spring, preloaded to keep the valve in the centre of its stroke  $x_v^\circ$  when the two electromagnets are not supplied, so that the elastic force  $F_{el,v}$ , due to the (equivalent) valve spring is  $F_{el,v} = -k_v(x_v - x_v^\circ)$ , with  $k_v$  the elastic coefficient of the (equivalent) linear spring. Considering the other forces acting on the valve of mass  $m_v$ , namely the viscous friction  $F_{fr,v} = -b_v \dot{x}_v$ , the disturbance force  $F_d$  due to the exhaust gases exiting the cylinder, and the force  $F_{c,v}$  due to constraint given by the valve seat, the valve dynamics are

$$\begin{aligned}
 m_v \ddot{x}_v &= F_{el,v} + F_{fr,v} + F_d + F_{c,v} \\
 &= -k_v(x_v - x_v^\circ) - b_v \dot{x}_v + F_d + F_{c,v}
 \end{aligned} \tag{10}$$

with  $b_v$  the viscous friction for the valve. The force  $F_d$  is a disturbance acting on the valve. It depends on the cam angle  $\vartheta_c$  and parameters, such as the valve equivalent area, the load, the gas turbulence, etc. A typical behaviour of  $F_d$  is given in figure 2 as a function of  $\vartheta_c$ , along with a valve lift reference. The model of  $F_d$  in a specific case will be given in the following sections. Note that  $F_{c,v}$  is always zero except for  $x_v = \rho$ .

*Mechanical dynamics.* By combining (9), (10) one gets the mechanical dynamics. As anticipated, we consider the case where the anchor is connected with the valve during its stroke, namely  $x_a = x_v$ , meaning that the considered position is that of the tip valve. Hence, adding (9) and (10) and considering (8), we work out

the mechanical dynamics

$$\begin{aligned} M\ddot{x}_v &= -kx_v - b\dot{x}_v + F_m + F_d + F_c, \\ F_m &= -\frac{1}{2}\left(\mathcal{R}'_1(x_v)\phi_{m1}^2 + \mathcal{R}'_2(x_v)\phi_{m2}^2\right) \end{aligned} \quad (11)$$

where  $M = m_a + m_v$ ,  $b = b_a + b_v$ ,  $k = k_a + k_v$ ,  $F_c = F_{c,a} + F_{c,v}$ . Here we have set the centre of the valve stroke  $x_v^\circ$  as the origin for  $x_v$ . Equation (11) holds for  $x_v \in [-\rho, \rho]$ , where  $\rho$  is the maximal displacement with respect to  $x_v^\circ$ . Note that  $F_c$  is always zero except for  $x_v = \pm\rho$ , where

$$\begin{aligned} F_c(\rho) &= k\rho + \frac{1}{2}\left(\mathcal{R}'_1(\rho)\phi_{m1}^2 + \mathcal{R}'_2(\rho)\phi_{m2}^2\right) - F_d \leq 0 \\ F_c(-\rho) &= -k\rho + \frac{1}{2}\left(\mathcal{R}'_1(-\rho)\phi_{m1}^2 + \mathcal{R}'_2(-\rho)\phi_{m2}^2\right) - F_d \geq 0. \end{aligned}$$

### 2.3 Mathematical model of the EMVS

From (11), (6), (7) one obtains the mathematical model of an EMVS

$$\left. \begin{aligned} \dot{x}_v &= v_v \\ \dot{v}_v &= \frac{1}{M}\left(-kx_v - b\dot{x}_v + F_m + F_d + F_c\right) \\ \dot{\phi}_{mj} &= \frac{R_j}{N^2}\left(I_{pj} - \mathcal{M}_{mj}(x_v, \phi_{mj})\right) + \frac{1}{N}u_{mj}, \quad j = 1, 2 \\ \dot{I}_{pj} &= -\frac{1}{L_{pj}}\left(R_{pj} + \frac{R_j}{N^2}\right)I_{pj} + \frac{R_j}{L_{pj}N^2}\mathcal{M}_{mj}(x_v, \phi_{mj}) \\ &\quad - \frac{1}{L_{pj}N}u_{mj}, \quad j = 1, 2 \end{aligned} \right\} \quad (12)$$

with  $F_m$  as in (11). In the following we suppose that all the state is available for measurement.

### 2.4 Hybrid model of the system and problem formulation

The mathematical model of the system is given by equation (12). The dynamics for the crankshaft angle  $\vartheta \in [0, 4\pi)$ , related with the cam's angle  $\vartheta_c = \vartheta/2 \in [0, 2\pi)$ , and velocity  $\omega$  have not been considered here for the sake of simplicity, even if the approach presented in the following sections can be applied for the complete engine dynamics. Therefore, in what follows we consider that the engine has reached a steady-state with  $\omega = \omega_0$  known.

The control problem is to determine a controller such that the valve is opened and closed following a desired valve lift reference trajectory (see figure 2), while the disturbance due to  $F_d$  is rejected. Due to the peculiarity of the present application, it is important to ensure exponential velocity of the convergence to the

valve lift reference, with velocity to be imposed by the designer.

A central aspect connected with the proper valve motion is the so-called soft landing of the valve. This problem comes into play either when the valve is closing or is reaching its maximum aperture, namely when the anchor is approaching one of the electromagnet surfaces (we have supposed the anchor and the valve rigidly connected). Clearly, this ensures also a soft landing for the valve in its seat during the closing phase, and avoids the valve chattering in the opening phase. Typical values of the valve velocity approaching the mechanical constraints (seating velocity) is 0.05 – 0.1 m/s.

One can translate the control problem with soft landing into a regulation problem for the valve stem, imposing that the valve position  $x_v$  tracks a desired reference  $x_r$ , while the disturbance due to  $F_d$  is rejected.

Various strategies can be followed to supply the electromagnets. For instance, some authors proposed that the closed-loop control is unnecessary when the anchor is in the intermediate position, since high currents would be necessary to impose an effectual force (Montanari *et al.* 2004). In order to reduce the control effort and for the sake of simplicity we will suppose that only one electromagnet is supplied when attracting the anchor. Therefore, when the magnetomotive force is positive (negative) only the electromagnet 1 (electromagnet 2) is supplied. This means that only one flux dynamic equation is forced by the input, while the other is in free evolution. Note that both the fluxes appear in  $F_m$ , so that the flux in free evolution determines a term which can be considered a perturbation to be compensated, see below.

This control strategy yields to a description of system (12) by means of a hybrid system with four discrete states  $q \in \mathcal{Q} = \{q_1, q_2, q_3, q_4\}$  of a finite state automaton (see figure 3): one for the closing phase, one when the valve is completely close, one for the opening phase, and finally one when the valve is completely open. The transitions among these states depend on the value of the system state  $x = (X_v \ V_v \ \Phi_m \ I_p)^T \in \mathbb{R}^4$  and on the value of the ‘‘perturbation’’ state  $x_w = (\Phi_w \ I_w)^T \in \mathbb{R}^2$ . The resulting hybrid system has continuous dynamics given by

$$\dot{x} = f_i(x, x_w, w, u) \quad (13)$$

$$\dot{x}_w = f_{wi}(x, x_w) \quad i \in \mathcal{I} = \{1, 2, \dots, N\} \quad (14)$$

$$e = h(x, w) \quad (15)$$

with  $N=4$ ,  $u(t) \in \mathbb{R}^m$  the input ( $m=1$ ),  $e \in \mathbb{R}^s$  the tracking error ( $s=1$ ),  $h = X_v - x_r(w)$  the output,  $x_r(w)$

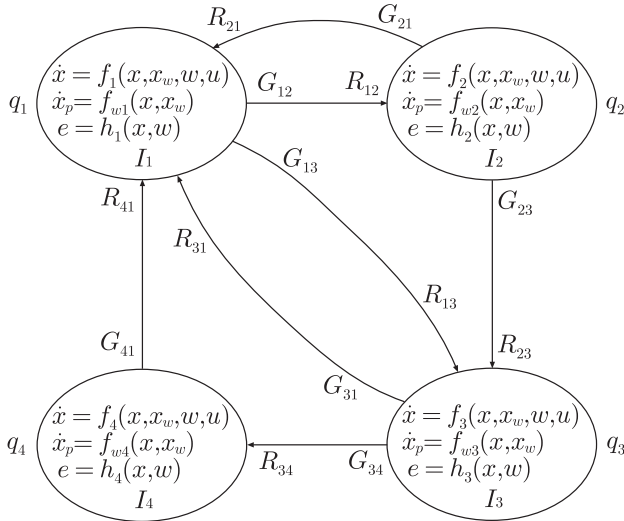


Figure 3. Hybrid system  $\mathcal{H}$  modelling the EMVS.

Table 1. Variables and parameters of functions (16).

$q_1$	$q_2$	$q_3$	$q_4$
$X_v = x_v$	$X_v = \rho$	$X_v = x_v$	$X_v = -\rho$
$V_v = v_v$	$V_v = 0$	$V_v = v_v$	$V_v = 0$
$\Phi_m = \phi_{m1}$	$\Phi_m = \phi_{m1}$	$\Phi_m = \phi_{m2}$	$\Phi_m = \phi_{m2}$
$\Phi_w = \phi_{m2}$	$\Phi_w = \phi_{m2}$	$\Phi_w = \phi_{m1}$	$\Phi_w = \phi_{m1}$
$I_p = I_{p1}$	$I_p = I_{p1}$	$I_p = I_{p2}$	$I_p = I_{p2}$
$I_w = I_{p2}$	$I_w = I_{p2}$	$I_w = I_{p1}$	$I_w = I_{p1}$
$U = u_{m1}$	$U = 0$	$U = u_{m2}$	$U = 0$
$\mathcal{R} = \mathcal{R}_1$	$\mathcal{R} = \mathcal{R}_1$	$\mathcal{R} = \mathcal{R}_2$	$\mathcal{R} = \mathcal{R}_2$
$\mathcal{N} = \mathcal{N}_1$	$\mathcal{N} = \mathcal{N}_1$	$\mathcal{N} = \mathcal{N}_2$	$\mathcal{N} = \mathcal{N}_2$
$\mathcal{R}_w = \mathcal{R}_2$	$\mathcal{R}_w = \mathcal{R}_2$	$\mathcal{R}_w = \mathcal{R}_1$	$\mathcal{R}_w = \mathcal{R}_1$
$\mathcal{N}_w = \mathcal{N}_2$	$\mathcal{N}_w = \mathcal{N}_2$	$\mathcal{N}_w = \mathcal{N}_1$	$\mathcal{N}_w = \mathcal{N}_1$
$R = R_1$	$R = R_1$	$R = R_2$	$R = R_2$
$R_p = R_{p1}$	$R_p = R_{p1}$	$R_p = R_{p2}$	$R_p = R_{p2}$
$L_p = L_{p1}$	$L_p = L_{p1}$	$L_p = L_{p2}$	$L_p = L_{p2}$
$R_w = R_2$	$R_w = R_2$	$R_w = R_1$	$R_w = R_1$
$R_{pw} = R_{p2}$	$R_{pw} = R_{p2}$	$R_{pw} = R_{p1}$	$R_{pw} = R_{p1}$
$L_{pw} = L_{p2}$	$L_{pw} = L_{p2}$	$L_{pw} = L_{p1}$	$L_{pw} = L_{p1}$
$F_c = 0$	$F_c = F_{c2}$	$F_c = 0$	$F_c = F_{c4}$

the output reference, and

$$f_i = \left. \begin{aligned} & V_v \\ & \frac{1}{M} \left( -kX_v - bV_v + F_m(x, x_w) + F_d(w) + F_c \right) \\ & \frac{R}{N^2} \left( I_p - \mathcal{M}_m(X_v, \Phi_m) + \frac{N}{R} U \right) \\ & - \frac{1}{L_p} \left( R_p + \frac{R}{N^2} \right) I_p + \frac{R}{L_p N^2} \mathcal{M}_m(X_v, \Phi_m) - \frac{1}{L_p N} U \end{aligned} \right\}$$

$$f_{wi} = \left. \begin{aligned} & \frac{R_w}{N^2} \left( I_w - \mathcal{M}_{mw}(X_v, \Phi_w) \right) \\ & - \frac{1}{L_{pw}} \left( R_{pw} + \frac{R_w}{N^2} \right) I_w + \frac{R_w}{L_{pw} N^2} \mathcal{M}_{mw}(X_v, \Phi_w) \end{aligned} \right\}. \tag{16}$$

