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On the fault diagnosis problem for non–linear systems: A fuzzy sliding–mode observer approach[1]

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Abstract. In this paper we propose a solution to the model–based fault diagnosis problem for the class of non–linear dynamic systems subjected to be described by a Takagi–Sugeno fuzzy model. A fuzzy observer is designed to estimate the system's state vector and to derive a diagnostic signal–residual. The residual is generated by the comparison of the measured and the estimated outputs. The proposed scheme has been satisfactorily tested in simulation and in a real–time benchmark given by a Two–Tank Hydraulic System.

Keywords: Fault diagnosis, Fuzzy system, Takagi–Sugeno fuzzy models, Fuzzy Observers.

1. Introduction

Although the automation of processes by means of automatic control has allowed the reduction of the exposition of human operators to potentially hazardous manual operations, repetitive tasks and unsafe environments, it does not avoid the appearance of fault events, since faults in their components are inherent problems associated with the physical nature of dynamic systems. An immediate consequence of the appearance of faults are the negative effects on the system performance. Thus, the availability, cost efficiency, reliability, operating safety and environmental protection are very important characteristics in modern control systems. For critical safety systems, the consequences of faults can be extremely serious in terms of human mortality, environmental impact and economic losses. Therefore, there is an increasing need of schemes of supervision and fault diagnosis to increase the reliability of such systems.

Many of the initial works related to fault diagnosis deal with fault detection in linear systems. A variety of techniques have been used to deal with the problem, e.g. non–linear approaches and artificial intelligence techniques. In the last decade, robust techniques of fault diagnosis have been studied and several applications can be found in the literature [4], [7], [6], [1], [14].

The different methods related to fault diagnosis can be gathered in three areas: signal analysis–based or statistical methods; input/output information knowledge–based methods and model–based methods [13], [16], [7], [8]. The *signal analysis–based methods* use statistical techniques or data mining. These have been used in applications of Power Electrical Systems, where a fault–free power system is compared in line with the

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current system. In the following, one can determine if the faults appear in the power electrical system by means of statistical analysis. The *Principal Component Analysis* (PCA) is the best known statistical technique, and has been widely used in the industrial process monitoring. This technique allows reducing the dimension of the plant model by using linear dependencies among the variables of the model [11]. The *input/output information knowledge methods* are classification methods. The most used example of this technique is the Artificial Neural Network (ANN). An ANN exhibits suitable characteristics to deal with the fault diagnosis problem, due to its learning capability and its ability of modeling an uncertain non–linear process. The *model–based approach* to fault diagnosis in dynamic processes has been receiving considerable attention since the beginning of the 1970s, both in the research context and in the domain of applications on real processes [4], [13]. The main idea of the model–based approach is the determination of faults, appearing in a dynamic system, from the comparison of available measurements to a prior information, represented by its mathematical model. From this process, comparison signals, known as *residuals,* are generated. These signals provide information about the faults in the system. In this paper, we follow this approach.

A structure commonly accepted for the model–based fault diagnosis is shown in Figure 1. In this scheme, the *residual generation* subsystem provides a diagnostic signal, the residual, which depends only on the faults

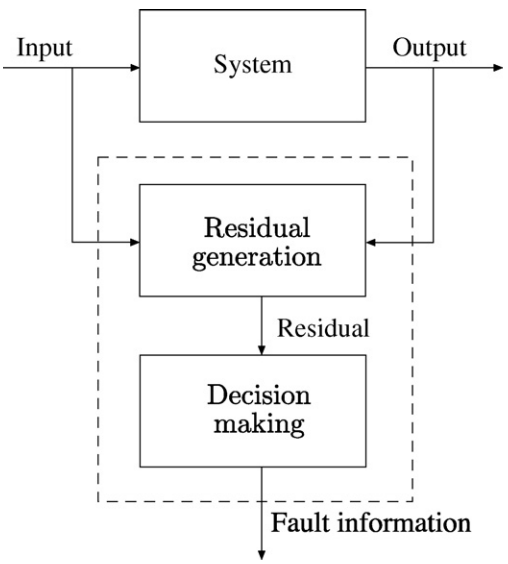

Fig. 1. General structure of the model–based fault diagnosis

and not on the inputs, while in the *decision making* subsystem, the residuals are examined regarding the likelihood of a fault. A decision rule is then applied to determine if the fault appears. A threshold value is generally used to guarantee robustness.

Most of the model–based fault diagnosis methods are based on linear system models and, traditionally, the fault diagnosis problem for non–linear dynamic systems is analyzed in two steps: first, the model is linearized around a desired operating point, and then a specific linear technique is applied to generate a diagnostic signal, e.g. Kalman filters, observers, parity relations, parameter estimation, etc. [4], [11]. However, since the behavior of many engineering systems exhibits nonlinearities, nonlinear models are more likely to be necessary for FDI purpose, since linear models are only valid in a local region around an equilibrium point. Motivated by this reason, in the last years many efforts have been made for the use of non–linear system techniques for fault diagnosis. For nonlinear systems, one difficulty results from the presence of non measured states, and two different approaches for dealing with this problem have been proposed, namely elimination and estimation. The estimation, through dynamical observers, has been addressed for example in [9], [17], [12], among many others. In particular, in [10] a decoupling strategy for a non–linear system is used to derive several subsystems, containing information about specific faults of the system in consideration. Once these subsystem are determined, a Luenberger fuzzy observer is implemented to generate the residuals. Parametric variations are not considered in this approach. In [6] a fuzzy observer is used to reconstruct the fault, rather than to detect its appearance through a residual signal. On the other hand, the results presented in [1] involve a fuzzy multiple observer, where the single components are fuzzy observers having the form presented in [19]. This multiple observer is capable of reconstructing the state and output vectors of a system, when some inputs are unknown. It is worth noting that, in this specific approach, the fault information is not distinguishable through the residuals.

A fault diagnosis method using only output information could give incorrect information on the faults, when the inputs of the system change. A way to circumvent this problem, affecting the model–based fault diagnosis methods, is to use the residual–generation concept, in which the inputs and outputs are used to generate a fault indicator.

In this paper, we propose a model–based approach with fuzzy observers in order to deal with the fault diagnosis problem. Roughly speaking, the proposed approach consists of obtaining a Takagi–Sugeno fuzzy model [18] of the non–linear system, and then designing fuzzy observers to estimate the system state vector. The diagnostic signal–residual is finally generated by the comparison between the measured and the estimated outputs. This approach allows determining a diagnostic signal which is insensible to parametric variations in a neighborhood of the nominal parameter values, and sensible only to the fault signal.

The paper is organized as follows: In section 2 some basic concepts on fault diagnosis theory, the problem definition and a way to solve it are shown. In section 3 we describe an application example. In section 4 we report the simulation and experimental results, and finally in section 5 we present some conclusions.

