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Structurally Stable Output Regulation Problem With Sampled-Output Measurements Using Fuzzy Immersions

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***Abstract*—In this paper, a fuzzy nonlinear ripple-free regulator is proposed to solve the sample-data structurally stable regulation problem for the case of nonlinear or generalized immersion. This regulator guarantees asymptotic tracking of time-varying references and rejection of disturbances while maintaining the closed-loop stability. Such a regulator is based on a continuous fuzzy error feedback controller, which updates its states at each sampling period and relies on the existence of an internal model. The internal model is obtained by determining, if possible, an observable generalized immersion of the exosystem dynamics. A key feature is that this immersion allows the generation of all the possible steady-state inputs for all admissible values of the system parameters. The robustness of the proposed fuzzy controller, under parameter uncertainties and changes on disturbance signals, is tested in an illustrative example.**

***Index Terms*—Fuzzy immersion, ripple-free tracking, robust regulation, sampled-data control.**

I. Introduction

A very interesting problem in control theory is the tracking of reference signals and the asymptotic rejection of disturbances. This problem, which is usually called the *servomechanism* or *output regulation problem*, involves the design of a controller guaranteeing that all the trajectories of the closed-loop system are bounded and that the output-tracking error decays asymptotically to zero [1]. The particular feature of the problem is the characterization of all possible exogenous inputs—disturbances, commands, and uncertain constant

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parameters—as the set of all possible solutions of a fixed differential equation: the so-called *immersion*. In this setting, any source of uncertainty is viewed as an uncertainty in the initial condition of a fixed autonomous finite-dimensional dynamical system, which is known as the *exosystem* (see, e.g., the survey paper [2]). In the particular case of linear systems, in [3], it has been shown that the solvability of a multivariable regulator problem is equivalent to the existence of a solution of two linear matrix equations. In the nonlinear case, Huang and Lin [4] and Huang [5] extended the results established by Francis to the general case of exosystems for the generation of time-varying references and disturbances, including the very important case of periodic signals. They proved that the solvability of the output regulation problem is equivalent to the solvability of a set of partial differential and algebraic equations, which are known as regulator or Francis–Isidori–Byrnes (FIB) equations. Additional studies have been conducted to explore the problems associated with the output regulation problem, and suitable solutions have been suggested [6]–[8]. Recently, some extensions of the regulator theory to the very interesting case of networked systems have been addressed, where the feedback signal can be obtained via a real-time shared media network (see [9] and [10] and references therein).

In all the aforementioned works, an important issue is that the controller guarantees the structural stability of the closed-loop system. Most efforts have been focused on the design of state or error feedback controllers in the presence of parameter uncertainties [4], [5], [11]–[14]. A new method to design asymptotic stabilizing and robust control laws for nonlinear systems relies upon the notions of system immersion and manifold invariance. In particular, in the outstanding works of Huang *et al.* and Isidori *et al.* [4], [5], [11], [15], [16], conditions to solve the structurally stable regulation problem have been given in terms of this immersion. Isidori [11] has also shown that when a linear immersion is found, then a linear controller solves the problem. Later, Huang [17] showed that a linear immersion exists only when the steady-state input is given by a polynomial of the exosystem states, while in [15], a general setting of the concept of nonlinear immersion for the output regulation problem has been formulated. In [18], the construction of a so-called generalized immersion, which is a time-varying linear immersion that depends explicitly on the exosystem states, has been proposed.

When dealing with controllers implemented by digital devices and zero-order holders, it is well known that the sampled version of the continuous time controller may introduce instability in the closed-loop system [19]. For the case of continuous systems, a discretized-model-based controller using a zero-order holder guarantees a zero-output-tracking error only at the sampling instants. In the intersampling time, the output-tracking error will generally present a ripple, due to the fact that the requested steady-state input cannot be reproduced with zero-order holders, except in the particular case of constant reference signals and few other cases. To ensure that the state trajectory remains on a zero-error submanifold also during the intersampling, Castillo-Toledo and Di Gennaro [14] have developed a structurally stable controller based on the discretized linear approximation of the original system. This controller consists of a digital compensator plus a continuous component, called the *exponential holder*, and allows the reproduction of the continuous internal model dynamics, i.e., the correct steady-state input needed to maintain the output-tracking error to zero. However, in general, this controller can be explicitly determined only when a linear immersion is available, since, for nonlinear immersions, an exact discretization is difficult or even impossible to obtain in closed form.

With the aim of overcoming this obstacle, we propose in this paper, a solution inspired to the fuzzy logic control technique. This technique, which has been proposed as an alternative to solve control problems, represents a way of collecting both human knowledge and expertise to deal with uncertainties in such systems. In particular, fuzzy methods, based on the computational rule of inference and approximation, have been extensively applied in a large number of dynamic systems [20]–[22]. In the past decade, the Takagi–Sugeno (T–S) fuzzy models have been incorporated into fuzzy control approaches to study stability issues. The problem of systematic analysis and design of fuzzy logic controllers to ensure stability has been addressed using different approaches [23]–[26]. More recently, the robustness analysis of a closed-loop system has been tackled with the so-called fuzzy dynamic model, which is composed by a family of local linear models smoothly connected through fuzzy membership functions. Moreover, the robustness properties have been further studied using various performance criteria, such as D stability, $\mathcal{H}_\infty$ attenuation, decay rate, etc. [27]–[29]. In both the continuous and the discrete setting, a common approach to study the stability of the overall fuzzy system is based on the existence of a positive-definite matrix, which results from a set of Lyapunov equations related to the local models. For the particular case of continuous fuzzy systems controlled using sample data, Nguang and Shi [30] have developed continuous fuzzy control algorithms using discrete measurements.