In figure 2 a possible reference for the output is indicated. The vector  $w \in \mathbb{R}^r$  represents the (bounded) reference and disturbance signals. Finally,  $F_m(x, x_w)$  is given by (11),

$$\begin{aligned} \mathcal{M}_m(X_v, \Phi_m) &= \mathcal{N}(\Phi_m) + \mathcal{R}(X_v)\Phi_m \\ \mathcal{M}_{mw}(X_v, \Phi_w) &= \mathcal{N}_w(\Phi_w) + \mathcal{R}_w(X_v)\Phi_w \end{aligned}$$

are the magnetomotive forces, while the state variables  $X_v, V_v, \Phi_m, I_p, \Phi_w, I_w$ , the controls  $U_1, U_2$  and the constraint force  $F_c$  are given in table 1, with  $F_{c2} = k\rho + (1/2)\mathcal{R}'_1(\rho)\phi_{m1}^2 - F_d(w) \leq 0$ ,  $F_{c4} = -k\rho + (1/2)\mathcal{R}'_2(-\rho)\phi_{m2}^2 - F_d(w) \geq 0$ . In particular, when the system is in the states  $q_2, q_4$ , the first two equations in (13) are identically verified, due to the constraint force  $F_c$ .

The transitions among these four models are forced when the invariant conditions, given by

$$\left. \begin{aligned} I_1: x \mid x_v \in [-\rho, \rho], F_m \geq 0 \quad \text{for } q_1 \\ I_2: x \mid x_v \equiv \rho, F_m \geq 0 \quad \text{for } q_2 \\ I_3: x \mid x_v \in [-\rho, \rho], F_m \leq 0 \quad \text{for } q_3 \\ I_4: x \mid x_v \equiv -\rho, F_m \leq 0 \quad \text{for } q_4 \end{aligned} \right\} \tag{17}$$

are violated (Lygeros *et al.* 1999). Note that  $x_v \equiv \pm\rho$  means that  $x_v$  is equal to  $\pm\rho$  for more than a time instant. These are conditions on the values of the extended state  $(x^T \ x_w^T)^T$ . Moreover, the transitions leaving a state  $q_i \in Q$  are regulated by the so-called guard conditions, namely rules stating when a certain transition can take place. Referring to figure 3, the guard conditions are

$$G_{jk}: x \text{ satisfies } I_k, \quad j, k = 1, 2, 3, 4.$$

Finally, after a transition,  $x$  can possibly undergo a reset of its components. In the case under study the reset functions  $R_{jk}$  are the identities.

**Remark 1:** Clearly, the resulting hybrid system  $\mathcal{H}$  depends on the control strategy we have considered here. Indeed, it is easy to determine an analogous hybrid model in the case of different control strategies, possibly taking into account different ways in which the electromagnets can be supplied.

Moreover, it is easy to incorporate in the model also valve bounces due to collisions of the valve in its seat

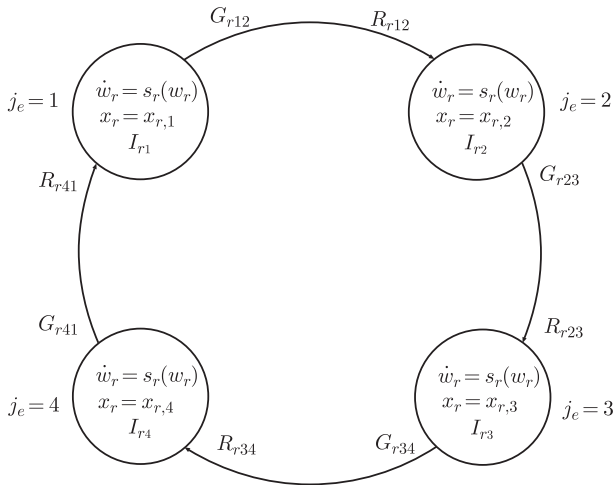


Figure 4. Reference hybrid system.

or of the anchor with the electromagnets. Indeed, in the case of partially elastic collisions, for instance, one can consider transitions with appropriate guard conditions (bounces are experienced for  $|v_v| \geq v_b$ , with  $v_b$  a certain minimum velocity) and reset functions ( $v$  is set to  $-cv$  after the bounce, with  $c \leq 1$  a certain coefficient).

### 3. The regulation problem for hybrid systems

In this section we will solve the regulation problem for a hybrid system  $\mathcal{H}$ , characterized by the continuous non-linear dynamics (13), (15), along with the autonomous non-linear dynamics

$$\dot{w} = s_{i_e}(w), \quad i_e \in \mathcal{I}_e = \{1, 2, \dots, N_e\} \quad (18)$$

$\mathcal{I}_e$  a finite index set,  $w(t) \in W_i \subset \mathbb{R}^r$  the state of the  $i_e$ th generator of bounded reference and disturbance signals, described by (18) and called exosystem (Isidori 1995).

As already mentioned, the control problem is to determine a controller such that the valve is opened and closed tracking (globally and exponentially) a desired valve lift reference trajectory  $x_r(w)$ , rejecting the disturbance due to  $F_d(w)$ . Since the state variables, references, disturbances, etc., in general may undergo discontinuities, we require the uniform exponential convergence to a  $\delta$ -ball of the origin, namely we require

$$\begin{aligned} |x_v - x_r(w)| &\leq k e^{-\lambda(t-t_0)} |x_v(t_0) - x_r(w(t_0))| + \delta, \\ &\forall x_v(t_0), \forall w(t_0) \end{aligned} \quad (19)$$

for appropriate  $k > 0, \lambda > 0$ . When  $\delta=0$  this implies that the the so-called regulator equations for

system (13) have to be solved (Isidori and Byrnes 1990, Isidori 1995).

In what follows we first consider the expression of the reference trajectory to be tracked and of the disturbance to be rejected, and then we solve the regulator equations for system (13) in the state  $q_i, i \in \mathcal{I}$ .

### 3.1 The reference trajectory and disturbances for a EMVS

The exosystem models the reference trajectory and the disturbances acting on the system. For the sake of simplicity and without loss of generality, some simplifications will be introduced in order to better illustrate the proposed approach. For instance, we consider that all the moving parts of the valve are rigid bodies, and we consider as disturbances only the pressure of the intake/exhaust gases. Flexibility in the valve mechanism, usually modeled as disturbance, and/or other perturbations could easily be taken into account at the expense of a more complicated presentation.

As far as the reference trajectory is concerned, we consider a trajectory ensuring the soft landing of the valve in its seat and of the anchor on the surfaces of the electromagnets. Such a trajectory is, for instance, the one imposed by a mechanical cam. In our case the EMVS allows more general references satisfying some condition of obvious physical interpretation. In fact, it is possible to impose a trajectory composed of four parts (see figure 2), depending on the cam's angle  $\vartheta_c = \vartheta/2 \in [0, 2\pi)$ , with  $\vartheta \in [0, 4\pi)$  the crankshaft's angle, solution of  $\dot{\vartheta} = \omega_0$  (since  $\omega_0$  is here considered constant, this dynamic equation has not been considered in the system model). We require that the reference be at least a  $C^4$  function of  $\vartheta_c$  (and hence of time). As it will be shown, this condition implies that the input ensuring tracking is a continuous function.

The first part of the reference (corresponding to  $j_e=1$  in figure 2), valid for  $\vartheta_c \in [\vartheta_{c0}, \vartheta_{c1})$ , is parameterized as

$$x_{r1}(\vartheta_c) = \sum_{k=0}^7 c_{r,1k} \frac{\vartheta_c^k}{k!} \quad (20)$$

with the coefficients in  $c_{r,1k}, k = 0, 1, \dots, 7$ , determined so that at  $\vartheta_c = \vartheta_{c0} = 0$  the closed valve starts to open with velocity, acceleration and jerk (namely up to the third derivative) equal to zero, and is closed (with zero velocity, acceleration and jerk) at



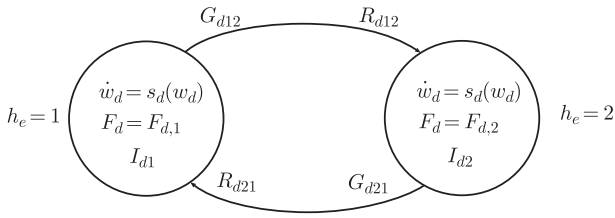


Figure 5. Disturbance hybrid system.

$\vartheta_c = \vartheta_{c1}$  (8 conditions, see appendix 2)

$$\left. \begin{aligned} x_{r1}(\vartheta_{c0}) &= \rho = \sum_{k=0}^7 c_{r,1k} \frac{\vartheta_{c0}^k}{k!}, \\ x_{r1}(\vartheta_{c1}) &= -\rho = \sum_{k=0}^7 c_{r,1k} \frac{\vartheta_{c1}^k}{k!}, \\ \frac{d^\ell x_{r1}}{d\vartheta_c^\ell} \Big|_{\vartheta_{c0}} &= 0 = \sum_{k=\ell}^7 c_{r,1k} \frac{\vartheta_{c0}^{k-\ell}}{(k-\ell)!}, \\ \frac{d^\ell x_{r1}}{d\vartheta_c^\ell} \Big|_{\vartheta_{c1}} &= 0 = \sum_{k=\ell}^7 c_{r,1k} \frac{\vartheta_{c1}^{k-\ell}}{(k-\ell)!}, \quad \ell = 1, 2, 3. \end{aligned} \right\} \quad (21)$$

In this way one obtains the expressions of the coefficients  $c_{r,1k}$ ,  $k = 0, 1, \dots, 7$ . In a similar way, it is easy to check that the second part of the reference (corresponding to  $j_e=2$  in figure 2), is simply  $x_{r2}(\vartheta_c) = -\rho$ , for  $\vartheta_c \in [\vartheta_{c3}, \vartheta_{c0} + 2\pi)$ . Here  $c_{r,20} = -\rho$ ,  $c_{r,21} = \dots = c_{r,25} = 0$ ,  $c_{r,40} = \rho$ ,  $c_{r,41} = \dots = c_{r,45} = 0$ , while  $c_{r,3k} = -c_{r,1k}$ ,  $k = 1, \dots, 7$ .