2. Observer–based fault diagnosis

In general, a *fault* will be considered as a change in the behavior of the system due to external inputs exceeding the limits of a pre-specified tolerance. Therefore, the fault diagnosis concept will be referred to as the problem of detecting and locating the fault, namely, not only merely recognize the presence of a fault, but also identify on which component of the system the fault has appeared. This is formally defined as the *Fault Detection and Isolation* (FDI) problem.

A traditional approach to fault diagnosis is based on hardware redundancy methods, which use multiple range sensors, actuators, computers and software to measure and/or control a particular variable. It is possible however to use different measured values and their combinations instead of duplicating each component individually to smooth the conflict between the reliability and the cost due to the additional components. This is the concept defined as functional or analytical redundancy or model based approach, because it takes advantage of the redundant analytical relationships between several measured variables in the monitored process [11].

The major advantage of the model–based approach is that no additional hardware is needed in order to perform the fault detection and identification algorithm since the analytical redundancy uses a mathematical model of the original system. The resulting signal $r(t) = y(t) - \tilde{y}(t)$ generated from the comparison of the measured and the estimated outputs is called *symptom* or *residual*. The absolute value of this residual should be close enough to zero when the system is in normal operating condition, namely should enter in a finite time T_B a ball $\mathcal{B}_B$ of radius B of the origin, while should diverge from zero, and leave $\mathcal{B}_B$ in finite time, when a fault $f(t)$ occurs in the system. Therefore, this property of the residual can be used to determine whether or not an abnormal behavior should be considered as a fault [5], [8], i.e. the residual must satisfy the following condition

$$
\begin{aligned}
|r(t)| &\approx 0 \qquad \text{if } f(t) = 0 \text{ (normal operation)} \\
|r(t)| &\gg 0 \qquad \text{if } f(t) \neq 0 \text{ (faulty operation).}
\end{aligned}
$$

We say that the residual is *sensitive* to a specific set of faults. In this sense, a desirable property of the residual is to be insensitive or robust to parametric variations in a neighborhood of the nominal values, namely, these parametric variations should not be confused with a fault.

Figure 1 shows that an essential problem in the model–based FDI is the generation of the diagnostic signal–residual, because if the algorithm that generates the residuals is not correctly designed, important information about the faults could be lost. This has motivated the study and proposal of different methods for the residuals generation, like Kalman filters, Luenberger observers and fuzzy observers [11]. In particular, the observer–based approach consists of the appropriate construction of an observer of the states of the system, generally in a fault–free situation, which provides an estimation of the output of the system that is compared with the measured output in order to generate the residuals to be used to detect a fault. The residual generation scheme is depicted in Figure 2. The basic idea is to eliminate every input component from the output so that the output depends only on the component related to the fault, i.e., to construct a fuzzy observer which provides a desirable estimation of the output of the system, to

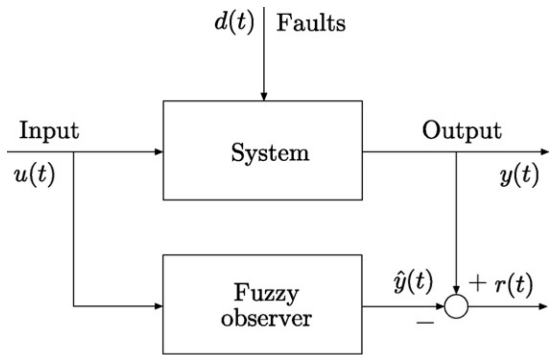

Fig. 2. Residual generation scheme with fuzzy observer

generate a residual which allows the proper identification of the fault and its localization by means of some suitable algorithm.

As discussed previously, the model–based approach needs a mathematical model of the system to be observed. In the case of non–linear dynamic systems, the design of observers is not in general an easy task due to the nonlinearities of the model. Several approaches have been proposed to deal with this important problem. One of the recent approaches consists of achieving an approximation of the non–linear system behavior in terms of an aggregation of linear dynamics calculated around some interesting points in the state–space, and then calculate an observer for each linear submodel. In this context, the Takagi–Sugeno (TS) fuzzy modeling provides a systematic way of obtaining a set of linear models that describe, at least locally, the behavior of the nonlinear dynamics [18].

In the following we develop a method based on output fuzzy observers. These observers are very useful and have several advantages, among which the possibility of working with reduced observation error dynamics, a finite time convergence for all the observable states and robustness under parameter variations. More precisely, let us consider nonlinear system described by

$$\begin{aligned} \dot{x} &= f(x, u, d, \mu) \\ y &= h(x, \mu) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state of the system, $u(t) \in \mathbb{R}^m$ is the input signal, $d(t) \in \mathbb{R}^q$ is an unknown input vector usually containing external disturbances or signals reflecting faults affecting the system, $y(t) \in \mathbb{R}^m$ is a measurable output signal, and $\mu \in \mathbb{R}^\nu$ is a vector of the system parameters subject to change.

The TS fuzzy model is described by a set of fuzzy IF–THEN rules which represent local linear input–output relations of a non–linear system. The main feature of a TS fuzzy model is the ability of expressing the local dynamic of each fuzzy implication (rule) by a linear subsystem [18]. In other words, suppose that it is possible to describe locally the input–output behavior of system (1) by a TS fuzzy dynamic model described by the following r rules

Plant rule i:

IF z_1 is M_{1i} and $\cdots$ and z_p is M_{pi}

$$\text{THEN } \Sigma_i\colon \begin{cases} \dot{x} = A_i x + B_i u + E_i d + \delta_i \\ y_i = C_i x + \Delta_i, \quad i = 1, \cdots, r \end{cases}$$

where $z_1, \cdots, z_p$ are measurable premise variables which may coincide with some states or a combination of them, δ_i, Δ_i are functions due to the variation of the parameter vector μ with respect to the nominal value μ_0, A_i, B_i, E_i, C_i are the nominal matrices, i.e. corresponding to $\mu = \mu_0$, M_{ji} are the fuzzy sets and the linear subsystems are obtained from some knowledge of the dynamics on the process.

For a given triplet (x, u, d), the aggregate fuzzy model is obtained by using a singleton fuzzifier, product inference and center of gravity defuzzifier, resulting in the following description

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{r} \omega_i A_i x + \sum_{i=1}^{r} \omega_i B_i u + \sum_{i=1}^{r} \omega_i E_i d + \sum_{i=1}^{r} \omega_i \delta_i \\ y &= \sum_{i=1}^{r} \omega_i C_i x + \sum_{i=1}^{r} \omega_i \Delta_i \end{aligned} \tag{2}$$

where ω_i is the normalized weight for each rule calculated from the membership functions for z_j in M_{ji} and satisfying $\omega_i = \omega_i(z) \geq 0$ and

$$\sum_{i=1}^{r} \omega_i(z) = 1 \tag{3}$$

with $z = \begin{pmatrix} z_1 & \cdots & z_p \end{pmatrix}^T$.