On the basis of the previous discussion, in this paper, we propose the T–S fuzzy modeling to solve the sample-data structurally stable regulation problem for systems, where only a nonlinear or a generalized immersion can be found. A nonlinear robust model-based control technique has been developed to provide a hybrid fuzzy error feedback controller, which ensures the boundedness of the output-tracking error within predefined bounds, while ensuring the stability of the closed-loop system. The proposed fuzzy controller has been determined for the case of the Lienard model, and its performance has been tested in the case of time-varying references and load disturbances.

This paper is organized as follows. In Section II, a brief overview of the structurally stable regulation theory is given. The main result of the paper is developed and presented in Section III, while in Section IV, the implementation of the fuzzy controller on the Lienard model is presented and discussed. Finally, some remarks conclude the paper.

II. Robust Regulation Problem for Nonlinear Systems

Let us consider the following nonlinear system:

$$\dot{x} = f(x, u, w, \mu) \quad (1a)$$

$$\dot{w} = s(w) \quad (1b)$$

$$e = h(x, w, \mu) \quad (1c)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ are the state and input variables of the plant, μ denotes a parameter vector, which takes values in a neighborhood $\mathcal{P} \subset \mathbb{R}^r$ of the nominal ones, and $w \in \mathbb{R}^q$ represents the state of an external signal generator—the exosystem—which models the reference and disturbance signals. Finally, (1c) describes the output-tracking error $e \in \mathbb{R}^p$.

The *structurally stable error feedback regulation problem (SSORP)* is defined as the problem of finding, if possible, a dynamic controller of the form

$$\dot{z} = \varphi(z, w, e)$$
$$u = \vartheta(z, w)$$

such that, for all $\mu \in \mathcal{P}$, the following conditions hold:

S) *Stability:* The equilibrium point $(x, z) = (0, 0)$ of the closed-loop system without disturbances

$$\dot{x} = f(x, \vartheta(z, 0), 0, \mu)$$
$$\dot{z} = \varphi(z, 0, h(x, 0, \mu))$$

is asymptotically stable in the first approximation.

R) *Regulation:* For each initial condition $(x(0), z(0), w(0))$ in a neighborhood of the origin, the solution of the closed-loop system

$$\dot{x} = f(x, \vartheta(z, w), w, \mu)$$
$$\dot{z} = \varphi(z, w, h(x, w, \mu))$$
$$\dot{w} = s(w)$$

satisfies the condition $\lim_{t\to\infty} e(t) = 0$.

A complete solution to this problem, given by the following result, has been provided by Huang and Lin [4], Huang [5], and Isidori [11].

Theorem 1: Assume that the equilibrium point $w = 0$ is stable in the Lyapunov sense, and all the eigenvalues of $S = \partial s/\partial w|_{w=0}$ lie on the imaginary axis. Then, the SSORP is solvable if and only if there exist mappings

$$x_{ss} = \pi(w, \mu) \qquad \text{and} \qquad u_{ss} = \gamma(w, \mu) = \begin{pmatrix} \gamma_1(w, \mu) \\ \vdots \\ \gamma_m(w, \mu) \end{pmatrix}$$

with $\pi(0, \mu) = 0$ and $\gamma(0, \mu) = 0$, both defined in a neighborhood of the origin, satisfying the equations

$$\frac{\partial \pi(w, \mu)}{\partial w} s(w) = f(\pi(w, \mu), \gamma(w, \mu), w, \mu) \tag{2a}$$
$$0 = h(\pi(w, \mu), w, \mu) \tag{2b}$$

for all (w, μ) and such that for each $i = 1, \ldots, m$, the exosystem is immersed into the system

$$\dot{\zeta} = \varphi(\zeta), \quad \zeta \in \mathbb{R}^d \tag{3a}$$
$$\gamma(w, \mu) = \psi(\zeta) \tag{3b}$$

defined on a neighborhood Ξ^0 of the origin, with $\varphi(0) = 0$ and $\psi(0) = 0$, and the matrices

$$\Phi = \left.\frac{\partial \varphi}{\partial \zeta}\right|_{\zeta=0}, \qquad H = \left.\frac{\partial \psi}{\partial \zeta}\right|_{\zeta=0} \tag{4}$$

are such that the pair (A_0, B_0) is stabilizable, and the pair

$$\begin{pmatrix} C_0 & 0 \end{pmatrix}, \begin{pmatrix} A_0 & -B_0 H \\ 0 & \Phi \end{pmatrix}$$

is detectable, where

$$A_0 = \left.\frac{\partial f(x, u, w, 0)}{\partial x}\right|_{\substack{x=0\\w=0\\u=0}}, \quad C_0 = \left.\frac{\partial h(x, w, 0)}{\partial x}\right|_{\substack{x=0\\w=0}}$$
$$B_0 = \left.\frac{\partial f(x, u, w, 0)}{\partial u}\right|_{\substack{x=0\\w=0\\u=0}}.$$

Moreover, the controller solving the problem is given by

$$\dot{z}_1 = (A_0 + B_0 K - G_1 C_0) z_1 + G_1 e$$
$$\dot{z}_2 = -G_2 C_0 z_1 + \varphi(z_2) + G_2 e$$
$$u(t) = K z_1 + \psi(z_2) \tag{5}$$

where K, G_1, and G_2 are calculated such that $(A_0 + B_0 K)$ and

$$\begin{pmatrix} A_0 - G_1 C_0 & -B_0 H \\ -G_2 C_0 & \Phi \end{pmatrix}$$

are Hurwitz.