It is clear that these four references can be generated by (recall that  $\vartheta_c = (\omega_0/2)t$ )

$$\begin{aligned} \dot{w}_0 &= 0 & w_0(0) &= 1 \\ \dot{w}_k &= \frac{\omega_0}{2} w_{k-1}, & w_k(0) &= 0, \quad k = 1, \dots, 7 \end{aligned}$$

namely by the linear system

$$\begin{aligned} \dot{w}_r &= s_r(w_r), \quad w_r = \begin{pmatrix} w_0 \\ \vdots \\ w_7 \end{pmatrix}, \\ s_r(w_r) &= \frac{\omega_0}{2} \begin{pmatrix} 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{pmatrix} w_r \end{aligned} \quad (22)$$

with the switching among the following different outputs

$$x_{r,j_e}(w) = \sum_{k=0}^7 c_{r,j_ek} w_k, \quad j_e = 1, \dots, 4. \quad (23)$$

The corresponding hybrid system generating the references is depicted in figure 4, where the invariant conditions are

$$\begin{aligned} I_{r1}: \vartheta_c \mid \vartheta_c \in [\vartheta_{c0}, \vartheta_{c1}) & \text{ for the state } j_e = 1 \\ I_{r2}: \vartheta_c \mid \vartheta_c \in [\vartheta_{c1}, \vartheta_{c2}) & \text{ for the state } j_e = 2 \\ I_{r3}: \vartheta_c \mid \vartheta_c \in [\vartheta_{c2}, \vartheta_{c3}) & \text{ for the state } j_e = 3 \\ I_{r4}: \vartheta_c \mid \vartheta_c \in [\vartheta_{c3}, \vartheta_{c0} + 2\pi) & \text{ for the state } j_e = 4 \end{aligned}$$

while the guard and reset conditions are

$$\begin{aligned} G_{r12}: \vartheta_c \geq \vartheta_{c1}, \quad G_{r23}: \vartheta_c \geq \vartheta_{c2}, \\ G_{r34}: \vartheta_c \geq \vartheta_{c3}, \quad G_{r41}: \vartheta_c \geq \vartheta_{c0} + 2\pi \\ R_{r12}, R_{r23}, R_{r34}, R_{r41}: w_0 = 1, \quad w_1 = \dots = w_7 = 0. \end{aligned}$$

As far as the disturbance is concerned, it can be generated by

$$\begin{aligned} \dot{w}_8 &= 0 & w_8(0) &= 1 \\ \dot{w}_9 &= \frac{\omega_0}{2} w_8 & w_9(0) &= 0 \end{aligned}$$

i.e., by the linear system

$$\begin{aligned} \dot{w}_d &= s_d(w_d), \quad w_d = \begin{pmatrix} w_8 \\ w_9 \end{pmatrix}, \\ s_d(w_d) &= \frac{\omega_0}{2} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} w_d \end{aligned} \quad (24)$$

with the switching among the following disturbance signals

$$F_{d,h_e}(w) = F_{d0} w_8 + c_{d,h_e} \frac{p_a w_8}{1 + (w_9 - p_c)^2 / p_b}, \quad h_e = 1, 2 \quad (25)$$

with

$$c_{d,h_e} = \begin{cases} 0 & \text{if } \vartheta_c \in [\vartheta_{c4} + \Delta_d - 2\pi, \vartheta_{c4}) \\ 1 & \text{if } \vartheta_c \in [\vartheta_{c4}, \vartheta_{c4} + \Delta_d), \end{cases}$$

where we have used the so-called Agnesi's versiera (or Agnesi's witch) to approximate the gas pressure profile during the combustion phase in the cylinder. The hybrid system generating the disturbance is depicted in figure 5, where the invariant conditions are

$$\begin{aligned} I_{d1}: \vartheta_c \mid \vartheta_c \in [\vartheta_{c4} + \Delta_d - 2\pi, \vartheta_{c4}) & \text{ for the state } h_e = 1 \\ I_{d2}: \vartheta_c \mid \vartheta_c \in [\vartheta_{c4}, \vartheta_{c4} + \Delta_d) & \text{ for the state } h_e = 2 \end{aligned}$$

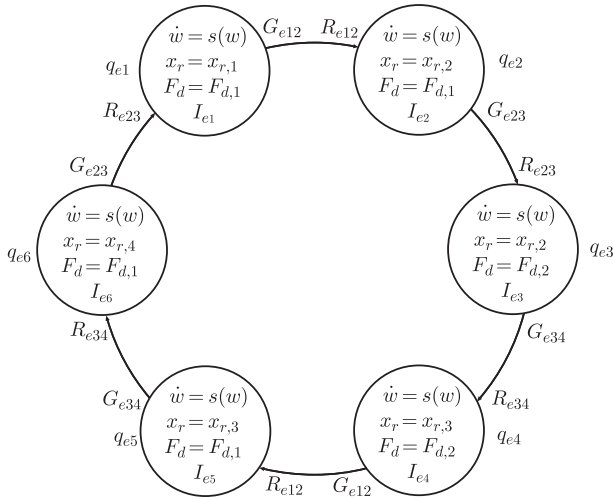


Figure 6. Exosystem hybrid system.

while the guard and reset conditions are

$$G_{d12}: \vartheta_c \geq \vartheta_{c4}, \quad G_{d21}: \vartheta_c \geq \vartheta_{c4} + \Delta_d, \\ R_{d12}: w_8 = 1, \quad w_9 = 0, \quad R_{d21}: I_d$$

with  $I_d$  the identity map.

The exosystem (18) is finally given by (22), (24), with

$$w = \begin{pmatrix} w_r \\ w_d \end{pmatrix}, \quad s_{i_e}(w) = \begin{pmatrix} s_r(w_r) \\ s_d(w_d) \end{pmatrix}, \\ i_e \in \mathcal{I}_e = \{1, \dots, 6\} \quad (26)$$

with outputs (23), (25) switching according to the hybrid system depicted in figure 6, where the invariant conditions are

$$I_{e1}: \vartheta_c \mid \vartheta_c \in [\vartheta_{c0}, \vartheta_{c1}) \quad \text{for } q_{e1}, \\ I_{e2}: \vartheta_c \mid \vartheta_c \in [\vartheta_{c1}, \vartheta_{c4}) \quad \text{for } q_{e2}, \\ I_{e3}: \vartheta_c \mid \vartheta_c \in [\vartheta_{c4}, \vartheta_{c2}) \quad \text{for } q_{e3}, \\ I_{e4}: \vartheta_c \mid \vartheta_c \in [\vartheta_{c2}, \vartheta_{c4} + \Delta_d) \quad \text{for } q_{e4}, \\ I_{e5}: \vartheta_c \mid \vartheta_c \in [\vartheta_{c4} + \Delta_d, \vartheta_{c3}) \quad \text{for } q_{e5}, \\ I_{e6}: \vartheta_c \mid \vartheta_c \in [\vartheta_{c3}, \vartheta_{c0} + 2\pi) \quad \text{for } q_{e6},$$

while the guard and reset conditions are

$$G_{e12}: \vartheta_c \geq \vartheta_{c1}, \quad G_{e23}: \vartheta_c \geq \vartheta_{c4}, \quad G_{e34}: \vartheta_c \geq \vartheta_{c2}, \\ G_{e45}: \vartheta_c \geq \vartheta_{c4} + \Delta_d, \quad G_{e56}: \vartheta_c \geq \vartheta_{c3}, \quad G_{e61}: \vartheta_c \geq \vartheta_{c0} + 2\pi \\ R_{e12}(w) = \begin{pmatrix} R_{r12}(w_r) \\ w_d \end{pmatrix}, \quad R_{e23}(w) = \begin{pmatrix} w_r \\ R_{d12}(w_d) \end{pmatrix}, \\ R_{e34}(w) = \begin{pmatrix} R_{r23}(w_r) \\ w_d \end{pmatrix}, \quad R_{e45}(w) = \begin{pmatrix} w_r \\ w_d \end{pmatrix}, \\ R_{e56}(w) = \begin{pmatrix} R_{r34}(w_r) \\ w_d \end{pmatrix}, \quad R_{e61}(w) = \begin{pmatrix} R_{r41}(w_r) \\ w_d \end{pmatrix}.$$

### 3.2 Solution of the regulator equations for the EMVS

In this section we solve the regulator equations for the hybrid system. In the determination of the solution of the regulator equations it should be stressed that each component of the centre manifold depends on the  $i$ th system (13), with output (15), and on the  $i_e$ th exosystem (26), with outputs (23), (25). For notational simplicity, we avoid indicating this dependence, which will be clear from the context.

Let us first consider the case in which system (13) is in the state  $q_1$ , where  $x_v \in [-\rho, \rho]$ ,  $F_m \geq 0$ . In this case the regulator equations are given by

$$\left. \begin{aligned} \mathcal{L}_s \pi_{x_v} &= \pi_{v_y} \\ \mathcal{L}_s \pi_{v_y} &= \frac{1}{M} \left( -k\pi_{x_v} - b\pi_{v_y} + \pi_{F_m} + F_{d,h_e}(w) \right) \\ \mathcal{L}_s \pi_{\phi_{m1}} &= \frac{R_1}{N^2} \left( \pi_{I_{p1}} - \mathcal{M}_{m1}(\pi_{x_v}, \pi_{\phi_{m1}}) + \frac{N}{R_1} c_{m1} \right) \\ \mathcal{L}_s \pi_{I_{p1}} &= -\frac{R_{p1}}{L_{p1}} \pi_{I_{p1}} - \frac{1}{L_{p1}} \mathcal{L}_s \pi_{\phi_{m1}} \\ 0 &= \pi_{x_v} - x_{r,j_e}(w), \end{aligned} \right\} \quad (27)$$

where  $\mathcal{L}$  stands for the classical Lie derivative, and  $s = s_{i_e}(w)$  is the  $i_e$ th exosystem function (26). Moreover,

$$\pi_{F_m}(w) = M \mathcal{L}_s^2 \pi_{x_v} + b \mathcal{L}_s \pi_{x_v} + k \pi_{x_v} - F_{d,h_e}(w) \\ = -\frac{1}{2} \mathcal{R}'_1(\pi_{x_v}) \pi_{\phi_{m1}}^2. \quad (28)$$

From the first and the last equations of (27) it is easy to work out

$$\pi_{x_v}(w) = x_{r,j_e} = \sum_{k=0}^7 c_{r,j_e k} w_k \\ \pi_{v_y}(w) = \mathcal{L}_s \pi_{x_v} = \frac{\omega_0}{2} \sum_{k=1}^7 c_{r,j_e k} w_{k-1}$$

while from the second equation

$$0 \leq -\frac{1}{2} \mathcal{R}'_1(\pi_{x_v}) \pi_{\phi_{m1}}^2 = \pi_{F_m} \\ = k \pi_{x_v} + b \mathcal{L}_s \pi_{x_v} + M \mathcal{L}_s^2 \pi_{x_v} - F_{d,h_e} \\ = \sum_{k=0}^7 C_{j_e k} w_k - F_{d,h_e} \\ C_{j_e k} = k c_{r,j_e k} + b c_{r,j_e k+1} \frac{\omega_0}{2} + M c_{r,j_e k+2} \frac{\omega_0^2}{4}, \\ k = 0, \dots, 7,$$