For this system, we can formulate the *Observed–Based Fault Detection and Isolation Problem* (OFDIP) which consists of finding an observer

$$\begin{aligned} \dot{\xi} &= \gamma(\xi, u, y) \\ \hat{y} &= \theta(\xi) \end{aligned}$$

such that the absolute values of the residuals converge in a finite time T_B to a ball $\mathcal{B}_B$ of radius B of the origin, when the system is in normal operating condition, while leave $\mathcal{B}_B$ in finite time when a fault occurs in the system. Moreover, the residuals have to be insensitive to parametric variations in a neighborhood of the nominal values of the parameters of the system. In the following section we will determine such an observer considering a family of sliding mode observers.

2.1. Sliding mode fuzzy observers for the Takagi–Sugeno model

According to the previous section we note that the requirement of observability of the nonlinear system is a necessary condition for generating the residuals. Taking advantage of the TS description of the dynamics of this nonlinear system, we will assume that it is possible to construct local observers for each linear subsystem. This motivates the following assumptions.

(H_1) The pairs $(C_i, A_i), i = 1, \cdots, r$, are detectable. $\diamond$

(H_2) There exist $\delta_{\max}$, $\Delta_{\max}$ such that $\|\delta_i\| \leq \delta_{\max}$, $\|\Delta_i\| \leq \Delta_{\max}$, for each $i = 1, \cdots, r$. $\diamond$

We propose the design of an aggregate sliding mode fuzzy observer structure based on local fuzzy observers, with each local observer associated with each fuzzy rule given as

Observer rule i:

IF z_1 is M_{1i} and $\cdots$ and z_p is M_{pi}

$$\text{THEN } \hat{\Sigma}_i : \begin{cases} \dot{\xi} = A_i\xi + B_i u + L_i(y - \hat{y}) + \varphi_i \\ \hat{y} = C_i\xi, \quad i = 1, \cdots, r \end{cases}$$

where φ_i is a discontinuous vector to be determined later on. We assume also that the premise variables do not depend on the estimated state variables.

We use the idea of the Parallel Distributed Compensation (PDC) proposed in [18], where the overall state estimation is a combination of individual local observer outputs. The overall observer dynamics will be then a weighted sum of individual linear observers, namely

$$\begin{aligned} \dot{\xi} &= \sum_{i=1}^{r} \omega_i A_i \xi + \sum_{i=1}^{r} \omega_i B_i u + \sum_{i=1}^{r} \omega_i L_i (y - \hat{y}) + \sum_{i=1}^{r} \omega_i \varphi_i \\ \hat{y} &= \sum_{i=1}^{r} \omega_i C_i \xi \end{aligned} \tag{4}$$

where the weights are the same as those used in the aggregate fuzzy model of the nonlinear system (2). To analyze the convergence of the fuzzy observer, the state estimation error is defined as $e = x - \xi$. Using (2) and (4) we get

$$\begin{aligned} \dot{e} = &\sum_{i=1}^{r}\sum_{j=1}^{r} \omega_i\omega_j \bar{A}_{ij} e + \sum_{i=1}^{r} \omega_i \Big[E_i d + \delta_i - \varphi_i \Big] \\ &- \sum_{i=1}^{r}\sum_{j=1}^{r} \omega_i\omega_j L_i \Delta_j \end{aligned} \tag{5}$$

with $\bar{A}_{ij} = A_i - L_i C_j$. The convergence of the estimation error is expressed in the following result.

Theorem 2.1. *Consider the TS fuzzy model (2), and suppose that there exist positive definite matrices N_i, F_i and P satisfying the following linear matrix inequalities (LMI's)*

$$\begin{aligned} &A_i^T P + PA_i - C_i^T N_i^T - N_i C_i < 0, && i = 1, \cdots, n \\ &A_i^T P + PA_i - C_j^T N_i^T - N_i C_j + A_j^T P + PA_j - C_i^T N_j^T - N_j C_i < 0, && i < j \leq r. \end{aligned} \tag{6}$$

Then, under (H_1), (H_2), if the faulty signal d satisfies the condition of normal operation

$$\|d\| \leq d_b \tag{7}$$

then there exists a sliding mode fuzzy observer (4), with

$$\varphi_i = \begin{cases} k\|E_i\| P^{-1} \bar{C}^T \dfrac{\bar{C}e}{\|\bar{C}e\|} & if\, \|\bar{C}e\| > \varepsilon \\ k\|E_i\| P^{-1} \bar{C}^T \dfrac{\bar{C}e}{\varepsilon} & if\, \|\bar{C}e\| \leq \varepsilon \end{cases} \tag{8}$$

$\varepsilon > 0$ fixed, $\bar{C} = \sum_{i=1}^{r} \omega_i C_i$, $\|\bar{C}e\| = \|y - \hat{y}\|$,

$$\begin{aligned} &k = \frac{D_{\max}}{2\kappa_{\min}\varepsilon} > 0, && D_{\max} = \kappa_{\max}\|\Lambda^{-1}\| d_b^2 + \|\Lambda_\delta^{-1}\|\delta_{\max}^2 + \kappa_{\Delta,\max}\|\Lambda_\Delta^{-1}\|\Delta_{\max}^2 \\ &\kappa_{\min} = \min_{i=1,\cdots,r} \|E_i\|, && \kappa_{\max} = \max_{i=1,\cdots,r} \|E_i\|^2, \qquad \kappa_{\Delta,\max} = \max_{i=1,\cdots,r} \|L_i\|^2 \end{aligned} \tag{9}$$

guaranteeing practical convergence of the estimation error in finite time

$$T_B = t_0 + \ln\left(\frac{r\delta_C}{B}\,\|e_0\|\right)^{1/\alpha}, \quad r = \sqrt{\frac{\lambda_{\max}^P}{\lambda_{\min}^P}}, \quad \delta_C = \sum_{i=1}^{r} \|C_i\| \tag{10}$$

where t_0 is the initial time instant, $\lambda_{\min}^P$, $\lambda_{\max}^P$ are the minimum and maximum eigenvalue of P, with estimation output error bound

$$B = r\varepsilon + \Delta_{\max}. \tag{11}$$

If the faulty signal d violates the normal operation condition (7), then estimation error exits the ball $\mathcal{B}_B$ in finite time and the residual detection is activated. ◇

Proof. Let us consider the following Lyapunov candidate function $V = e^T Pe$, with $P = P^T > 0$. Deriving this function and using (5) we get

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{r}\sum_{j=1}^{r} \omega_i\omega_j e^T(P\bar{A}_{ij} + \bar{A}_{ij}^T P)e \\ &\quad + 2\sum_{i=1}^{r} \omega_i \left[e^T PE_i d + e^T P\delta_i - e^T P\varphi_i\right] \\ &\quad - 2\sum_{i=1}^{r}\sum_{j=1}^{r} \omega_i\omega_j e^T PL_i\Delta_j. \end{aligned} \tag{12}$$