In general, finding a nonlinear immersion for a nonlinear system is a very difficult task. However, Huang [17] has shown that a linear immersion of the form

$$\dot{\zeta} = \Phi \zeta, \qquad \zeta \in \mathbb{R}^r$$
$$\gamma(w, \mu) = H\zeta \tag{6}$$

can be readily found if the steady-state input is given by a polynomial of the exosystem state w. In a recent paper, Castillo-Toledo *et al.* [18] has shown that this limitation can be relaxed by allowing the immersion to explicitly depend on the exosystem state w. This way, an extended class of functions, including trigonometric ones, can be also taken into account when devising immersion calculation procedures. This immersion has been called *generalized immersion*.

When dealing with sample-data measurements, a discretized version of the controller should be implemented, provided that the exact discretization of the linear approximation of the system plus the controller can be found. This scheme, which is called the *ripple-free structurally stable regulation problem* (RFSRP), can be formulated as the problem of finding a dynamic controller, which guarantees the conditions of stability and regulation similar to S) and R) but using sample data, i.e., $e(k\delta) = h(x(k\delta), w(k\delta), \mu)$, where δ is the sampling time, and k is a nonnegative integer. Castillo-Toledo and Di Gennaro [14] have shown that if an linear immersion similar to (6) for system (1) is found, then the RFRRP is solvable by the following controller:

$$z_{d1}((k+1)\delta)) = (A_{d0} + B_{d0} K_d - G_{d1} C_{d0}) z_{d1}(k\delta) + G_{d1} e_d(k\delta)$$
$$z_{d2}((k+1)\delta)) = -G_{d2} z_{d1}(k\delta) + e^{\Phi\delta} z_{d2}(k\delta) + G_{d2} e_d(k\delta)$$
$$u(k\delta + \theta) = K_d z_{d1}(k\delta) + H e^{\Phi\theta} z_{d2}(k\delta) \tag{7}$$

where $\theta \in [0, \delta]$ and K_d, $G_d = (G_{d1} \quad G_{d2})^T$ render stable the matrices

$$(A_{d0} + B_{d0} K_d), \quad \begin{pmatrix} A_{d0} & -M_{d0} \\ 0 & \Phi_d \end{pmatrix} - \begin{pmatrix} G_{d1} \\ G_{d2} \end{pmatrix} \begin{pmatrix} C_{d0} & 0 \end{pmatrix}$$

respectively, where

$$M_{d0} = \int_0^\delta e^{A_0 \tau} H e^{\Phi(\delta - \tau)}\, d\tau, \qquad \Phi_d = e^{\Phi\delta}$$

and A_{d0}, B_{d0}, and C_{d0} are given by

$$A_{d0} = e^{A_0 \delta}, \qquad B_{d0} = \int_0^\delta e^{A_0 \tau}\, d\tau\, B_0, \qquad C_{d0} = C_0.$$

Here, the stabilizability and detectability of the pairs

$$(A_{d0}, B_{d0}) \quad \text{and} \quad \left[\begin{pmatrix} A_{d0} & -M_{d0} \\ 0 & \Phi_d \end{pmatrix}, \begin{pmatrix} C_{d0} & 0 \end{pmatrix}\right]$$

is a necessary condition.

Note that the term $e^{\Phi\theta}$ in (7) is the internal model exponential holder, which allows the production of the exact continuous steady-state input. When only a nonlinear or a generalized immersion is available, the discretized controller can be constructed if the discretization of the immersion is exactly calculated. Since this is not always possible, in this paper, we propose the use of the T–S model and fuzzy observer design techniques to generate the continuous steady-state input when only sample-data error measurements and nonlinear or generalized immersions are available.

III. Regulation Problem Based on Fuzzy Immersion and Sampled-Error Measurements

The following assumptions are instrumental for the main result of the paper.

H2: There exists a solution $\pi(w,\mu)$, $\gamma(w,\mu)$ to the FIB equations (2).
H3: There exists a generalized immersion

$$\dot{\zeta} = \phi(\zeta, w), \qquad \zeta \in \mathbb{R}^d \tag{8a}$$

$$u_{ss} = \gamma(w,\mu) = \psi(\zeta, w). \tag{8b}$$

We assume that the nonlinear immersion (8a) and (8b) can be described by a T–S fuzzy model, namely that the immersion is described by a suitable aggregation of local linear subsystems, which is defined as follows:

Plant rule i: IF Z_1 is F_{1i} and $\cdots$ and Z_p is F_{pi}

$$\text{THEN} \sum : \begin{cases} \dot{\zeta}(t) = \Phi_i \zeta(t) \\ u_{ss,i} = H_i \zeta(t), \quad i = 1,\dots,r \end{cases}$$

where $\zeta \in \mathbb{R}^n$ is the state, $u_{ss,i}$ is the output, and $Z_1(\cdot),\dots,Z_p(\cdot)$ are measurable premise variables, which may coincide with the state vector ζ or a partial set of this vector through the output signals $u_{ss,i}(\cdot)$. In addition, F_{1i} are the corresponding fuzzy sets. Thus, for a given $\zeta(\cdot)$, the aggregate fuzzy model may be obtained by using a singleton fuzzifier, product inference, and center of gravity defuzzifier, namely

$$\dot{\zeta}(t) = \sum_{i=1}^{r} m_i(\zeta, w)\Phi_i \zeta(t) \tag{9a}$$

$$u_{ss} = \sum_{i=1}^{r} m_i(\zeta, w) H_i \zeta(t) \tag{9b}$$

where $m_i(\zeta, w)$ represents the membership function of each one of the r fuzzy subsets.

The linear subsystems can be obtained using different approaches, for example, linearizing at some point of interest near the equilibrium point leads to an approximated description, or using the well–known sector nonlinearity approach [27] leads to an exact representation of the nonlinear system (8). In this paper, we will take the later approach.

When considering the fuzzy model of the nonlinear immersion, the structure of the continuous time controller is similar to that given by (5), namely

$$\dot{z}_1(t) = (A_0 + B_0 K - G_1 C_0) z_1 - G_1 e \tag{10}$$

$$\dot{z}_2(t) = -G_2 C_0 + \sum_{i=1}^{r} m_i(z_2, w)\Phi_i z_2(t) + G_2 e \tag{11}$$

$$u = K z_1(t) + \sum_{i=1}^{r} m_i(z_2, w) H_i z_2 \tag{12}$$

provided that for each $i = 1,2,\dots,r$, the following pair of matrices is detectable [27]:

$$\begin{pmatrix} C_0 & 0 \end{pmatrix}, \begin{pmatrix} A_0 & -B_0 H_i \\ 0 & \Phi_i \end{pmatrix}. \tag{13}$$

Since we are interested in the error feedback regulation using sample data with a sampling period δ, and the discretization of fuzzy system (10)–(12) may be still a difficult task, in this paper, we propose the design of a continuous fuzzy observer, which updates its states at the sampling instants. The *regulation problem with error measurement* (RPEM) can be thus recast as the problem to find a controller

$$\dot{z} = \varphi(z, w, e_k)$$

$$u = \vartheta(z, w)$$

such that conditions S) and R) hold. To propose a solution to this problem, we introduce a preliminary result, which is based on the results of [30].

Lemma 2: Consider the fuzzy system

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{r} m_i(z)\tilde{A}_i x(t) \tag{14a}$$

$$\tilde{y}(k\delta) = C\tilde{x}(k\delta) \tag{14b}$$

where $\tilde{x} \in \mathbb{R}^n$ is the state vector, $\tilde{y} \in \mathbb{R}^p$ is the output vector measured at each sampling instant δ, m_i, $i = 1,\dots,r$ are the membership functions, and z are the premise variables. Then, the observer

$$\dot{\xi}(t) = \sum_{i=1}^{r} m_i(z)\tilde{A}_i \xi(t), \quad t \neq k\delta \tag{15a}$$

$$\xi(k\delta^+) = \xi(k\delta) - G\Big[\tilde{y}(k\delta) - C\xi(k\delta)\Big], \quad t = k\delta \tag{15b}$$

where $\xi(k\delta^+) := \lim_{\varepsilon \to 0} \xi(k\delta + \varepsilon)$ guarantees that $\lim_{t\to\infty}[\tilde{x}(t) - \xi(t)] = 0$ if there exist matrices $Q > 0$ and R and a constant γ, which solve the linear matrix inequalities (LMIs)

$$0 \leq \begin{pmatrix} Q - (\beta + \gamma\sigma)I & Q + RC \\ Q + C^T R^T & Q \end{pmatrix} \tag{16}$$

$$\gamma\sigma I < Q \tag{17}$$

$$\tilde{A}_i^T Q + Q\tilde{A}_i \leq \gamma[I + \sigma(\tilde{A}_i^T + \tilde{A}_i)] \tag{18}$$

$$i = 1,\dots,r$$

where

$$\sigma \geq \frac{e^{\alpha\delta} - 1}{\alpha}, \qquad \alpha \geq \sum_{i=1}^{r} \left|\lambda_{\max}\left(\tilde{A}_i^T + \tilde{A}_i\right)\right| \tag{19}$$

and $\beta > 0$ is an arbitrary constant. Moreover, the matrix G can be chosen as $G = Q^{-1}R$.

Proof: See the Appendix. ■

Remark 3: Note that updating the observer states according to (15b) is equivalent to resetting the initial condition of the continuous-time observer at each sampling instant.

Using the result of Lemma 2, it is now possible to design a controller having the structure of the fuzzy observer (15) to solve the RPEM. The following, which is the main result of the paper, gives the conditions for the existence of such a controller.