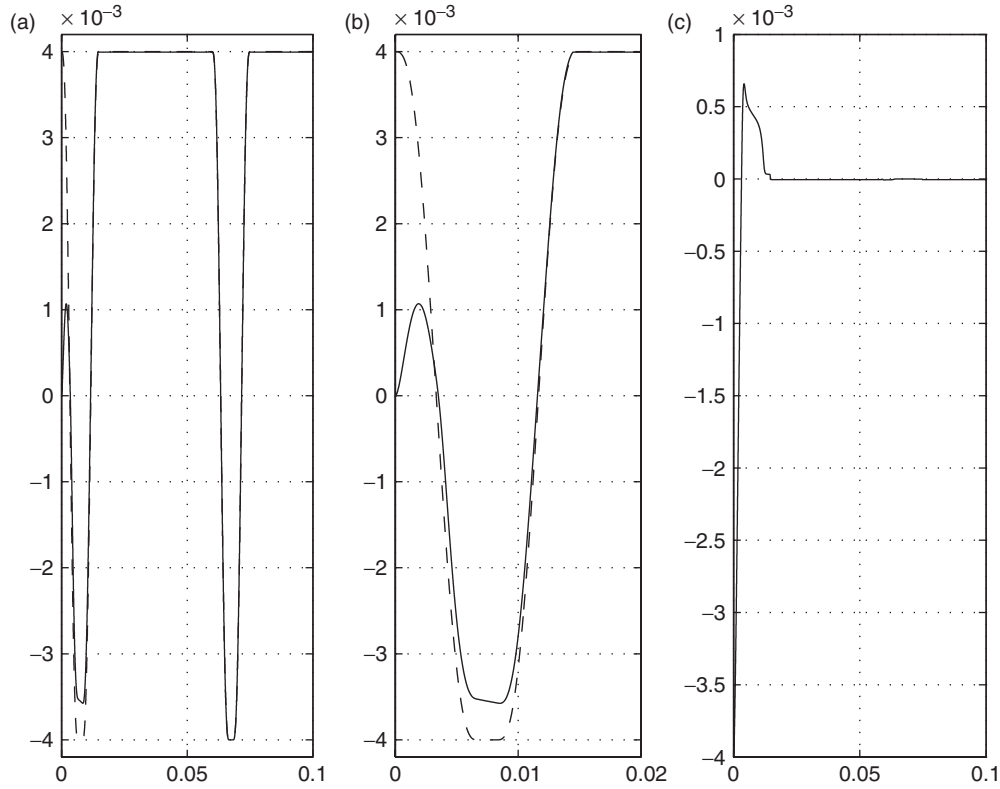


Figure 7. (a) Valve position  $x_v$  and reference  $x_r = \pi_{x_v}(w)$  [m]; (b) detail of  $x_v, x_r$  [m]; (c) error  $x_v - x_r$  [m].

where  $c_{r,j_8} = c_{r,j_9} = 0$ . Hence, one obtains

$$\pi_{\phi_{m1}}(w) = \begin{cases} \sqrt{\alpha_1} & \text{if } \alpha_1 \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\alpha_1(w) = 2 \frac{\pi_{F_m}(w)}{-\mathcal{R}'_1(\pi_{x_v})} = 2 \frac{\pi_{F_m}(w)}{|\mathcal{R}'_1(\pi_{x_v})|}, \quad (29)$$

where  $\mathcal{R}'_j(\pi_{x_v}) \neq 0, j=1,2$ . The fourth equation of (27) admits a solution  $\pi_{I_{p1}}$  since the corresponding differential equations (4) have eigenvalues  $-(R_{p1}/L_{p1})$  with negative real parts. This solution could possibly be calculated in an approximated way. Alternatively, one can consider a numeric resolution; the problem is that the initial conditions are unknown. Therefore, using the following observer

$$\mathcal{L}_s \hat{\pi}_{I_{p1}} = -\frac{R_{p1}}{L_{p1}} \hat{\pi}_{I_{p1}} - \frac{1}{L_{p1}} \mathcal{L}_s \pi_{\phi_{m1}}$$

where  $\pi_{\phi_{m1}}$  is given in (29), and  $\hat{\pi}_{I_{p1}}(0)$  is known, one obtains an estimate  $\hat{\pi}_{I_{p1}}(w)$  which exponentially tends to  $\pi_{I_{p1}}(w)$ .

Finally, from the third equation of (27) one works out the steady-state input

$$\hat{c}_{m1}(w) = -\frac{R_1}{N} \left( \hat{\pi}_{I_{p1}} - \mathcal{M}_{m1}(\pi_{x_v}, \pi_{\phi_{m1}}) \right) + N \mathcal{L}_s \pi_{\phi_{m1}}.$$

This control exponentially tends to

$$c_{m1}(w) = -\frac{R_1}{N} \left( \pi_{I_{p1}} - \mathcal{M}_{m1}(\pi_{x_v}, \pi_{\phi_{m1}}) \right) + N \mathcal{L}_s \pi_{\phi_{m1}}$$

satisfying the third equation of (27).

When the system (13) is in the state  $q_3$ , where  $x_v \in [-\rho, \rho], F_m \leq 0$ , in a similar way one determines

$$\pi_{x_v}(w) = x_{r,j_e} = \sum_{k=0}^7 c_{r,j_e k} w^k$$

$$\pi_{v_v}(w) = \mathcal{L}_s \pi_{x_v} = \frac{\omega_0}{2} \sum_{k=1}^7 c_{r,j_e k} w^{k-1}$$

$$\pi_{\phi_{m2}}(w) = \begin{cases} \sqrt{\alpha_2} & \text{if } \alpha_2 \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\alpha_2 = 2 \frac{-\pi_{F_m}}{\mathcal{R}'_2(\pi_{x_v})} = 2 \frac{-\pi_{F_m}}{|\mathcal{R}'_2(\pi_{x_v})|}$$

$$\hat{c}_{m2}(w) = -\frac{R_2}{N} \left( \hat{\pi}_{I_{p2}} - \mathcal{M}_{m2}(\pi_{x_v}, \pi_{\phi_{m2}}) \right) + N \mathcal{L}_s \pi_{\phi_{m2}}$$

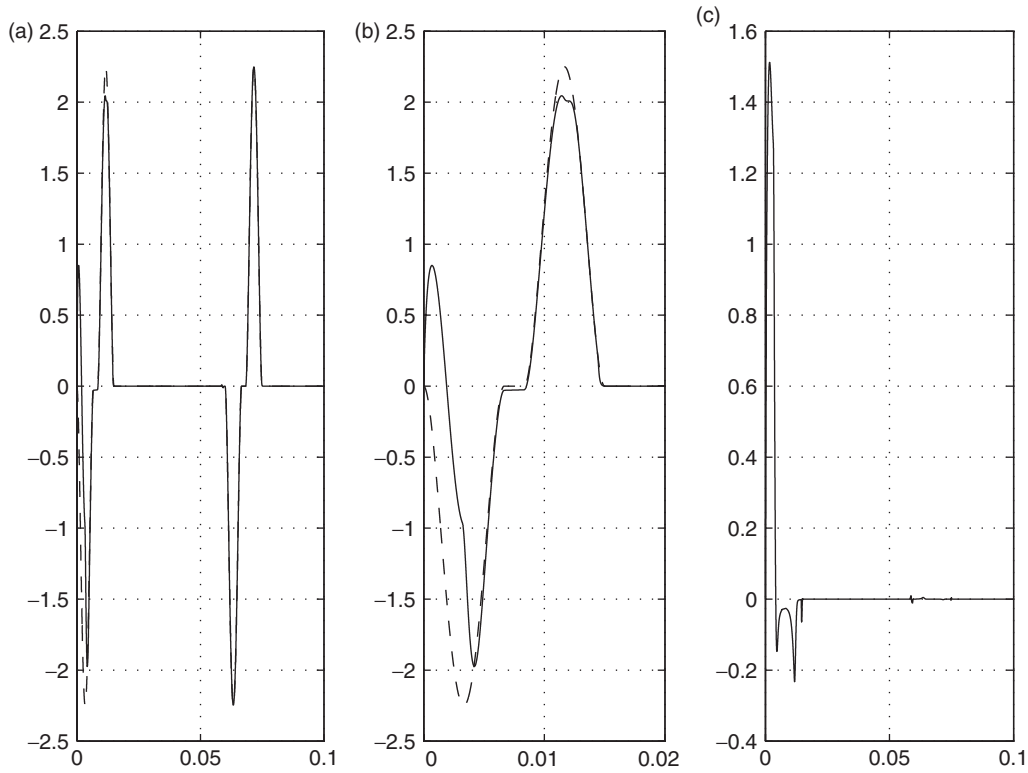


Figure 8. (a) Valve velocity  $v_v$  and reference  $v_r = \pi_v(w)$  [m/s]; (b) detail of  $v_v$ ,  $v_r$  [m/s]; (c) error  $v_v - v_r$  [m/s].

where an estimate  $\hat{\pi}_{I_{p2}}(w)$  exponentially converging to  $\pi_{I_{p2}}(w)$  is given by the solution of

$$\mathcal{L}_s \hat{\pi}_{I_{p2}} = -\frac{R_{p2}}{L_{p2}} \hat{\pi}_{I_{p2}} - \frac{1}{L_{p2}} \mathcal{L}_s \pi_{\phi_{m2}}$$

with  $\hat{\pi}_{I_{p2}}(0)$  known.

Let us finally consider the case in which system (13) is in the state  $q_2$  ( $x_v \equiv \rho$ ,  $F_m \geq 0$ , i.e.,  $j=1$ ) or  $q_4$  ( $x_v \equiv -\rho$ ,  $F_m \leq 0$ , i.e.,  $j=2$ ). In this case the regulator equations are

$$\begin{aligned} \mathcal{L}_s \pi_{x_v} &= 0 \\ \mathcal{L}_s \pi_{v_v} &= 0 \\ \mathcal{L}_s \pi_{\phi_{mj}} &= \frac{R_j}{N^2} \left( \pi_{I_{pj}} - \mathcal{M}_{mj}(\pm\rho, \pi_{\phi_{mj}}) + \frac{N}{R_j} c_{mj} \right), \quad j = 1, 2 \\ \mathcal{L}_s \pi_{I_{pj}} &= -\frac{R_{pj}}{L_{pj}} \pi_{I_{pj}} - \frac{1}{L_{pj}} \mathcal{L}_s \pi_{\phi_{mj}} \\ 0 &= \pm\rho - x_r \end{aligned} \quad (30)$$

and it results

$$\pi_{x_v} = \pm\rho, \quad \pi_{v_v} = 0$$

while  $\pi_{\phi_{mj}}$ ,  $j=1, 2$ , remain undetermined. It is hence possible to choose

$$\begin{aligned} q_2: \pi_{\phi_{m1}} &= \begin{cases} \sqrt{\alpha_1} & \text{if } \alpha_1 \geq 0 \\ 0 & \text{otherwise} \end{cases} \\ q_4: \pi_{\phi_{m2}} &= \begin{cases} \sqrt{\alpha_2} & \text{if } \alpha_2 \geq 0 \\ 0 & \text{otherwise} \end{cases} \\ \alpha_1 &= 2 \frac{k\rho - F_{d,h_e}}{-\mathcal{R}'_1(\rho)} = 2 \frac{\pi_{F_m}}{|\mathcal{R}'_1(\rho)|} \\ \alpha_2 &= 2 \frac{k\rho + F_{d,h_e}}{\mathcal{R}'_2(-\rho)} = 2 \frac{-\pi_{F_m}}{|\mathcal{R}'_2(-\rho)|}. \end{aligned}$$

Moreover, for  $\pi_{I_{pj}}$  the same reasoning holds, and hence it is possible to consider estimates  $\hat{\pi}_{I_{pj}}$ , solutions of

$$\mathcal{L}_s \hat{\pi}_{I_{pj}} = -\frac{R_{pj}}{L_{pj}} \hat{\pi}_{I_{pj}} - \frac{1}{L_{pj}} \mathcal{L}_s \pi_{\phi_{mj}}, \quad j = 1, 2$$

while the steady-state controls are given by

$$\hat{c}_{mj} = N \mathcal{L}_s \pi_{\phi_{mj}} - \frac{R_j}{N} \hat{\pi}_{I_{pj}} + \frac{R_j}{N} \mathcal{M}_{mj}(\pm\rho, \pi_{\phi_{mj}}), \quad j = 1, 2.$$

#### 4. Full information output regulation for a EMVS

In this section we will determine a control law ensuring that the reference is followed, rejecting the disturbance. This control will need the knowledge of the whole state.

The importance of such a control relies on the fact that it represents the first step in the design of a dynamic controller from the measured variables (typically not including the valve position).

First we consider the classical regulation problem for the single state  $q_i$ ,  $i = 1, \dots, 4$  of  $\mathcal{H}$  (system (13)), and then we prove that the whole hybrid system fulfills the requirements of the regulation problem expressed by (19).