In the following we use the Young inequality

$$X^T Y + Y^T X \leq X^T \Lambda X + Y^T \Lambda^{-1} Y$$

for matrices $X, Y \in \mathbb{R}^{n\times k}$, and $\Lambda \in \mathbb{R}^{k\times k}$, $\Lambda = \Lambda^T > 0$ [21]. Hence, from (12) one gets

$$\begin{aligned}\dot{V} &\leq \sum_{i=1}^{r}\sum_{j=1}^{r} \omega_i\omega_j\, e^T(P\bar{A}_{ij} + \bar{A}_{ij}^T P)e \\ &\quad + e^T P(\Lambda + \Lambda_\delta + \Lambda_\Delta)Pe \\ &\quad + \sum_{i=1}^{r} \omega_i d^T E_i^T \Lambda^{-1} E_i d + \sum_{i=1}^{r} \omega_i^T \delta_i \Lambda_\delta^{-1}\delta_i \\ &\quad + \sum_{i=1}^{r}\sum_{j=1}^{r} \omega_i\omega_j \Delta_j^T L_i^T \Lambda_\Delta^{-1} L_i\Delta_j \\ &\quad - 2e^T P\sum_{i=1}^{r} \omega_i\varphi_i \end{aligned} \tag{13}$$

Λ, Λ_δ, Λ_Δ symmetric and positive definite. Under (H_1) and setting $L_i = P^{-1}N_i$, by (6) the first term on the right–hand side of equation (13) is negative definite. Let us set

$$A_i^T P + PA_i - C_i^T N_i^T - N_i C_i = -Q_{ij}$$

with $Q_{ij} = Q_{ij}^T > 0$ thanks to the first of (6).

Let us consider first the case of system normal operation, in which the faulty signal d satisfies (7). Setting

$$\lambda = \gamma_{\min} - \rho, \quad \rho = \|P(\Lambda + \Lambda_\delta + \Lambda_\Delta)P\|$$

with $\gamma_{\min} = \min_{i,j=1,\cdots,r} \lambda_{\min}^{Q_{ij}}$, $\lambda_{\min}^{Q_{ij}}$ the minimum eigenvalue of Q_{ij}, using (3), from (13) one works out

$$\dot{V} \leq -\lambda\|e\|^2 + D_{\max} - 2e^T P\sum_{i=1}^{r} \omega_i\varphi_i$$

where (H_2) and (9) have been used. Note that one can fix the matrices Λ, Λ_δ, Λ_Δ so that $\rho < \gamma_{\min}$.

We have now to distinguish two cases, due to the fact that the observer (4) may determine an error e such that $\|\bar{C}e\| > \varepsilon$ or $\|\bar{C}e\| \leq \varepsilon$. In the first case, choosing the functions φ_i as in (8), for $\varepsilon < \|\bar{C}e\| \leq \|\bar{C}\|\|e\|$

$$\begin{aligned}\dot{V} &\leq -\lambda\|e\|^2 + D_{\max} - 2k\sum_{i=1}^{r} \omega_i\|E_i\|e^T\bar{C}^T\frac{\bar{C}e}{\|\bar{C}e\|} \\ &\leq -\lambda\|e\|^2 + D_{\max} - 2k\kappa_{\min}\varepsilon = -\lambda\|e\|^2. \end{aligned}$$

Therefore, using standard arguments [15], for $\|e\| > \varepsilon/\|\bar{C}\|$ one obtains that the error satisfies

$$\|e\| \leq r\left(\|e_0\|e^{-\alpha(t-t_0)} + \frac{\varepsilon}{\|\bar{C}\|}\right)$$

where $\alpha = \lambda/(2\lambda_{\max}^P)$, e_0 is the error at time t_0. Hence, during the normal operation and under bounded parameter variations, the error is uniformly ultimately bounded with bound $b = r\varepsilon/\|\bar{C}\|$, i.e. one obtains the practical convergence of the estimation error. It is easy to check that the estimation error enters the ball of radius b in a finite time $t_b = t_0 + \ln(\|\bar{C}\|\|e_0\|/\varepsilon)^{1/\alpha}$. Note that $\|\bar{C}\|\|e_0\|/\varepsilon > 1$. Finally, from (2), (4) and using (H_3)

$$\begin{aligned}\|\bar{C}e\| = \|y - \hat{y}\| &= \left\|\sum_{i=1}^{r} \omega_i C_i e + \sum_{i=1}^{r} \omega_i\Delta_i\right\| \\ &\leq r\left(\|\bar{C}\|\|e_0\|e^{-\alpha(t-t_0)} + \varepsilon\right) + \Delta_{\max} \end{aligned}$$

namely, during the normal operation and under bounded parameter variations, $\bar{C}e = y - \hat{y}$ is uniformly ultimately bounded with bound (11), after a finite time

$$\begin{aligned}\bar{T}_B &= t_0 + \ln\left(\frac{r\|\bar{C}\|}{r\varepsilon + \Delta_{\max}}\,\|e_0\|\right)^{1/\alpha} \leq t_0 \\ &\quad + \ln\left(\frac{r\delta_C}{r\varepsilon + \Delta_{\max}}\,\|e_0\|\right)^{1/\alpha} \\ &= T_B, \quad \|\bar{C}\| \leq \delta_C \end{aligned}$$

and hence after a finite time (10). Note that $T_B > t_0$ for $\Delta_{\max} < r(\delta_C \|e_0\| - \varepsilon)$ (recall that here we consider $\|e_0\| > \varepsilon/\|\bar{C}\| > \varepsilon/\delta_C$). This means that the maximal parameter variation has to be small enough so that the ball $\mathcal{B}_B$ does not contain the point e_0.