Theorem 4: Assume immersion (9) exists and the pairs

$$(A_0, B_0), \quad (C, A_i),\ i = 1,2,\dots,r$$

with

$$C = \begin{pmatrix} C_0 & 0 \end{pmatrix}, \quad A_i = \begin{pmatrix} A_0 & -B_0 H_i \\ 0 & \Phi_i \end{pmatrix}$$

are stabilizable and detectable, respectively. Let us assume also that there exist matrices $Q > 0$, R, and a constant γ, which solve the LMIs (16)–(18) for matrices C and A_i. Then, the RPEM is solvable. Moreover, a controller is given by

$$\dot{z}_1(t) = (A_0 + B_0 K) z_1(t), \quad t \neq k\delta \tag{20}$$

$$\dot{z}_2(t) = \sum_{i=1}^{r} m_i(z_2, w)\Phi_i z_2(t), \quad t \neq k\delta$$

$$z_1(k\delta^+) = (I + G_1 C_0) z_1(k\delta) - G_1 e(k\delta), \quad t = k\delta$$
$$z_2(k\delta^+) = z_2(k\delta) + G_2 C_0 z_1(k\delta) - G_2 e(k\delta), \quad t = k\delta$$
$$u = K z_1(t) + \sum_{i=1}^{r} m_i(z_2, w) H_i z_2(t) \tag{21}$$

where G is calculated as follows:

$$G = \begin{pmatrix} G_1 \\ G_2 \end{pmatrix} = Q^{-1} R.$$

Proof: To show that the controller solves the RPEM, it will be shown first that the closed-loop system is asymptotically stable when $w = 0$. The first approximation of the closed-loop system (1a) plus the controller (20) and (21) is given by

$$\dot{x}(t) = A_0 x + B_0 K z_1 + B_0 \sum_{i=1}^{r} m_i(z_2, w) H_i z_2$$
$$\dot{z}_1(t) = (A_0 + B_0 K) z_1, \quad t \neq k\delta$$
$$\dot{z}_2(t) = \sum_{i=1}^{r} m_i(z_2, w) \Phi_i z_2(t), \quad t \neq k\delta$$
$$z_1(k\delta^+) = (I + G_1 C_0) z_1(k\delta) - G_1 e(k\delta), \quad t = k\delta$$
$$z_2(k\delta^+) = z_2(k\delta) + G_2 C_0 z_1(k\delta) - G_2 e(k\delta), \quad t = k\delta$$

which can be written in compact form as follows:

$$\dot{\eta}(t) = \sum_{i=1}^{r} m_i(\eta, w) \Psi_i \eta(t), \quad t \neq k\delta$$
$$\eta(k\delta^+) = \Gamma \eta(k\delta), \quad t = k\delta, \ k = 0, 1, 2, \ldots \tag{22}$$

where Γ and Ψ_i are given by

$$\Psi_i = \begin{pmatrix} A_0 & B_0 K & B_0 H_i \\ 0 & A_0 + B_0 K & 0 \\ 0 & 0 & \Phi_i \end{pmatrix}, \quad i = 1, 2, \ldots, r$$
$$\Gamma = \begin{pmatrix} I & 0 & 0 \\ -G_1 C_0 & I + G_1 C_0 & 0 \\ -G_2 C_0 & G_2 C_0 & I \end{pmatrix}.$$

Considering the nonsingular transformation

$$\begin{pmatrix} I & 0 & 0 \\ -I & I & 0 \\ 0 & 0 & I \end{pmatrix}$$

the ith pair of matrices (Γ, Ψ_i) is shown to be similar to the ith pair

$$\left(\begin{array}{c|cc} I & 0 & 0 \\ \hline 0 & I + G_1 C_0 & 0 \\ 0 & G_1 C_0 & I \end{array}\right) \left(\begin{array}{c|cc} A_0 + B_0 K & B_0 K & B_0 H_i \\ \hline 0 & A_0 & -B_0 H_i \\ 0 & 0 & \Phi_i \end{array}\right).$$

By assumption, the matrix $(A_0 + B_0 K)$ is Hurwitz. Now, if there exist matrices $Q > 0$ and R and a constant γ such that the LMIs (16)–(18) hold for the subsystem

$$\dot{\tilde{\eta}}(t) = \sum_{i=1}^{r} m_i(\eta, w) A_i \tilde{\eta}(t), \quad t \neq k\delta$$
$$\tilde{\eta}(k\delta^+) = \left[\begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} + \begin{pmatrix} G_1 \\ G_2 \end{pmatrix} \begin{pmatrix} C_0 & 0 \end{pmatrix}\right] \tilde{\eta}(k\delta)$$

then, by Lemma 2, this subsystem has an asymptotically stable equilibrium point $\tilde{\eta} = 0$. Thus, the aggregated dynamics (22) have an asymptotically stable equilibrium point $\eta = 0$. For the tracking requirement, it follows from the FIB equations (2) that when z_1 goes asymptotically to zero, the dynamics of z_2 tens to the immersion dynamics (9), and the steady-state input is equal to

$$\sum_{i=1}^{r} m_i(\xi_2, w) H_i \xi_2(t) = \psi(\xi_2, w) = \gamma(w, \mu)$$

which is the necessary input making invariant the zero-tracking error submanifold. ■

Remark 5: Note that the main feature guaranteeing the zero-output-tracking error is the immersion (9a), which incorporates the nonlinearity of the steady-state input.