#### 4.1 Output regulation for single states of $\mathcal{H}$

The design of the desired control law is based on the Lyapunov approach. Let us determine the control law when system (13) is in the state  $q_1$ . For the mechanical subsystem (first two equations of the EMVS model)

$$\begin{aligned} \begin{pmatrix} \dot{x}_v \\ \dot{v}_v \end{pmatrix} &= A \begin{pmatrix} x_v \\ v_v \end{pmatrix} + B(F_m + F_{d,h_e}), \\ A &= \begin{pmatrix} 0 & 1 \\ -\frac{k}{M} & -\frac{b}{M} \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ \frac{1}{M} \end{pmatrix} \end{aligned} \quad (31)$$

with  $F_m$  given by (11), let us consider the following Lyapunov function candidate for the ‘‘mechanical’’ dynamics

$$\mathcal{V}_{sm} = \frac{1}{2} \left\| \begin{pmatrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{pmatrix} \right\|_{P_s}^2, \quad P_s = P_s^T > 0$$

where  $\|z\|_P^2$  stands for  $z^T P z$ . Moreover, note that

$$\begin{pmatrix} \mathcal{L}_s \pi_{x_v} \\ \mathcal{L}_s \pi_{v_v} \end{pmatrix} = A \begin{pmatrix} \pi_{x_v} \\ \pi_{v_v} \end{pmatrix} + B(\pi_{F_m} + F_{d,h_e})$$

with  $\pi_{F_m}$  as in (28). Considering the derivative of  $\mathcal{V}_{sm}$  along the trajectories of (31), one obtains

$$\dot{\mathcal{V}}_{sm} = - \left\| \begin{pmatrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{pmatrix} \right\|_{Q_s}^2 + \begin{pmatrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{pmatrix}^T P_s B (F_m - \pi_{F_m})$$

with  $P_s$  solution of  $(P_s A + A^T P_s)/2 = -Q_s$ ,  $Q_s = Q_s^T > 0$ , and

$$\begin{aligned} F_m - \pi_{F_m} &= -\frac{1}{2} \mathcal{R}'_1(x_v) (\phi_{m1}^2 - \kappa_{s1} \pi_{\phi_{m1}}^2) - \frac{1}{2} \mathcal{R}'_2(x_v) \phi_{m2}^2, \\ \kappa_{s1}(x_v, \pi_{x_v}) &= \sqrt{\mathcal{R}'_1(\pi_{x_v}) / \mathcal{R}'_1(x_v)}. \end{aligned}$$

Recall that  $\mathcal{R}'_j(x_v) \neq 0$ ,  $j = 1, 2$ ,  $\forall x_v$ . In the following it will be clear that  $x_w = (\phi_{m2} \ I_{p2})^T$ , acting as a disturbance, goes exponentially to zero, so that no compensation is necessary.

Therefore, consider the subsystem given by the mechanical dynamics plus the dynamics of  $\phi_{m1}$ ,  $I_{p1}$  and  $\hat{\pi}_{I_{p1}}$ , and the following Lyapunov function candidate

$$\mathcal{V}_s = \mathcal{V}_{sm} + \mathcal{V}_{se} \quad (32)$$

where the ‘‘electromagnetic’’ part is given by

$$\begin{aligned} \mathcal{V}_{se} &= \frac{N^2}{2R_1} (\phi_{m1} - \kappa_{s1} \pi_{\phi_{m1}})^2 \\ &+ \frac{1}{2R_{p1}} (\phi_{m1} + L_{p1} I_{p1} - (\pi_{\phi_{m1}} + L_{p1} \pi_{I_{p1}}))^2 \\ &+ \frac{\gamma_{g1}}{4\lambda} (\zeta_w^T \mathcal{P} \zeta_w)^2 + \gamma_{g2} \frac{L_{p1}}{2R_{p1}} (\pi_{I_{p1}} - \hat{\pi}_{I_{p1}})^2 \end{aligned}$$

with  $\gamma_{g1}, \gamma_{g2} > 0$ , and

$$\begin{aligned} \zeta_w &= \begin{pmatrix} \phi_{m2} \\ \phi_{m2} + L_{p2} I_{p2} \end{pmatrix} = T x_w, \\ \mathcal{P} &= \begin{pmatrix} \frac{1}{L_{p2}} \frac{N^2}{R_2} & 0 \\ 0 & \frac{1}{L_{p2} R_{p2}} \end{pmatrix} > 0, \quad T = \begin{pmatrix} 1 & 0 \\ 1 & L_{p2} \end{pmatrix}. \end{aligned}$$

Notice that (see the appendix 1)

$$\begin{aligned} L_{p1}^2 |I_{p1} - \pi_{I_{p1}}|^2 &\leq (\phi_{m1} + L_{p1} I_{p1} - (\pi_{\phi_{m1}} + L_{p1} \pi_{I_{p1}}))^2 \\ &\leq 3(\phi_{m1} - \kappa_{s1} \pi_{\phi_{m1}})^2 \\ &+ 3L_{p1}^2 |I_{p1} - \pi_{I_{p1}}|^2 + 3\beta_{sj}^2 \left\| \begin{pmatrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{pmatrix} \right\|^2 \end{aligned}$$

$\beta_{sj} = \alpha_{sj} \max_w |\pi_{\phi_{m1}}(w)|$ . The derivative of the Lyapunov function candidate is given by

$$\begin{aligned} \dot{\mathcal{V}}_s &= - \left\| \begin{pmatrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{pmatrix} \right\|_{Q_s}^2 \\ &+ (\phi_{m1} - \kappa_{s1} \pi_{\phi_{m1}}) \left[ -\frac{1}{2} \mathcal{R}'_1(x_v) (\phi_{m1} \right. \\ &+ \kappa_{s1} \pi_{\phi_{m1}}) B^T P_s \begin{pmatrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{pmatrix} \\ &+ I_{p1} - \kappa_{s1} \pi_{I_{p1}} - (\mathcal{N}_1(\phi_{m1}) - \kappa_{s1} \mathcal{N}_1(\pi_{\phi_{m1}}) \\ &+ \mathcal{R}_1(x_v) \phi_{m1} - \kappa_{s1} \mathcal{R}_1(\pi_{x_v}) \pi_{\phi_{m1}}) \\ &+ \frac{N}{R_1} (u_{m1} - \kappa_{s1} c_{m1}) - \frac{N^2}{R_1} \dot{\kappa}_{s1} \pi_{\phi_{m1}} \left. \right] \\ &- \frac{1}{2} \mathcal{R}'_2(x_v) \phi_{m2}^2 B^T P_s \begin{pmatrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{pmatrix} \\ &- (\phi_{m1} - \pi_{\phi_{m1}}) (I_{p1} - \pi_{I_{p1}}) - L_{p1} (I_{p1} - \pi_{I_{p1}})^2 \\ &- \frac{\gamma_{g1}}{\lambda} \zeta_w^T \mathcal{P} \zeta_w \left( \frac{1}{L_{p2}} \mathcal{M}_{m2}(x_v, \phi_{m2}) \phi_{m2} + \hat{I}_{p2}^2 \right) \\ &- \gamma_{g2} (\pi_{I_{p1}} - \hat{\pi}_{I_{p1}})^2, \end{aligned}$$

where

$$\dot{\kappa}_{s1} = \frac{1}{2\kappa_{s1}} \frac{\mathcal{R}'_1(\pi_{x_v})\mathcal{R}'_1(x_v)\pi_{v_v} - \mathcal{R}'_1(\pi_{x_v})\mathcal{R}''_1(x_v)v_v}{\mathcal{R}'_1(x_v)^2}.$$

Since  $\mathcal{R}_2(x_v) \geq 0$

$$\begin{aligned} & -\frac{1}{L_{p2}} \mathcal{M}_{m2}(x_v, \phi_{m2})\phi_{m2} \\ &= -\frac{1}{L_{p2}} \left( \mathcal{N}_{02} e^{\eta_2 \phi_{m2}} \phi_{m2}^2 + \mathcal{R}_2(x_v) \phi_{m2}^2 \right) \\ &\leq -\frac{\mathcal{N}_{02}}{L_{p2}} \phi_{m2}^2 \leq -\phi_{m2}^2 \\ -\zeta_w^T \mathcal{P} \zeta_w &= -\begin{pmatrix} \phi_{m2} \\ I_{p2} \end{pmatrix}^T \bar{\mathcal{P}} \begin{pmatrix} \phi_{m2} \\ I_{p2} \end{pmatrix} \leq -\bar{\lambda} (\phi_{m2}^2 + I_{p2}^2), \\ \bar{\mathcal{P}} &= T^T \mathcal{P} T, \end{aligned}$$

where  $\bar{\lambda} > 0$  is the minimum eigenvalue of  $\bar{\mathcal{P}}$ , and  $\mathcal{N}_{02}/L_{p2} > 1$ , as shown in the simulation section. Hence,

$$\begin{aligned} \dot{V}_s &\leq -\left\| \begin{matrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{matrix} \right\|_{Q_s}^2 + (\phi_{m1} - \kappa_{s1} \pi_{\phi_{m1}}) \\ &\times \left[ -\frac{1}{2} \mathcal{R}'_1(x_v) (\phi_{m1} + \kappa_{s1} \pi_{\phi_{m1}}) B^T P_s \begin{pmatrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{pmatrix} \right. \\ &+ I_{p1} - \kappa_{s1} \pi_{I_{p1}} - \left( \mathcal{N}_1(\phi_{m1}) - \kappa_{s1} \mathcal{N}_1(\pi_{\phi_{m1}}) \right) \\ &+ \mathcal{R}_1(x_v) \phi_{m1} - \kappa_{s1} \mathcal{R}_1(\pi_{x_v}) \pi_{\phi_{m1}} \\ &\left. + \frac{N}{R_1} (u_{m1} - \kappa_{s1} c_{m1}) - \frac{N^2}{R_1} \dot{\kappa}_{s1} \pi_{\phi_{m1}} \right] \\ &- \frac{1}{2} \mathcal{R}'_2(x_v) \phi_{m2}^2 B^T P_s \begin{pmatrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{pmatrix} \\ &- (\phi_{m1} - \pi_{\phi_{m1}})(I_{p1} - \pi_{I_{p1}}) - L_{p1}(I_{p1} - \pi_{I_{p1}})^2 \\ &- \gamma_{g1} (\phi_{m2}^2 + I_{p2}^2) - \gamma_{g2} (\pi_{I_{p1}} - \hat{\pi}_{I_{p1}})^2. \end{aligned}$$

Therefore, when system (13) is in the state  $q_1$ , the control

$$\begin{aligned} u_{m1} &= \kappa_{s1} \hat{c}_{m1} + \frac{R_1}{N} \left[ \frac{1}{2} \mathcal{R}'_1(x_v) (\phi_{m1} + \kappa_{s1} \pi_{\phi_{m1}}) \right. \\ &\times B^T P_s \begin{pmatrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{pmatrix} - k_3 (\phi_{m1} - \kappa_{s1} \pi_{\phi_{m1}}) \\ &- (I_{p1} - \kappa_{s1} \hat{\pi}_{I_{p1}}) + \mathcal{N}_1(\phi_{m1}) - \kappa_{s1} \mathcal{N}_1(\pi_{\phi_{m1}}) \\ &\left. + \mathcal{R}_1(x_v) \phi_{m1} - \kappa_{s1} \mathcal{R}_1(\pi_{x_v}) \pi_{\phi_{m1}} + \frac{N^2}{R_1} \dot{\kappa}_{s1} \pi_{\phi_{m1}} \right] \end{aligned} \quad (33)$$