In the second case, when $\|\bar{C}e\| \leq \varepsilon$, taking φ_i as in (8)

$$\begin{aligned}\dot{V} &\leq -\lambda\|e\|^2 + D_{\max} - 2k\sum_{i=1}^{r}\omega_i\|E_i\|e^T\bar{C}^T\frac{\bar{C}e}{\varepsilon}\\ &\leq -\lambda\|e\|^2 + D_{\max} - 2k\kappa_{\min}\frac{\|\bar{C}e\|^2}{\varepsilon}\\ &= -(1-\vartheta)\lambda\|e\|^2\\ &\quad + \frac{D_{\max}}{\varepsilon^2}\left[\varepsilon^2 - e^T\left(\bar{C}^T\bar{C} + \frac{\varepsilon^2\vartheta\lambda}{D_{\max}}I_{m\times m}\right)e\right]\\ &\leq -(1-\vartheta)\lambda\|e\|^2\end{aligned}$$

for $\|e\| \geq \frac{\varepsilon}{\sqrt{\lambda_{\max}^R}}$, with $\vartheta \in (0, 1)$, and $\lambda_{\max}^R$ the maximum eigenvalue of the matrix

$$R = \bar{C}^T\bar{C} + \frac{\varepsilon^2\vartheta\lambda}{D_{\max}}I_{m\times m}.$$

Therefore, during the normal operation and under bounded parameter variations, the error e is uniformly ultimately bounded with bound $b' = r\varepsilon/\sqrt{\lambda_{\max}^R} \leq b$, as well as the output error $\bar{C}e = y - \hat{y}$ is uniformly ultimately bounded with bound $B' = r\,\varepsilon\|\bar{C}\|/\sqrt{\lambda_{\max}^R} + \Delta_{\max} \leq B$, since $\|\bar{C}\|/\sqrt{\lambda_{\max}^R} \leq 1$.

In conclusion, in both cases $\|\bar{C}e\| > \varepsilon$, $\|\bar{C}e\| \leq \varepsilon$ the estimation output error remains bounded with bound (11) during the normal operation. During faulty operation, condition (7) is violated, and the convergence analysis is not valid anymore, and it is not possible to ensure that the error enters or remains in the ball $\mathcal{B}_B$, and the fault can be detected after a finite time (10). □

Remark. The proof of this theorem allows the construction of the fuzzy observer (4). As clear from the proof, the design parameters are be used to determine a suitable bound d_b for the the external signal signal $d(t)$ in order to be considered a fault. Moreover, if we consider that $d(t)$ can also take into account variations in the system parameters, it is clear that the observer (4) is robust against bounded parameter variations satisfying the normal parameter operation. ◇

3. Model–based fault detection for two–tank hydraulic system

The proposed method for fault diagnosis has been tested on a two–tank hydraulic laboratory system shown in Figure 3. For comparison purposes, numerical

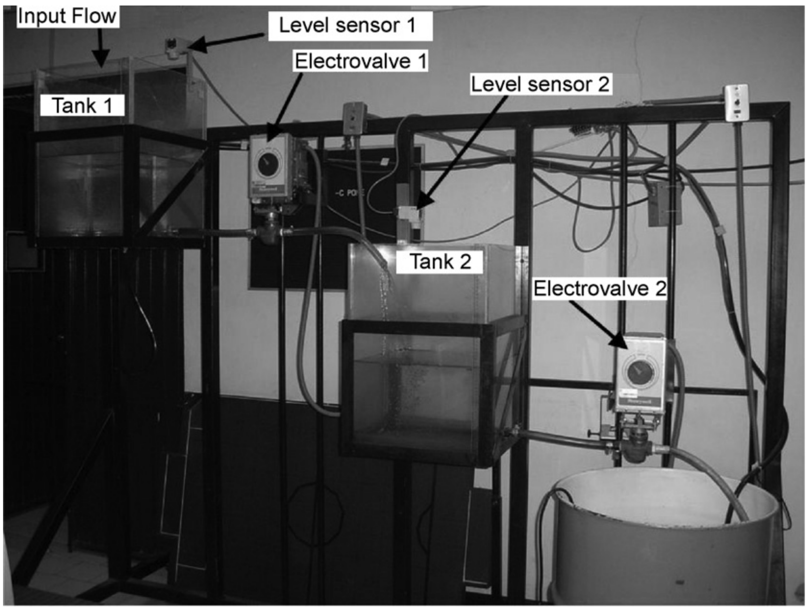

Fig. 3. The two–tank hydraulic system

simulations have been carried out using the mathematical model of the system, and real–time experiments have been performed on the laboratory system. This system consists of two interconnected tanks, two ultrasonic level sensors, two industrial electro–valves connected at the output of the tanks and a pump providing constant supply rate.

The hydraulic system has as inputs the voltage supplied to each one of the electro–valves, and as outputs the liquid levels of each tank. The mathematical model of the system is described by [2]

$$\begin{aligned}
\dot{h}_1 &= \frac{1}{A_t}\left(\phi_e - w_1\sqrt{(1+f_{s1})h_1}\right)\\
\dot{h}_2 &= \frac{1}{A_t}\left(w_1\sqrt{(1+f_{s1})h_1} - w_2\sqrt{(1+f_{s2})h_2}\right)\\
\dot{w}_1 &= \frac{1}{\tau}(k_{e1}v_1 - w_1)\\
\dot{w}_2 &= \frac{1}{\tau}(k_{e2}v_2 - w_2)
\end{aligned} \tag{14}$$

where h_i, w_i and v_i, $i = 1, 2$, are the liquid level in the i^{th} tank, the opening ratio of the i^{th} electro–valve, and the voltage input to the i^{th} electro–valve, respectively; A_t is the transversal area of each tank; τ, k_{e1}, k_{e2} are the time constant and the static gains of the electro–valves respectively; ϕ_e is a constant input flow to the tank 1 and f_{s1} and f_{s2} model the faults in the level sensors 1 and 2, respectively.

For this system, we have considered faults in both the level sensors, and the proposed method has been tested.

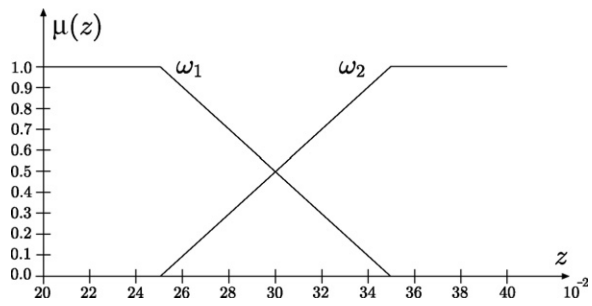

Fig. 4. Membership functions of the fuzzy model

3.1. Takagi–Sugeno fuzzy model

When modeling a nonlinear system by a Takagi–Sugeno representation, the number of rules is normally determined by the required accuracy. In general a large number of rules leads to a higher accuracy. However, the complexity of the resulting model should also be considered. In order to evaluate the fault diagnosis technique proposed in this paper we have considered two rules corresponding to the two reference liquid levels: Rule 1 associated with $h_{01} = 0.25$ m, $h_{02} = 0.35$ m, and Rule 2 associated with $h_{01} = 0.35$ m, $h_{02} = 0.25$ m.

For the fuzzy sets associated to the premise variables $z(t)$, we have chosen the membership functions illustrated in Figure 4, where the vector $z(t)$ is formed by the state variables $z_1 = h_1$ and $z_2 = h_2$.