IV. Illustrative Example

Let us consider the Lienard model

$$\ddot{y} + f(y, \mu)\dot{y} + g(y, d, \mu) = u \tag{23}$$

where $f(y, \mu)$ and $g(y, d, \mu)$ are smooth functions of the output y, μ is an uncertain parameter vector, $u \in \mathbb{R}$ is the input, and $d \in \mathbb{R}$ represents a disturbance signal. In particular, we will take $g(y, d, \mu) = -a^2 y + \mu_1 + \mu_2 d$ and

$$f(y, \mu) = \frac{1}{\mu_3 + y^2}$$

with a a known constant. In state variables, this model can be recast as follows:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -g(x_1, d, \mu) - f(x_1, \mu) x_2 + u.$$

In order to analyze the performance of the proposed controller, x_1 has to track a reference y_r. The exosystem dynamics are chosen to be

$$\dot{w} = s(w)$$
$$d = R_1 w$$
$$y_r = R_2 w$$

where $w \in \mathbb{R}^5$, $s(w) = (w_2 \quad w_3 \quad -c_1^2 w_2 \quad w_5 \quad -c_2 w_4)^T$, $R_1 = (1 \quad 0 \quad 0 \quad 0 \quad 0)$ and $R_2 = (0 \quad 0 \quad 0 \quad 1 \quad 0)$. Note that the exosystem describes both the disturbance and the reference signals. It is also assumed that the signals (w_1, w_2, w_3) have known oscillation frequency with possible unknown amplitude satisfying $c_1 > 0$. However, the signals (w_4, w_5) are assumed to be measurable with $c_2 \neq c_1$.

The tracking error for this system is defined as $e = x_1 - w_4$ so that, when $e = 0$, the steady-state submanifold is described by

$$x_{1ss} = w_4 =: \pi_1(w)$$
$$x_{2ss} = w_5 =: \pi_2(w)$$
$$u_{ss} = -(a^2 + c_2^2) w_4 + \mu_1 + \mu_2 w_1 + \frac{w_5}{\mu_3 + w_4^2}.$$

where the uncertain parameters are $\mu = (\mu_1, \mu_2, \mu_3)$, with $\mu_3 > 0$. Since the term $-(a^2 + c_2^2) w_4$ is known, one chooses to construct an immersion only for the term

$$\hat{\gamma}(w, \mu) = \mu_1 + \mu_2 w_1 + \frac{w_5}{\mu_3 + w_4^2}.$$

An observable immersion for this term can be given by the system

$$\dot{\zeta} = \Phi(w_4, \zeta)$$
$$\hat{\gamma}(w, \mu) = H\zeta$$

where $\Phi(w_4, \zeta) = \mathrm{col}\{c_1\zeta_2, c_1\zeta_3, -c_1\zeta_2, c_2\zeta_5 - 2w_4\zeta_4^4, -c_2\zeta_4 - 2w_4\zeta_4\zeta_5\}$ and $H = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \end{pmatrix}$, while $\zeta = \mathrm{col}\{\mu_1 + \mu_2 w_1,\ \mu_2 w_2/c_1,\ \mu_2 w_3/c_1^2,\ w_5/(\mu_3 + w_4^2),\ -c_2 w_4/(\mu_3 + w_4^2)\}$. Note that the nonlinearities appear only in the time derivatives of the states ζ_4 and ζ_5, which represent the immersion for the term $w_5/(\mu_3 + w_4^2)$. Through an analysis of the magnitude for the nonlinear term $w_4\zeta_4$, it results that it is bounded by $-c_2 < w_4\zeta_4 < c_2$. Then, the fuzzified version of the immersion can be constructed as follows:

$$\dot{\zeta}(t) = \sum_{i=1}^{2} m_i(\zeta, w)\Phi_i\zeta(t)$$
$$\hat{\gamma}(w, \mu) = \sum_{i=1}^{2} m_i(\zeta, w)H\zeta(t)$$

where

$$\Phi_i = \begin{pmatrix} 0 & c_1 & 0 & 0 & 0 \\ 0 & 0 & c_1 & 0 & 0 \\ 0 & -c_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(-1)^i c_2 & c_2 \\ 0 & 0 & 0 & -c_2 & 2(-1)^i c_2, \end{pmatrix} \quad i = 1, 2$$
$$m_1(\zeta, w) = \frac{c_2 + w_4\zeta_4}{2c_2}, \qquad m_2(\zeta, w) = \frac{c_2 - w_4\zeta_4}{2c_2}.$$

However, the linearized matrices for the error observer around the point (x_{1s}, x_{2s}) are as follows:

$$A_0 = \begin{pmatrix} 0 & 1 \\ a^2 & -\frac{1}{\mu_{3\mathrm{nom}}} \end{pmatrix}, \quad B_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad C_0 = \begin{pmatrix} 1 & 0 \end{pmatrix}.$$

Then, it is possible to design a controller (20) and (21) with

$$u(t) = Kz_1(t) - (a^2 + c_2^2)w_4 + \sum_{i=1}^{2} m_i(z_2, w)Hz_2(t)$$

where K, G_{1i}, and $G_{2i}, i = 1, 2$ are calculated to satisfy the conditions given in Theorem 4.