$k_3 > 0$  (recall that  $u_{m2} = 0$ ), is such that

$$\begin{aligned} \dot{V}_s &\leq -\left\| \begin{matrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{matrix} \right\|_{Q_s}^2 - k_3 (\phi_{m1} - \kappa_{s1} \pi_{\phi_{m1}})^2 \\ &- \frac{1}{2} \mathcal{R}'_2(x_v) \phi_{m2}^2 B^T P_s \begin{pmatrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{pmatrix} \\ &+ |\phi_{m1} - \kappa_{s1} \pi_{\phi_{m1}}| |I_{p1} - \pi_{I_{p1}}| \\ &+ \beta_{sj} |I_{p1} - \pi_{I_{p1}}| \left\| \begin{matrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{matrix} \right\| - L_{p1} (I_{p1} - \pi_{I_{p1}})^2 \\ &- \gamma_{g1} (\phi_{m2}^2 + I_{p2}^2) - \gamma_{g2} (\pi_{I_{p1}} - \hat{\pi}_{I_{p1}})^2, \end{aligned}$$

where we have considered that (see the appendix 1)

$$\begin{aligned} &- (\phi_{m1} - \kappa_{s1} \pi_{\phi_{m1}} + (\kappa_{s1} - 1) \pi_{\phi_{m1}}) (I_{p1} - \pi_{I_{p1}}) \\ &\leq |\phi_{m1} - \kappa_{s1} \pi_{\phi_{m1}}| |I_{p1} - \pi_{I_{p1}}| + \beta_{sj} |I_{p1} - \pi_{I_{p1}}| \left\| \begin{matrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{matrix} \right\|. \end{aligned}$$

Since  $\mathcal{R}'_2(x_v)$  is bounded,  $k_3, \gamma_{g1}, \gamma_{g2}$  can be made large enough to make the right hand term negative. This shows that  $x = (x_v \ v_v \ \phi_{m1} \ I_{p1})^T$  exponentially tends to  $\pi = (\pi_{x_v} \ \pi_{v_v} \ \pi_{\phi_{m1}} \ \pi_{I_{p1}})^T$  since  $\kappa_{s1}$  tends to 1. Finally, it is easy to check that  $x_w = (\phi_{m2} \ I_{p2})^T$  exponentially tends to  $\pi_w = (\pi_{\phi_{m2}} \ \pi_{I_{p2}})^T = 0$ .

Analogously, when (13) is in  $q_3$  ( $u_{m1} = 0$ ) the control

$$\begin{aligned} u_{m2} &= \kappa_{s2} \hat{c}_{m2} + \frac{R_2}{N} \left[ \frac{1}{2} \mathcal{R}'_2(x_v) (\phi_{m2} + \kappa_{s2} \pi_{\phi_{m2}}) \right. \\ &B^T P_s \begin{pmatrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{pmatrix} - k_3 (\phi_{m2} - \kappa_{s2} \pi_{\phi_{m2}}) \\ &- (I_{p2} - \kappa_{s2} \hat{\pi}_{I_{p2}}) + \mathcal{N}_2(\phi_{m2}) - \kappa_{s2} \mathcal{N}_2(\pi_{\phi_{m2}}) \\ &\left. + \mathcal{R}_2(x_v) \phi_{m2} - \kappa_{s2} \mathcal{R}_2(\pi_{x_v}) \pi_{\phi_{m2}} + \frac{N^2}{R_2} \dot{\kappa}_{s2} \pi_{\phi_{m2}} \right] \end{aligned} \quad (34)$$

is such that  $x = (x_v \ v_v \ \phi_{m2} \ I_{p2})^T$  exponentially tends to  $\pi = (\pi_{x_v} \ \pi_{v_v} \ \pi_{\phi_{m2}} \ \pi_{I_{p2}})^T$ , while  $x_w = (\phi_{m1} \ I_{p1})^T$  exponentially tends to  $\pi_w = (\pi_{\phi_{m1}} \ \pi_{I_{p1}})^T = 0$ .

The exponential stability when (13) is in  $q_2$  or  $q_4$  can be shown analogously.

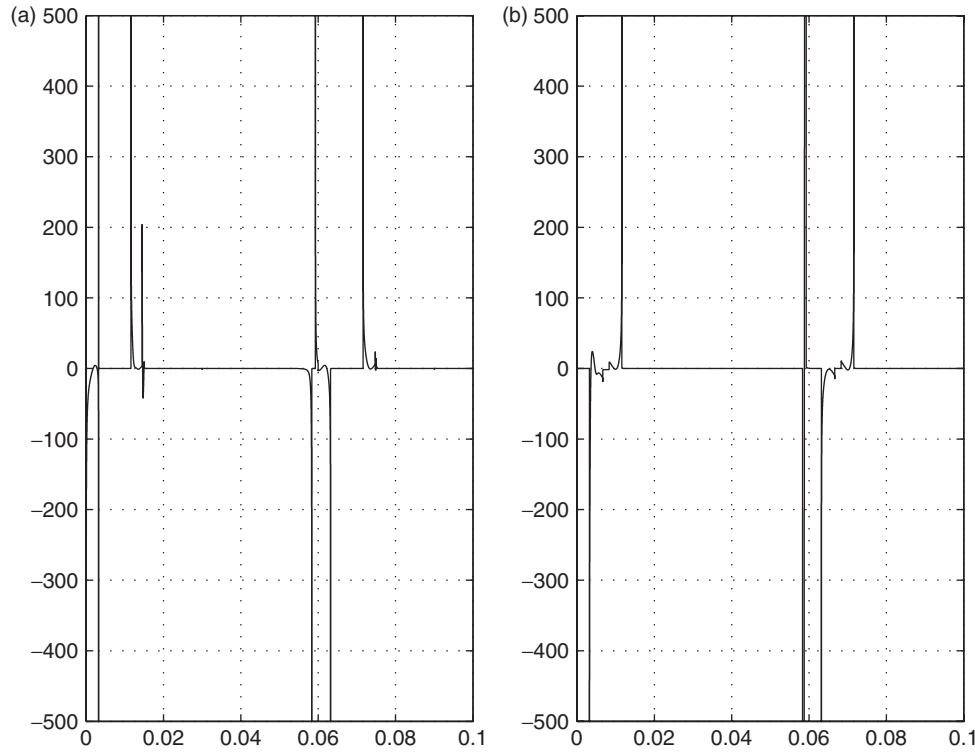


Figure 9. (a) Input  $u_{m1}$  [V]; (b) input  $u_{m2}$  [V].

**4.2 Solution of the regulation problem for the hybrid system  $\mathcal{H}$**

The solvability of the regulation problem for  $\mathcal{H}$ , expressed by (19), can be proven on the basis of simple considerations about the so called dwell time (Liberzon 2003). In particular, we will show that the  $i$ th system remains in the state  $q_i$  at least for a certain time  $\tau_{d,i}$ , namely for a time interval  $T_i = [t_i, t'_i = t_i + \tau_{d,i}]$ . In fact,  $\mathcal{H}$  leaves  $q_i$  when the invariant condition  $I_i$  in (17) is not fulfilled. These conditions depend on the values of  $x_v$  and  $F_m$ . The absence of Zeno behaviour like chattering is physically obvious in the present application, and can be proved mathematically showing that  $x_v$  and  $F_m$  are uniformly continuous. Note that the control inputs  $u_{mi}(x, w)$ ,  $i = 1, \dots, 4$ , designed in §4.1 are continuous functions of their arguments  $x, w$ . Then the system (13) in closed loop, rewritten as

$$\dot{x} = f_i(x, x_w, w, u_i(x, w)) = \bar{f}_i(t, x)$$

is such that  $\bar{f}_i$  is continuous in its arguments. Moreover, since only polynomial and exponential terms appear in  $\bar{f}_i$ ,  $\partial \bar{f}_i / \partial x$  is continuous. Therefore,  $\bar{f}_i$  is Lipschitz and hence its solution  $x(t)$  is locally uniformly continuous in a compact set. Hence,  $x_v(t)$  and

$F_m(x_v(t), \phi_{m1}(t), \phi_{m2}(t))$  in (11) are uniformly continuous. This implies that no Zeno behaviours can occur, even for  $t \rightarrow \infty$ . In fact, if a function  $\Psi(t)$  is uniformly continuous, for a generic  $k_1 > 0$  let  $\bar{t}_i$  be the time instant such that  $|\Psi(\bar{t}_i)| = k_1$ . For the uniform continuity, there exists a positive constant  $\tau_{d,i}$  such that  $|\Psi(t + \theta) - \Psi(t)| < k_1/2$  for all  $t \geq \bar{t}_i$  and for all  $\theta \in [0, \tau_{d,i}]$ . Hence,

$$\begin{aligned} |\Psi(t)| &= |\Psi(t) - \Psi(\bar{t}_i) + \Psi(\bar{t}_i)| \geq |\Psi(\bar{t}_i)| \\ &\quad - |\Psi(t) - \Psi(\bar{t}_i)| > k_1 - \frac{k_1}{2} = \frac{k_1}{2} = \delta_{k_1} \end{aligned}$$

for all  $t \in [\bar{t}_i, \bar{t}_i + \tau_{d,i}]$ . Since  $\Psi(t)$  maintains its sign in a neighbourhood of  $\bar{t}_i$ , which does not depend on  $t$ , chattering can not occur.

On the basis of definitions (17) and of the previous discussion, instantaneous switchings can not occur, and  $\mathcal{H}$  remains in a state  $q_i$  at least for a finite time interval  $T_i = [t_i, t'_i = t_i + \tau_{d,i}]$ , with  $\tau_{d,i}$  that does not go to zero. This shows the existence of a minimum dwell time.

The solvability of the regulation problem for  $\mathcal{H}$  is now proved as follows. Let us suppose that  $\mathcal{H}$  is in the generic  $i$ th discrete state  $q_i$ , and consider a generic switching sequence  $\mathcal{I} = i, i_2, \dots, j, \dots, i_{v-1}, i$  driving  $\mathcal{H}$  again in  $q_i$  (cycle). Since the discrete structure of  $\mathcal{H}$  in our case is a finite state machine, this sequence always exists and

can not be infinite. Moreover, when  $\mathcal{H}$  is in  $q_i$  we have determined in the previous section a Lyapunov function  $\mathcal{V}_{s,i} = \mathcal{V}_{sm,i} + \mathcal{V}_{se,i}$  such that

$$\begin{aligned}
 k_{1,i} \begin{Bmatrix} \|z_m\|^2 \\ \|z_{e,i}\|^2 \\ \|z_{w,i}\|^2 \end{Bmatrix} &\leq \mathcal{V}_{sm,i} + \mathcal{V}_{se,i} \leq k_{2,i} \begin{Bmatrix} \|z_m\|^2 \\ \|z_{e,i}\|^2 \\ \|z_{w,i}\|^2 \end{Bmatrix} \\
 \dot{\mathcal{V}}_{sm,i} + \dot{\mathcal{V}}_{se,i} &\leq -\lambda_{s,i} \begin{Bmatrix} \|z_m\|^2 \\ \|z_{e,i}\|^2 \\ \|z_{w,i}\|^2 \end{Bmatrix} \\
 z_m &= \begin{Bmatrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{Bmatrix}, \quad z_{e,i} = \begin{Bmatrix} \phi_{mj} - \kappa_{sj}\pi_{\phi_{mj}} \\ I_{pj} - \pi_{I_{pj}} \end{Bmatrix} \\
 z_{w,i} &= \begin{Bmatrix} \phi_{m\ell} \\ I_{p\ell} \end{Bmatrix}, \quad \ell = (j+1) \bmod 2
 \end{aligned}$$

for appropriate values  $k_{1,i}, k_{2,i}$ . Note that  $\lambda_{s,i}$  can be chosen arbitrarily large. Therefore,

$$\dot{\mathcal{V}}_{s,i} \leq -\lambda_i \mathcal{V}_{s,i}$$

$\lambda_i = \lambda_{s,i}/k_{2,i}$ , so that  $\mathcal{V}_{s,i}(t) \leq e^{-\lambda_i(t-t_i)} \mathcal{V}_{s,i}(t_i)$ ,  $t \in T_i = [t_i, t'_i]$ , and setting  $\tau_d = \min_{i \in \mathcal{I}} \tau_{d,i}$

$$\begin{aligned}
 \mathcal{V}_{s,i}(t'_i) &= \mathcal{V}_{sm,i}(t'_i) + \mathcal{V}_{se,i}(t'_i) \leq \mathcal{V}_{s,i}(t_i + \tau_d) \\
 &\leq e^{-\lambda_i \tau_d} \mathcal{V}_{s,i}(t_i) = e^{-\lambda_i \tau_d} (\mathcal{V}_{sm,i}(t_i) + \mathcal{V}_{se,i}(t_i)).
 \end{aligned}$$