From the linearization of the nonlinear model (14) around the operation points, system (1) can be described by means a TS fuzzy model (2) with

$$A_i = \begin{pmatrix} -\frac{w_{01}}{2A_t\sqrt{h_{01}}} & -\frac{\sqrt{h_{01}}}{A_t} & 0 & 0\\ 0 & -\frac{1}{\tau} & 0 & 0\\ \frac{w_{01}}{2A_t\sqrt{h_{01}}} & \frac{\sqrt{h_{01}}}{A_t} & -\frac{w_{02}}{2A_t\sqrt{h_{02}}} & -\frac{\sqrt{h_{02}}}{A_t}\\ 0 & 0 & 0 & -\frac{1}{\tau} \end{pmatrix}, \quad B_i = \begin{pmatrix} 0 & 0\\ \frac{k_{e1}}{\tau} & 0\\ 0 & 0\\ 0 & \frac{k_{e2}}{\tau} \end{pmatrix}$$

$$C_i = \begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad E_i = \begin{pmatrix} f_{s1}\\ 0\\ f_{s2}\\ 0 \end{pmatrix}.$$

Taking now $T = 2.6525$ s, $A_t = 0.16$ m^2 and the following set of parameters for each operation points $(\mathrm{op}_1, \mathrm{op}_2)$

$$\mathrm{op}_1 = \begin{pmatrix} h_{01} = 0.25 & w_{01} = 0.20795 \times 10^{-3} & h_{02} = 0.35 \\ w_{02} = 0.20795 \times 10^{-3} & k_{e1} = 0.03528 \times 10^{-3} & k_{e2} = 0.03923 \times 10^{-3} \end{pmatrix}$$

$$\mathrm{op}_2 = \begin{pmatrix} h_{01} = 0.35 & w_{01} = 0.17574 \times 10^{-3} & h_{02} = 0.25 \\ w_{02} = 0.20795 \times 10^{-3} & k_{e1} = 0.03083 \times 10^{-3} & k_{e2} = 0.02860 \times 10^{-3} \end{pmatrix}$$

we get

$$A_1 = \begin{pmatrix} -0.00129 & -3.12500 & 0 & 0 \\ 0 & -0.377 & 0 & 0 \\ 0.00129 & 3.125 & -0.001290 & -3.1250 \\ 0 & 0 & 0 & -0.377 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0 & 0 \\ 0.000013302 & 0 \\ 0 & 0 \\ 0 & 0.000014789 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} -0.00092 & -3.69754 & 0 & 0 \\ 0 & -0.377 & 0 & 0 \\ 0.00092 & 3.69754 & -0.00092 & -3.69754 \\ 0 & 0 & 0 & -0.377 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 \\ 0.000011624 & 0 \\ 0 & 0 \\ 0 & 0.000010783 \end{pmatrix}$$

$$C_1 = C_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad E_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad E_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

The fuzzy observer (4) is designed to estimate the system output and generate the diagnostic signal–residual that indicates whether or not a fault appears. In this case two local observers are required in the aggregate fuzzy observer. With A_1, A_2, C_1, C_2, we solve the LMI's (6) and obtain N_1, N_2 and P, where the gains L_i are obtained from $L_i = P^{-1}N_i$, $i = 1, 2$, according with Theorem 1.

Finally, the *structured* residual set [4] are designed to be sensitive to a certain group of faults and insensitive to others. The sensitivity and insensitivity properties make the faults isolation possible. The ideal situation is to make each residual sensitive only to a particular fault and insensitive to all others. For example, system (14) is sensitive to certain faults (f_{s1}, f_{s2}) which cause that the diagnostic signal–residual to be active or not. This response pattern is known as *fault signature* or *fault code* and is basically a characteristic of the faults in the system [11]. For the hydraulic system, the response pattern is shown in table 1. We observe that a fault in sensor 1 will only activate the residual 1, and a fault in sensor 2 will activate the residuals 1 and 3. Residuals 2 and 4 correspond to faults (f_{w1}, f_{w2}) in the electro–valves 1 and 2, which are not studied here and therefore in the fault signature matrix their diagnostic signal–residual is set to 0.

The fuzzy observer–based method has been tested in the two–tank system both in *simulation* and *real–time*. The initial condition for the system has been chosen as $x_0 = \begin{pmatrix} 0.25 & 0.0 & 0.25 & 0.0 \end{pmatrix}^T$ and has been carried out around the operating point. In this work we have considered the case when the sensor measurements are abruptly interrupted, due possibly to a sensor breakdown.

3.2. Simulations

Using the TS fuzzy model and the proposed fuzzy observer, the following three cases have been simulated

Table 1
Fault signatures

		Faults			
Residual	f_{s1}	f_{s2}	f_{w1}	f_{w2}	Notes:
r_1	1	1	0	0	f_{s1} fault in sensor 1
r_2	0	0	0	0	f_{s2} fault in sensor 2
r_3	0	1	0	0	f_{w1} fault in actuator 1
r_4	0	0	0	0	f_{w2} fault in actuator 2

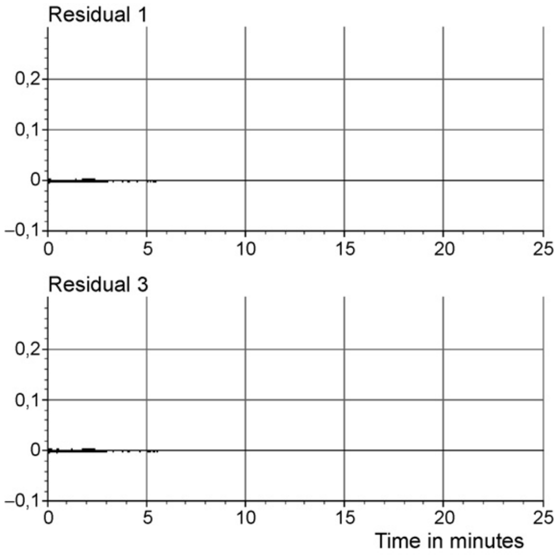

Fig. 5. Simulations: Residuals for a fault–free system

Case 1: *Fault–free system*. A 25 minutes running time has been simulated. Figure 5 shows the response of the residuals for this case. As it can be observed, the residuals have not been activated, as expected, indicating that the system is indeed free of faults.

Case 2: *System with faults*. Figure 6 shows the behavior of residual signals 1 and 3 when an abrupt fault is introduced in sensor 1 at 10 minutes. As predicted by the fault signature matrix, only the residual 1 (r_1) is activated. Note that the magnitude of the residual 3 is small, therefore we can assume that the fault appears only in the sensor 1. In the same way, a fault in sensor 2 has been simulated and the results in Figure 7 show that the respective residual (r_3) is activated.