A. Numerical Simulation Results

The following parameters values have been chosen: $\delta = 0.25$ s, $\alpha = 9$, $\sigma = 47.1541$, $\beta = 1$, and $a = 4\pi/5\ \mathrm{s}^{-1}$. The nominal values of the uncertain parameters are $\mu = (1, 0.4, 0.2)$. The reference has been taken as $y_r = \sin c_1 t$, while the disturbance has been set equal to $1 - 0.5\sin c_2 t$, with $c_1 = 2\pi/15\ \mathrm{s}^{-1}$ and $c_2 = \pi/5\ \mathrm{s}^{-1}$. The gain matrices K, G_1, and G_2 have been calculated as follows:

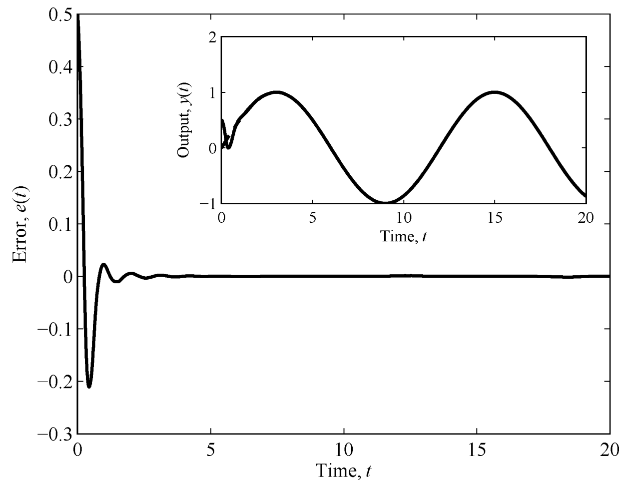

Fig. 1. Error for nominal simulation with fuzzy controller.

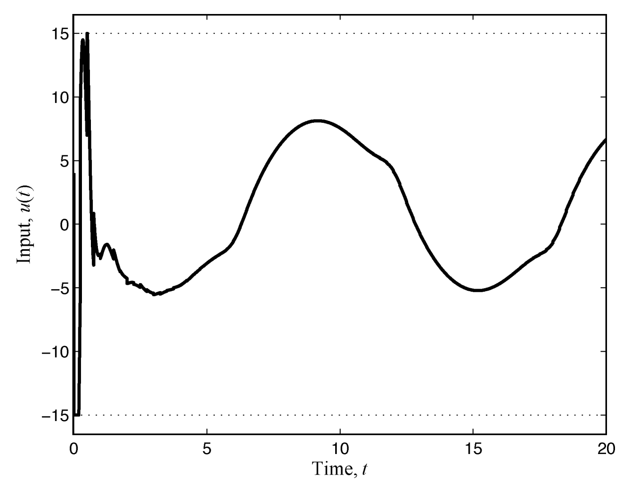

Fig. 2. Input for nominal simulation.

$$K = \begin{pmatrix} -86.3165 & -3.0000 \end{pmatrix}$$

$$G_1 = \begin{pmatrix} -99.40 \\ -5.211 \end{pmatrix} \times 10^{-2}, \quad G_2 = \begin{pmatrix} -4.56 \\ 1.201 \\ 0.7940 \\ -4.230 \\ 1.855 \end{pmatrix} \times 10^{-3}.$$

Figs. 1 and 2 show the error and input signals for the closed-loop system. As expected, the performance of the proposed fuzzy scheme is satisfactory.

Figs. 3 and 4 illustrate the performance of the proposed fuzzy control scheme under the influence of parametric variations of 7%, −5%, and 7% on the nominal values of μ_1, μ_2, and μ_3, respectively, which were introduced at $t = 0$ s. In addition, at $t = 10$ s, a variation of −5% on the value of μ_3, at $t = 20$ s, a variation of 10% on the value of μ_1, and at $t = 30$ s, a variation of −5% on the value of μ_2, have been introduced. Fig. 3 shows the output-tracking error, and Fig. 4 shows the control signal. As it can be observed, the proposed scheme is able to compensate the parametric variations.

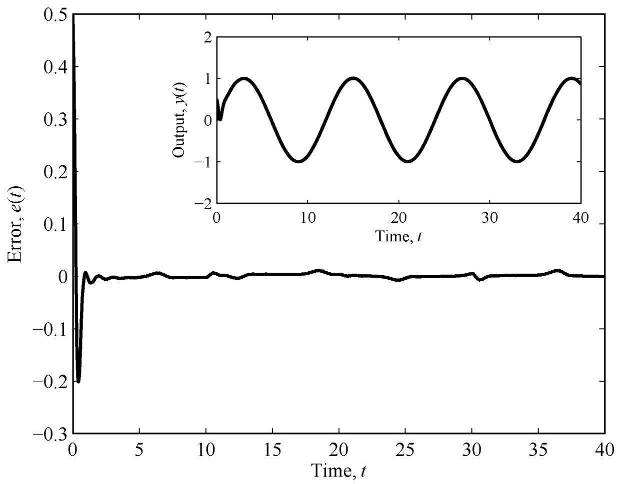

Fig. 3. Error for parametric variation test.