When  $\mathcal{H}$  switches in  $q_{i+1}$  at  $t_{i+1} = t'_i$ , from the definitions of  $\mathcal{V}_{sm}, \pi_{x_v}, \pi_{v_v}, x_v(w)$ , one checks that  $\mathcal{V}_{sm,i}(t_{i+1}) = \mathcal{V}_{sm,i}(t'_i)$ , so that

$$\begin{aligned}
 \mathcal{V}_{sm,i}(t_{i+1}) &\leq e^{-\lambda_i \tau_d} \mathcal{V}_{sm,i}(t_i) + \Delta_{e,i}, \\
 \Delta_{e,i} &= e^{-\lambda_i \tau_d} \mathcal{V}_{se,i}(t_i) - \mathcal{V}_{se,i}(t'_i).
 \end{aligned}$$

On the contrary, at the switching instant  $\mathcal{V}_{se,i}(t_{i+1}) \neq \mathcal{V}_{se,i}(t'_i)$ . In fact, when in  $q_i$ ,  $\phi_{mj} \rightarrow \pi_{\phi_{mj}}, I_{pj} \rightarrow \pi_{I_{pj}}, \hat{\pi}_{I_{pj}} \rightarrow \pi_{I_{pj}}, \phi_{m\ell} \rightarrow 0, I_{p\ell} \rightarrow 0, \ell = (j+1) \bmod 2$ , while when in  $q_{i+1}$ ,  $\phi_{m\ell} \rightarrow \pi_{\phi_{m\ell}}, I_{p\ell} \rightarrow \pi_{I_{p\ell}}, \hat{\pi}_{I_{p\ell}} \rightarrow \pi_{I_{p\ell}}, \phi_{mj} \rightarrow 0, I_{pj} \rightarrow 0$ . Moreover, at the switching  $\pi_{\phi_{m\ell}}(t_{i+1}) \neq \pi_{\phi_{mj}}(t'_i)$ . This will determine the presence of a term  $\delta$  in (19).

If  $\lambda_{\min}^{P_{s,i}}, \lambda_{\max}^{P_{s,i}}$  are the minimum and maximum eigenvalues of  $P_{s,i}$ ,  $\lambda_{\min}^{P_{s,i}}/2 \|z_m\|^2 \leq \mathcal{V}_{sm,i} \leq \lambda_{\max}^{P_{s,i}}/2 \|z_m\|^2$ , and

$$\mathcal{V}_{sm,i+1}(t_{i+1}) \leq k_i \mathcal{V}_{sm,i}(t_{i+1}) \leq k_i e^{-\lambda_i \tau_d} \mathcal{V}_{sm,i}(t_i) + k_i \Delta_{e,i}$$

with  $k_i = \lambda_{\max}^{P_{s,i+1}}/\lambda_{\min}^{P_{s,i}}$ . Hence, it is easy to check that

$$\left. \begin{aligned}
 \mathcal{V}_{sm,i+1}(t_{i+1}) &\leq k_i e^{-\lambda_i \tau_d} \mathcal{V}_{sm,i}(t_i) + k_i \delta_{e,i} \\
 \mathcal{V}_{sm,i+2}(t_{i+2}) &\leq k_i k_{i+1} e^{-(\lambda_i + \lambda_{i+1}) \tau_d} \mathcal{V}_{sm,i}(t_i) \\
 &\quad + k_{i+1} \delta_{e,i+1} \\
 \mathcal{V}_{sm,i+3}(t_{i+3}) &\leq k_i k_{i+1} k_{i+2} e^{-(\lambda_i + \lambda_{i+1} + \lambda_{i+2}) \tau_d} \mathcal{V}_{sm,i}(t_i) \\
 &\quad + k_{i+2} \delta_{e,i+2} \\
 &\quad \vdots \\
 \mathcal{V}_{sm,i}(t_{i+v}) &= \mathcal{V}_{sm,i+v}(t_{i+v}) \\
 &\leq k_i k_{i+1} \dots k_{i+v-1} e^{-(\lambda_i + \lambda_{i+1} + \dots + \lambda_{i+v-1}) \tau_d} \mathcal{V}_{sm,i}(t_i) \\
 &\quad + k_{i+v-1} \delta_{e,i+v-1}
 \end{aligned} \right\} \tag{35}$$

with  $|\mathcal{I}| = v + 1$  the cardinality of  $\mathcal{I}$ , and

$$\begin{aligned}
 \delta_{e,i} &= \Delta_{e,i} \\
 \delta_{e,i+1} &= k_i e^{-\lambda_{i+1} \tau_d} \Delta_{e,i} + \Delta_{e,i+1} \\
 \delta_{e,i+2} &= k_i k_{i+1} e^{-(\lambda_{i+1} + \lambda_{i+2}) \tau_d} \Delta_{e,i} \\
 &\quad + k_{i+1} e^{-\lambda_{i+2} \tau_d} \Delta_{e,i+1} + \Delta_{e,i+2} \\
 &\quad \vdots \\
 \Delta_{e,i+h} &= e^{-\lambda_{i+h} \tau_d} \mathcal{V}_{se,i+h}(t_{i+h}) - \mathcal{V}_{se,i+h}(t'_{i+h}), \\
 h &= 0, \dots, v-1.
 \end{aligned}$$

It is now sufficient to show that for every pair of time intervals  $T_i, T_{i+v}$ , in which  $\mathcal{H}$  is in  $q_i$ , one has

$$\mathcal{V}_{s,i}(t_{i+v}) - \mathcal{V}_{s,i}(t_i) \leq -W_i(t_i)$$

for a (family) of positive definite continuous functions  $W_i, i \in \mathcal{I}$ . For instance, if one takes  $W_i(t_i) = \epsilon \mathcal{V}_{sm,i}(t_i)$ , for an arbitrarily small  $\epsilon > 0$ , the inequality

$$\begin{aligned}
 &\mathcal{V}_{sm,i}(t_{i+v}) - \mathcal{V}_{sm,i}(t_i) \\
 &\leq \left( k_i k_{i+1} \dots k_{i+v-1} e^{-(\lambda_i + \lambda_{i+1} + \dots + \lambda_{i+v-1}) \tau_d} - 1 \right) \\
 &\quad \times \mathcal{V}_{sm,i}(t_i) + k_{i+v-1} \delta_{e,i+v-1} \\
 &\leq -\epsilon \mathcal{V}_{sm,i}(t_i) + k_{i+v-1} \delta_{e,i+v-1}
 \end{aligned}$$

is verified if

$$k_i k_{i+1} \dots k_{i+v-1} e^{-(\lambda_i + \lambda_{i+1} + \dots + \lambda_{i+v-1}) \tau_d} - 1 \leq -\epsilon.$$

Since  $\epsilon > 0$  is arbitrarily small, one ensures that the previous inequality is verified when

$$k_i k_{i+1} \dots k_{i+v-1} e^{-(\lambda_i + \lambda_{i+1} + \dots + \lambda_{i+v-1}) \tau_d} - 1 < 0,$$



i.e.,

$$\lambda_i + \lambda_{i+1} + \dots + \lambda_{i+v-1} > \frac{1}{\tau_d} \ln(k_i k_{i+1} \dots k_{i+v-1}).$$

If we choose  $\lambda_{s,i} = \dots = \lambda_{s,i+v-1} = \lambda_s$ , under the condition

$$\lambda_s > \frac{1}{\tau_d \Delta_i} \ln(k_i k_{i+1} \dots k_{i+v-1}),$$

$$\Delta_i = \frac{1}{k_{2,i}} + \frac{1}{k_{2,i+1}} + \dots + \frac{1}{k_{2,i+v-1}}$$

we ensure the uniform exponential convergence of  $z_m$  to zero in a practical sense. Finally, to show that the regulation problem is solved, note that for  $t \in T_{i+v} = [t_{i+v}, t'_{i+v}]$ ,  $\dot{\mathcal{V}}_{s,i+v} \leq -\lambda_{i+v} \mathcal{V}_{s,i+v}$ , so that, with the same arguments

$$\begin{aligned} k_{1,i+v} \|z_m(t)\|^2 &\leq \mathcal{V}_{sm,i+v}(t) \leq \mathcal{V}_{s,i+v}(t) \leq e^{-\lambda_{i+v}(t-t_{i+v})} \\ &\quad \times (\mathcal{V}_{sm,i+v}(t_{i+v}) + \mathcal{V}_{se,i+v}(t_{i+v})) \\ &\leq e^{-\lambda_{i+v}(t-t_{i+v})} (\bar{k} \mathcal{V}_{sm,i}(t_i) + \delta_{e,i+v}) \\ &\leq e^{-\lambda_{i+v}(t-t_{i+v})} (\bar{k} k_{2,i} \|z_m(t_i)\|^2 + \delta_{e,i+v}), \end{aligned}$$

where the last of (35) has been used,  $\bar{k} = k_i k_{i+1} \dots k_{i+v-1} e^{-(\lambda_i + \lambda_{i+1} + \dots + \lambda_{i+v-1})\tau_d}$ ,  $\delta_{e,i+v} = k_{i+v-1} \delta_{e,i+v-1} + \mathcal{V}_{se,i+v}(t_{i+v})$ . Hence, using the Bernoulli's inequality (see appendix 1), for  $t \in T_{i+v}$

$$\begin{aligned} \|z_m(t)\| &\leq \sqrt{\frac{\delta_{e,i+v}}{k_{1,i+v}}} e^{-(\lambda_{i+v}/2)(t-t_{i+v})} \left(1 + \frac{\bar{k} k_{2,i}}{\delta_{e,i+v}} \|z_m(t_i)\|^2\right)^{1/2} \\ &\leq e^{-\lambda(t-t_{i+v})} (k \|z_m(t_i)\| + \delta) \leq k e^{-\lambda(t-t_{i+v})} \|z_m(t_i)\| + \delta \end{aligned}$$

$k = (1/2)(\bar{k} k_{2,i})/(\delta_{e,i+v})\delta \|z_m(t_i)\|$ ,  $\lambda = (\lambda_{i+v})/2$ ,  $\delta = \sqrt{(\delta_{e,i+v})/(k_{1,i+v})}$ , where  $t_i$  is the initial time instant. Note that if  $t_{i+v} = \infty$ , then one can prove the uniform exponential stability.