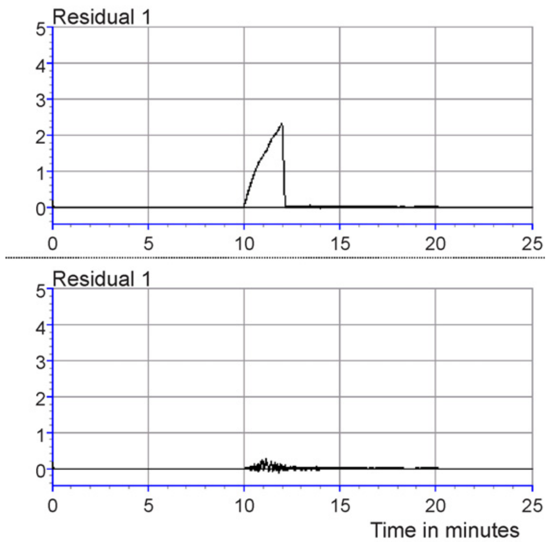

Fig. 6. Simulations: Residuals corresponding to a fault in sensor 1

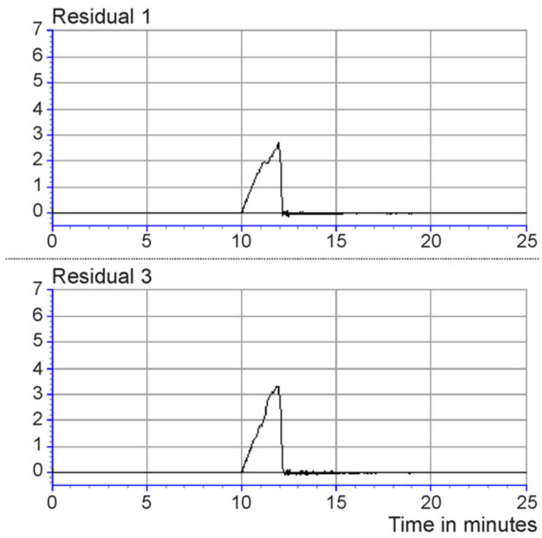

Fig. 7. Simulations: Residuals corresponding to a fault in sensor 2

Case 3: *Parametric variations*. Here, parametric changes on the values of A_i and B_i have been introduced. We observe in Figure 8 that

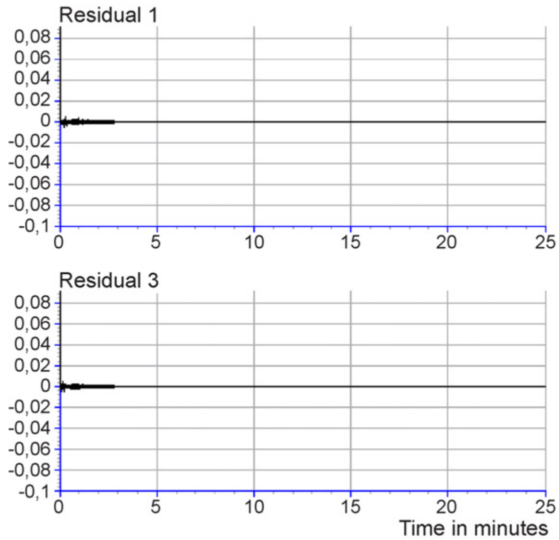

Fig. 8. Simulations: Residuals of a fault–free system with parametric variations

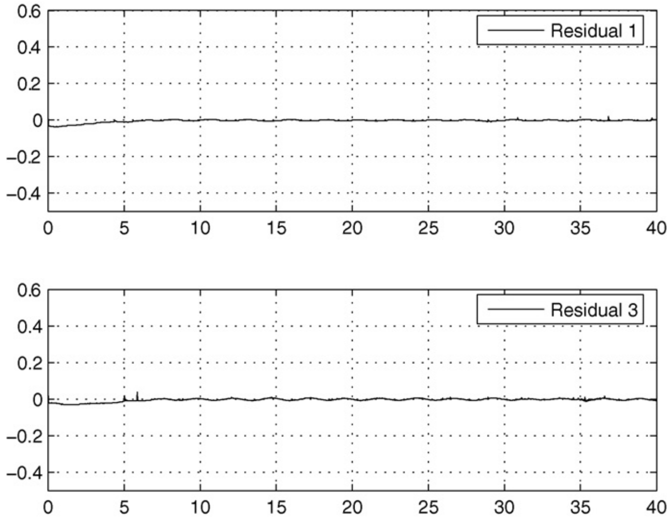

Fig. 9. Experimental results: Residuals of the fault–free two–tank system

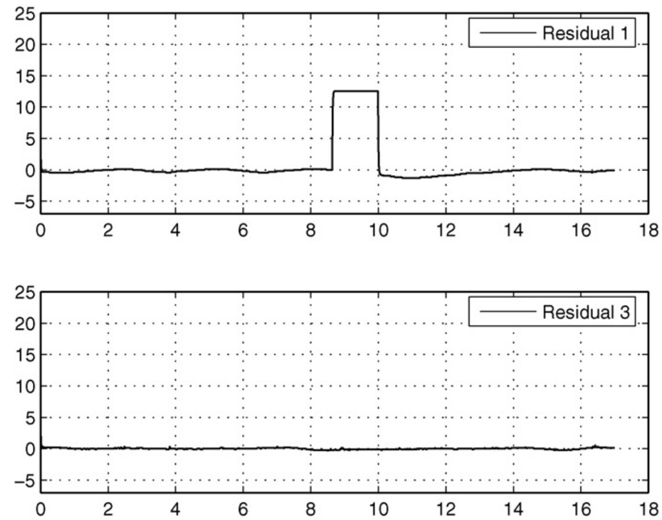

Fig. 10. Experimental results: Residuals corresponding to a fault in sensor 1

none of the residuals are activated, showing that the proposed scheme is robust in face of parametric variations in a neighborhood of the nominal values. This is in accordance with Theorem 1, as already explained in Remark 1.

3.3. Experimental results

We have reproduced in the experimental setup the three cases taken into account in the simulation section. In the case of fault–free, the residuals have not been activated while the system operates in normal conditions, as shown in Figure 9.

In the second case, faults in sensors 1 and 2 have been determined by disconnecting them for a short period. Figures 10 and 11 show that the respective residuals activate correctly, allowing the correct detection of the fault.