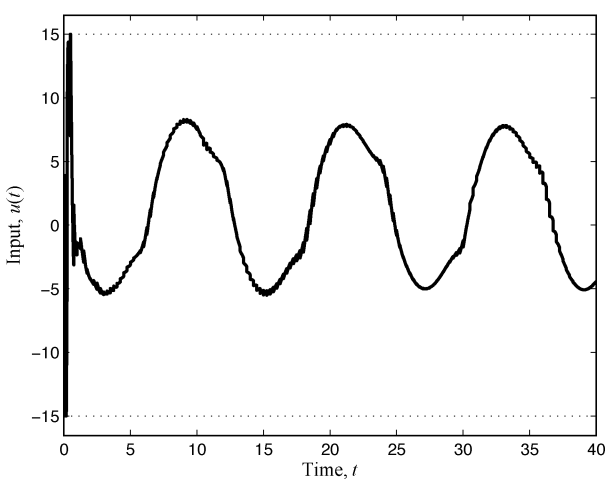

Fig. 4. Input for parametric variation simulation.

V. Conclusion

The main contribution of this paper is the proposal of a structurally stable regulator when only sample-data output-tracking error measurements are available. This scheme provides an alternative to the classical continuous-time approach or the discretized regulator, which are not readily applicable in this situation. The controller design, based on the concept of fuzzy immersions and composed of an error feedback controller and a fuzzy estimator, allows elimination of the ripple in the intersampling instants and guarantees the stability of the closed-loop system. The performance of the regulator has been tested with numerical simulations, under modeling errors, various uncertainties/variations, and external disturbances, showing a remarkable performance. These results suggest the validity of the proposed approach.

Appendix

Proof of Lemma 2

Proof: Since $x(k\delta^+) = x(k\delta)$, the observer error $e = x - \xi$ has the dynamics

$$\dot{e}(t) = \sum_{i=1}^{r} m_i(z) A_i e(t), \quad t \neq k\delta$$

$$e(k\delta^+) = (I + GC)\, e(k\delta), \quad t = k\delta.$$

Given the matrix $P > 0$ and defining the Lyapunov function $V(e(t)) = e^T(t)Pe(t)$, it follows that for $t \in (k\delta^+, (k\delta + \delta))$

$$\int_{k\delta^+}^{t} \frac{d}{d\lambda} V(e(\lambda))\, d\lambda = V(e(t)) - V(e(k\delta^+))$$

$$= \int_{k\delta^+}^{t} \sum_{i=1}^{r} m_i(z) e^T(\lambda)\big(PA_i + A_i^T P\big) e(\lambda)\, d\lambda. \quad (24)$$

Let us consider now that there exists a constant γ (which may be even positive) such that

$$PA_i + A_i^T P \leq \gamma I \qquad \forall i = 1, 2, \ldots, r \quad (25)$$

then

$$V(e(t)) - V(e(k\delta^+)) \leq \gamma \int_{k\delta^+}^{t} e^T(\lambda) e(\lambda)\, d\lambda.$$

Let us consider the Lyapunov function at the sampling instants

$$V(e(k\delta^+)) - V(e(k\delta)) = e^T(k\delta^+) P e(k\delta^+) - e^T(k\delta) P e(k\delta)$$

and combining this equation with (24), one gets

$$V(e(t)) - V(e(k\delta)) \leq \gamma \int_{k\delta^+}^{t} e^T(\lambda) e(\lambda)\, d\lambda + e^T(k\delta^+) P e(k\delta^+) - e^T(k\delta) P e(k\delta). \quad (26)$$

Additionally, since $\|e(t)\|^2 = e^T(t)e(t)$, then

$$\frac{d}{dt}\|e(t)\|^2 = \sum_{i=1}^{r} m_i(z) e^T(t)\big(A_i^T + A_i\big) e(t).$$

Let us define α as in (19), namely $\alpha \geq \sum_{i=1}^{r} |\lambda_{\max}(A_i^T + A_i)|$. Therefore, we get

$$\frac{d}{dt}\|e(t)\|^2 \leq \alpha \|e(t)\|^2$$

and $\|e(t)\|^2 \leq \|e(k\delta^+)\|^2 e^{\alpha(t - k\delta^+)}$ for $t \in (k\delta, (k+1)\delta)$. Then, the integral in (26) satisfies the inequality

$$\int_{k\delta^+}^{t} e^T(\lambda) e(\lambda)\, d\lambda \leq \frac{e^{\alpha(t - k\delta^+)} - 1}{\alpha} \|e(k\delta^+)\|^2.$$

When $t = (k+1)\delta$, this inequality becomes

$$\int_{k\delta^+}^{(k+1)\delta} e^T(\lambda) e(\lambda)\, d\lambda \leq \sigma \|e(k\delta^+)\|^2$$

where σ is given as in (19). Therefore, (26) can be written as follows:

$$V(e((k+1)\delta)) - V(e(k\delta)) = e^T(k\delta)[(I + GC)^T(\gamma\sigma I + P)(I + GC) - P] e(k\delta).$$

Then, the Lyapunov function decreases in each sampling period if, given a positive constant β, the inequality

$$(I + GC)^T(\gamma\sigma I + P)(I + GC) - P \leq -\beta I \quad (27)$$

holds, guaranteeing that $\lim_{k\to\infty}[x(k\delta) - \xi(k\delta)] = 0$, which implies that $\lim_{t\to\infty}[x(t) - \xi(t)] = 0$. Letting us define $Q = \gamma\sigma I + P > 0$, (27) is equivalent to

$$(I + GC)^T Q(I + GC) - Q \leq -(\beta + \gamma\sigma)I$$

which (using the Schur complement) produces the LMI (16), with $R = QG$. Finally, since, by assumption, (25) holds, in terms of Q, this inequality becomes (18). ■

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