### 5. Simulation results

The control law has been implemented on a digital computer to test the performance. A maximal current of 30 A, and a maximal dissipated power of 1 kW have been considered. In the following we give the values of

the system's parameters.

$$\begin{aligned} k &= 1.17 \times 10^5 \text{ N/m} & b &= 6 \text{ Ns/m} \\ M &= 0.1054 \text{ Kg} & N &= 50 \\ L_{p,1} &= 4.8000 \times 10^{-6} & L_{p,2} &= 8.0745 \times 10^{-6} \text{ H} \\ R_{p,1} &= 0.0451 \Omega & R_{p,2} &= 0.0234 \Omega \\ R_1 &= 0.2040 \Omega & R_2 &= 0.2440 \Omega \\ r_{l,1} &= 0.57813 & r_{l,2} &= 0.452 \\ \mathcal{N}_{j0} &= 31.66 & n_j &= 8.64 \times 10^3 \\ a_j &= b_j p_{j1} e^{-p_{j2} r_{l,j} \rho} & b_j &= (-1)^j r_{l,j} p_{j2} \\ c_j &= (-1)^j r_{l,j} p_{j3} & d_j &= p_{j1} + r_{l,j} p_{j3} \\ \rho &= 0.004 \text{ m} \end{aligned}$$

$j = 1, 2$ , and (in Nm/Wb<sup>2</sup>)

$$\begin{aligned} p_{11} &= 1.5167 \times 10^6, & p_{21} &= 1.5330 \times 10^6 \\ p_{12} &= 1.1473 \times 10^3, & p_{22} &= 996.5755 \\ p_{13} &= 3.8869 \times 10^8, & p_{23} &= 1.8226 \times 10^8. \end{aligned}$$

The control law has been determined setting  $Q = 2 \times 10^2 I$ ,  $k_3 = 10^{10}$ . Figures 7–9 summarize the simulation results. After a short transient, the tracking error for  $x_v$  is of the order  $10^{-6}$  m, and the seating velocity results to be about 0.01 m/s. This implies the respect of the control requirements.

### 6. Conclusions

In this paper a controller has been designed for a camless engine, in which the main control problem is represented by the so-called soft landing. The approach follows the regulation theory. The main limitation of the derived controller is the use of the whole state. In fact, a technological request is the elimination of the valve position sensor. We are currently working on an extension of the proposed controller to a dynamic one, which only uses output measurements.

### Appendix 1

In this appendix we prove some useful relations about  $\kappa_{sj}$ ,  $j = 1, 2$ . Let us consider first

$$\begin{aligned} 0 < \frac{\mathcal{R}'(y_1)}{\mathcal{R}'(y_2)} &= \frac{ae^{-by_1} + c}{ae^{-by_2} + c} = 1 + z, \\ z &= \frac{e^{-by_1} - e^{-by_2}}{e^{-by_2} + c/a} > -1 \end{aligned}$$

$y_1, y_2 \in [-\rho, \rho]$ . Note that

$$\begin{aligned} e^{-by_1} - e^{-by_2} &= \sum_{k=1}^{\infty} (-1)^k \frac{b^k}{k!} (y_1^k - y_2^k) \\ &= -b(y_1 - y_2) \sum_{k=1}^{\infty} (-1)^{k-1} \frac{b^{k-1}}{(k-1)!} \frac{S_{k-1}}{k} \\ &\leq |b| |y_1 - y_2| \sum_{i=0}^{\infty} \frac{|b|^i}{i!} \rho^i \\ &= |b| |y_1 - y_2| e^{|b|\rho} \end{aligned}$$

with  $S_{k-1} = \sum_{h=0}^{k-1} y_1^{k-1-h} y_2^h$ ,  $|S_{k-1}| \leq k\rho^{k-1}$ . Hence, the Bernoulli's inequality

$$\begin{aligned} (1+z)^r &\leq 1+rz, \quad \forall r \in [0, 1] \subset \mathbb{R}, \\ \forall z > -1, \quad z \in \mathbb{R} \end{aligned}$$

can be applied with  $r = 1/2$ , obtaining

$$\begin{aligned} (1+z)^{1/2} &\leq 1 + \frac{1}{2}z \leq 1 + \alpha_s |y_1 - y_2| \leq 1 + \alpha_s \left\| \begin{matrix} y_1 - y_2 \\ \dot{y}_1 - \dot{y}_2 \end{matrix} \right\|, \\ \alpha_s &= \frac{1}{2} \frac{|b| e^{|b|\rho}}{e^{-|b|\rho} + c/a}. \end{aligned}$$

Using this relation, if  $\kappa_{sj} \geq 1$  (this happens when  $\pi_{x_v} \geq x_v$  if  $j=1$ , and  $\pi_{x_v} \leq x_v$  if  $j=2$ )

$$\begin{aligned} \kappa_{sj} &= \left[ \frac{\mathcal{R}'_j(\pi_{x_v})}{\mathcal{R}'_j(x_v)} \right]^{1/2} = \left[ 1 + \frac{e^{-b_j \pi_{x_v}} - e^{-b_j x_v}}{e^{-b_j x_v} + c_j/a_j} \right]^{1/2} \\ &\leq 1 + \alpha_{sj} \left\| \begin{matrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{matrix} \right\| \end{aligned}$$

namely

$$\kappa_{sj} - 1 \leq \alpha_{sj} \left\| \begin{matrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{matrix} \right\|$$

while if  $\kappa_{sj} < 1$  (this happens when  $\pi_{x_v} < x_v$  if  $j=1$ , and  $\pi_{x_v} > x_v$  if  $j=2$ )

$$\begin{aligned} \kappa_{sj} &= \frac{1}{\left[ \frac{\mathcal{R}'_j(x_v)}{\mathcal{R}'_j(\pi_{x_v})} \right]^{1/2}} = \frac{1}{\left[ 1 + \frac{e^{-b_j x_v} - e^{-b_j \pi_{x_v}}}{e^{-b_j \pi_{x_v}} + c_j/a_j} \right]^{1/2}} \\ &\geq \frac{1}{1 + \alpha_{sj} \left\| \begin{matrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{matrix} \right\|} \end{aligned}$$

so that

$$1 - \kappa_{sj} \leq \frac{\alpha_{sj} \left\| \begin{matrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{matrix} \right\|}{1 + \alpha_{sj} \left\| \begin{matrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{matrix} \right\|} \leq \alpha_{sj} \left\| \begin{matrix} x_v - \pi_{x_v} \\ v_v - \pi_{v_v} \end{matrix} \right\|.$$

### Appendix 2

The coefficients in (20), satisfying (21), are given by

$$\begin{aligned} c_{r,10} &= -\rho \left( 1 - 70 \frac{\vartheta_0^4}{\Delta^4} - 168 \frac{\vartheta_0^5}{\Delta^5} - 140 \frac{\vartheta_0^6}{\Delta^6} - 40 \frac{\vartheta_0^7}{\Delta^7} \right) \\ c_{r,11} &= -\frac{280}{\Delta} \rho \frac{\vartheta_0^3 (\Delta + \vartheta_0)^3}{\Delta^3} \\ c_{r,12} &= \frac{840}{\Delta^2} \rho \frac{\vartheta_0^2}{\Delta^2} \left( 1 + 4 \frac{\vartheta_0}{\Delta} + 5 \frac{\vartheta_0^2}{\Delta^2} + 2 \frac{\vartheta_0^3}{\Delta^3} \right) \\ c_{r,13} &= -\frac{1680}{\Delta^3} \rho \left( \frac{\vartheta_0}{\Delta} + 6 \frac{\vartheta_0^2}{\Delta^2} + 10 \frac{\vartheta_0^3}{\Delta^3} + 5 \frac{\vartheta_0^4}{\Delta^4} \right) \\ c_{r,14} &= \frac{1680}{\Delta^4} \rho \left( 1 + 12 \frac{\vartheta_0}{\Delta} + 30 \frac{\vartheta_0^2}{\Delta^2} + 20 \frac{\vartheta_0^3}{\Delta^3} \right) \\ c_{r,15} &= -\frac{20160}{\Delta^5} \rho \left( 1 + 5 \frac{\vartheta_0}{\Delta} + 5 \frac{\vartheta_0^2}{\Delta^2} \right) \\ c_{r,16} &= \frac{100800}{\Delta^6} \rho \left( 1 + 2 \frac{\vartheta_0}{\Delta} \right) \\ c_{r,17} &= -\frac{201600}{\Delta^7} \rho \end{aligned}$$

with  $\Delta = \vartheta_{c1} - \vartheta_{c0}$ .

### References

M.S. Ashhab, A.G. Stefanopoulou, J.A. Cook and M. Levin, "Camless engine control for robust unthrottled operation", *SAE*, New York, SAE Paper 960 581, 1996.  
 C. Gray, "A review of variable engine valve timing", *SAE*, New York, SAE Paper 880 386, 1988.  
 W. Hoffmann and A.G. Stefanopoulou, "Iterative learning control of electromechanical camless valve actuator", *IEEE Trans. Contr. Syst. Technol.*, 11, pp. 174-184, 2003.  
 A. Isidori, *Nonlinear Control Systems*, Third Edition, London: Springer-Verlag, 1995.  
 A. Isidori and C.I. Byrnes, "Output regulation of nonlinear systems", *IEEE Trans. Autom. Contr.*, 35, pp. 131-140, 1990.  
 D. Liberzon, *Switching in Systems and Control*, Boston, USA: Birkhäuser, 2003.  
 J. Lygeros, C. Tomlin and S. Sastry, "Controllers for reachability specifications for hybrid systems", *Automatica*, Special Issue on Hybrid Systems, 35, pp. 349-370, 1999.  
 M. Marchi, A. Palazzi and M. Panciroli, "Innovative valve control (IVC) model", *Variable Ventilsteuerung: Ein Verfahren zur Reduzierung von Kraftstoffverbrauch und Emissionen*, S. Pischinger, Ed., Expert Verlag, Renningen, pp. 114-129, Expert Verlag, Renningen, 2002.  
 M. Montanari, F. Ronchi, C. Rossi and A. Tonielli, "Control of a camless engine electromechanical actuator: position reconstruction

- and dynamic performance analysis", *IEEE Trans. Ind. Electr.*, 51, pp. 299–311, 2004.
- K. Peterson, A. Stefanopoulou, Y. Wang and M. Haghgoie, "Nonlinear self-tuning control for soft landing of an electromechanical valve actuator", *Proceedings of the 2nd IFAC Conference on Mechatronic Systems*, Berkeley, California, USA, 9–11 December 2002, pp. 207–212, 2002a.
- K. Peterson, A. Stefanopoulou, T. Megli and M. Haghgoie, "Output observer based feedback for soft landing of electromechanical camless valvetrain actuator", *Proceedings of the American Control Conference 2002*, 2, pp. 1413–1418, 2002b.
- K. Peterson, A. Stefanopoulou and Y. Wang, "Control of electromechanical actuators: valves tapping in rhythm", in *Multidisciplinary Research in Control: The Mohammed Dahleh Symposium 2002*, Lecture Notes in Control and Information Sciences, Vol. 289, Berlin/Heidelberg: Springer, 2003.
- M. Pischinger, W. Salber, F. van der Staay, H. Baumgarten and H. Kemper, "Benefits of electromechanical valve train in vehicle operation", *SAE*, New York, SAE Paper 2000-01-1223, 2000.
- F. Ronchi, C. Rossi and A. Tilli, "Sensing devices for camless engine electromagnetic actuators", *IEEE Conference of the Industrial Electronics Society*, 2, pp. 1669–1674, 2002.
- M. Schechter and M.B. Levin, "Camless engine", *SAE*, New York, SAE Paper 860 581, 1996.
- C. Tai, A. Stubbs and T.-C. Tsao, "Modeling and controller design of an electromagnetic engine valve", *Proceedings of the 2001 American Control Conference*, Arlington VA, USA, 25–27 June 2001, pp. 2840–2895.
- Y. Wang, T. Megli, M. Haghgoie, K.S. Peterson and A.G. Stenopoulou, "Modeling and control of electromechanical valve actuator", *SAE*, New York, SAE Paper 2002-01-1106, 2002.