Finally, a parametric variation has been induced by introducing an object into tank 2 and changing the input flow. Figure 12 shows that the residuals, as expected, have not been activated, showing that the proposed fault

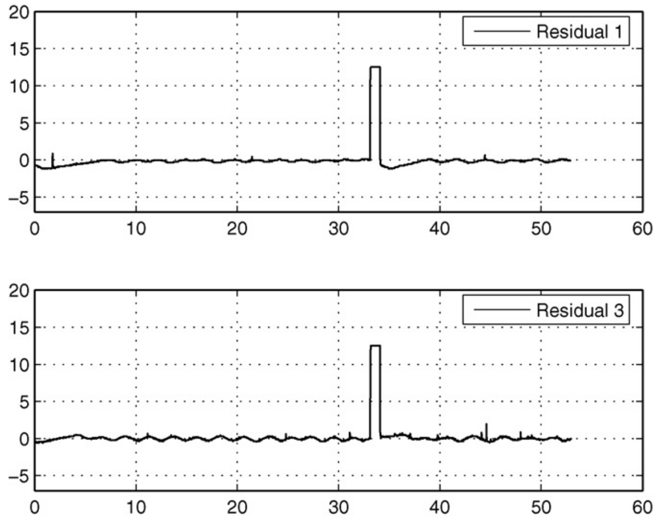

Fig. 11. Experimental results: Residuals corresponding to a fault in sensors 1 and 2

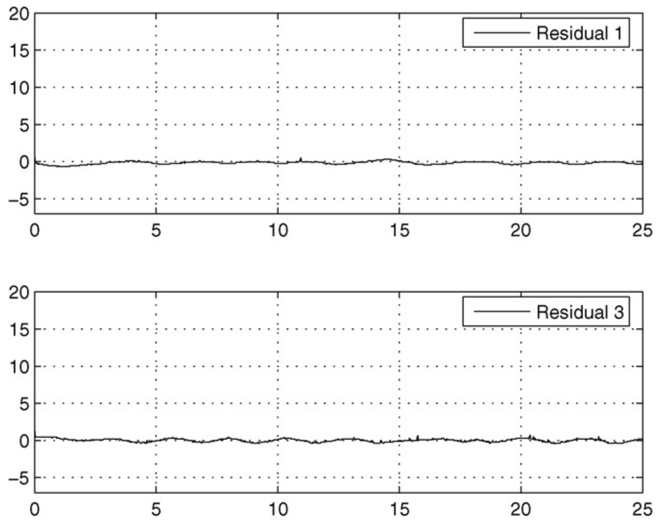

Fig. 12. Experimental results: Residuals of the two–tank system with parametric variations

diagnosis algorithm is insensitive to bounded parametric variations.

4. Conclusions

In this paper, a robust fault detection scheme by means of fuzzy observers has been proposed. The model based approach allows constructing such observers and introduce bounds on the external signals to be considered a fault. The results obtained through the application on a laboratory system, suggest the validity of the proposed scheme to be robust in face of parametric variations on the system, i.e. the diagnostic signal–residual only depends on external inputs or faults appearing in the system.

References

[1] A. Akhenak, M. Chadli, D. Maquin and J. Ragot, Sliding mode multiple observer for fault detection and isolation, *Proceedings*

of the 42^{th} *Conference on Decision and Control*, USA, pp. 953–958, 2003.
[2] O. Begovich, E.N. Sánchez and M. Maldonado, Real–Time nonlinear system regulation via TS fuzzy control, *Proceedings* 18^{th} *International Conference of North American*, Fuzzy Information Processing Society, USA, pp. 645–649, 1999.
[3] S. Boyd, L.E. Ghaoui, E. Feron and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, Society for Industrial Applied Mathematics (SIAM), Philadelphia, 1994.
[4] J. Chen and R. J. Patton, *Robust Model–Based Fault Diagnosis for Dynamic Systems*, Kluwer Academic Publishers, 1999.
[5] E.Y. Chow and A.S. Willsky, Analytical Redundancy and the Design of Robust Failure Detection Systems, *IEEE Transactions on Automatic Control*, Vol. 29, No. 7, pp. 603–614, 1984.
[6] C. Edwards, S.K. Spurgeon and R.J. Patton, Sliding mode observer for fault detection and isolation, *Automatica*, Vol. 36, pp. 541–553, 2000.
[7] C. Edwards and S.K. Spurgeon, *Sliding Mode Control: Theory and Applications*, Taylor & Francis Ltd, 1998.
[8] P.M. Frank, Fault Diagnosis in Dynamic Systems Using Analytical and Knowledge–Based Redundancy – A Survey and some New Results, *Automatica*, Vol. 26, No. 3, pp. 459–474, 1990.
[9] E.A. Garcia and P.M. Frank, Deterministic Nonlinear Observed–Based Approaches to Fault Diagnosis: a Survey, *Control Engineering Practice*, Vol. 5 No. 5, pp. 663–670, 1997.
[10] E.A. Garcia and S.S. Flores, Fault Detection and Isolation based on Takagi–Sugeno modeling, *Proceedings of the 2003 IEEE ISIC*, USA, pp. 673–678, 2003.
[11] J.J. Gertler, *Fault Detection and Diagnosis in Engineering Systems*, Marcel Dekker, 1998.
[12] H. Hammouri, M. Kinnaert and E.H. El Yaagoubi, Application of Nonlinear Observers to Fault Detection and Isolation, *Lecture Notes in Control and Information Sciences*, Nijmeier and Fossen Eds., pp. 423–443, 1999.
[13] R. Isermann, Process Fault Detection Based on Modeling and Estimation Methods – A survey, *Automatica*, Vol. 20, No. 4, pp. 387–404, 1984.
[14] B. Jiang, M. Staroswiecki and V. Cocquempot, Fault Estimation in Nonlinear Uncertain Systems using Robust/Sliding–mode Observer, *IEE Proceedings Control Theory Appl*, Vol. 151, No. 1, 2004.
[15] H.K. Khalil, *Nonlinear Systems*, Third Edition, Prentice Hall, Upper Saddle River, New Jersey, USA, 2002.
[16] R.J. Patton, P.M. Frank and R. K. Clark, *Fault Diagnosis in Dynamic Systems: Theory and Applications*, Prentice Hall, 1989.
[17] R. Seliger and P. M. Frank, Fault diagnosis by Disturbance Decoupled Nonlinear Observers, *Proceedings of the* 32^{nd} *Conference on Decision and Control*, pp. 2248–2253, 1991.
[18] K. Tanaka and H.O. Wang, *Fuzzy Control Systems Design and Analysis: a Linear Matrix Inequality Approach*, John Wiley & Sons, INC., 2001.
[19] B.L. Walcott and S.H. Zak, State observation of Nonlinear Uncertain Dynamical Systems, *IEEE Transactions on Automatic Control*, Vol. AC–32, No. 2, 1987.
[20] D.W. Gu and F.W. Poon, A Robust State Observer Scheme, *IEEE Transactions on Automatic Control*, Vol. 46, No. 12, pp. 1958–1963, 2001.
[21] L. Xie and C.E. de Souza, Robust H_∞ Control for Nonlinear Systems with Norm–Bounded Time–Varying Uncertainty, *IEEE Transactions on Automatic Control*, Vol. 37, No. 8, pp. 1188–1191, 1992.
[22] Z. Yi and P.A. Heng, Stability of Fuzzy Control Systems with Bounded Uncertain Delays, *IEEE Transactions on Fuzzy Systems*, Vol. 10, No. 1, pp. 92–96, 2002.