Magnetization of multiply connected superconductors with and without π -junctions loops

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Abstract— The magnetic behavior of Multiply Connected Superconductors (MCS) can be described analyzing the simplest loop structures containing Josephson junctions: conventional loops with all conventional Josephson junctions and π -loops with an odd subset of π -junctions. These last are unconventional Josephson junctions in which the coupling has the reversed sign and appears in the ceramic materials as consequence of *d*-pairing. Among MCS the magnetic behavior of large β two-dimensional Josephson Junction Arrays (JJA) is based on the single loop behavior. Solving full mutual inductance Josephson junction square array equations with and without π -loops show that the mutual inductance coupling influence the distribution of π /conventional loops without altering substantially their single loop magnetization. The JJA mean magnetic behavior in low field can be recovered using a simple energy approach based on the single loop solutions avoiding the solution of the array full equations. Also we draw some consequences on the behavior of more complex MCS as high-T_C ceramics and their observed paramagnetic susceptibilities (Paramagnetic Meissner Effect).

INTRODUCTION

A two-dimensional Josephson junction array (JJA) is a special realization of a two-dimensional multiply connected superconductor (MCS). Classical arrays are made of a mesh of *N* identical loops with a fixed number of Josephson junctions per loop [1-3] (see Fig.1). Many studies have modeled high- T_c superconducting ceramics by means of low- T_c Josephson junction arrays founding their assumptions on the granular nature of such superconductors [4,5]. The magnetization properties between JJA and high- T_c ceramics could be related at least qualitatively [6]. Anyway the disordered nature of most high- T_c ceramics would make difficult a quantitative comparison of the magnetization properties. A more realistic MCS model of high- T_c ceramics would be not made of all identical loops or with loops with the same number of junctions.

An example of qualitative relation is the Paramagnetic Meissner Effect (PME) that was found first in field cooled HTC ceramic materials [7], then in low-T_c samples [8] and finally in the JJA [9]. PME also raised a debate on the role of *d*-wave superconductivity in the paramagnetic high-T_c ceramics. The paramagnetic response was ascribed to the presence of unconventional junctions, i.e., π -junctions, due to the *d*-wave pairing [10]. In ref.s [11-12] we have shown by means of numerical simulations how there is no need of π -junctions to obtain a paramagnetic response from a JJA: is the MCS nature of array that generates the paramagnetic response. On the other hand we have simulated array with π -junctions showing that there are at best only subtle differences between a conventional array and an array containing a mixture of conventional and π -junctions (see Fig.1) [13].

These results however show that the magnetization of the array is strictly connected to the properties of single loop made of p Josephson junctions [11-13]. In this paper we will show how we can obtain magnetization properties of a JJA making some simple assumption on the energy behavior of single loop

states in the mean field generated by other loops. We also briefly discuss the implications on the magnetic behavior of high- T_c materials that could explain some of the phenomena at the basis of their macroscopic mean magnetization.

II. THE SINGLE LOOP SOLUTIONS

We assume that a single loop is made of *p* Josephson junction with equal critical current I_0 and total selfinductance *L*. Under such hypothesis the flux quantization rule and the total flux expression $\Phi = \Phi_x + Li$ implies that static current states are solutions of the equation [14]:

$$\gamma_n = \sin\left(\frac{1}{p} \left(2\pi n - \pi k - 2\pi f - \beta \gamma_n\right)\right) \tag{1}$$

here β is the SQUID parameter equal to $2\pi LI_0/\Phi_0$ with Φ_0 the flux quantum; $\gamma_n = i/I_0$ is the normalized current; *n* is the quantum number [15]; *f* is the frustation equal to Φ_x/Φ_0 with Φ_x the external flux; finally *k* is an integer which is 1 if the array contains an odd number of unconventional π -junction. In the following we assume that β is large. Numerically we set it to 30 as in ref.s [9,11]. This assumption will simplify the search for the solutions of Eq.(1) because implies that lowest energy (*n*=0,1) stable solutions can be written in the form:

$$\gamma_{\rm n} = -A_{\rm n}f + B_{\rm n} \tag{2a}$$

with

$$A_{n} = \frac{2\pi/p}{1+\beta/p}$$

$$B_{n} = n \frac{2\pi/p}{1+\beta/p} \quad (k=0)$$

$$B_{n} = (-1)^{n} \frac{\pi/p}{1+\beta/p} \quad (k=1)$$
(2b)

Solutions can be classified in diamagnetic or paramagnetic according to the sign of current: negative current implies $\Phi < \Phi_x$ so the solution is diamagnetic, positive current implies $\Phi > \Phi_x$ and the solution is paramagnetic. For conventional loops n=0 is a diamagnetic solution and n=1 is paramagnetic. Lowest energy states are always diamagnetic if k=0, i.e., there are no π -junctions, and f<1/2. As illustration of this we show in Fig.2a the two lowest energy solutions for the p=4 loop. For f<1/2 the diamagnetic solution have the lower current and so the lower energy. For f>1/2 the situation is reversed [11]. The addition of a π -junction to the four-junction loop changes the solutions as shown in the same Fig.2a. Now at zero frustation there are two energy degenerate spontaneous current [16] solutions given by $\pm B_n/2$. With f different from zero the paramagnetic solutions. For f>1/2 the solutions n=0, which is diamagnetic, is the lowest energy solution [13]. In Fig.2b and Fig.2c we report the analogous states for the three, six, twelve and twenty-four junction loops.

III. A MEAN FIELD ANALYSIS OF SINGLE LOOP DISTRIBUTION

We study small values of frustation 0 < f < 1. In this region the behavior of an array with all normal (or all π -junctions) roughly follows the single loop behavior permitting a better comparison with above single loop theory, which is periodic in *f* [9,11]. On general basis if we have N_0 diamagnetic loops and N_1 paramagnetic loops their total energy is:

$$E = N_0 E_0 + N_1 E_1 + E_{\rm I} \tag{3}$$

where E_0 is the diamagnetic loop energy and E_1 is the paramagnetic loop energy. E_1 is the interaction energy between diamagnetic and paramagnetic loops. For large values of β both energies E_n can be put in the following simple form [14]:

$$E_{\rm n} = (1/2)(\beta/\pi)(1 + (\beta/4\pi)(1 - N_{\rm n}\zeta))\gamma_{\rm n}^{2}$$
(4)

The diamagnetic-diamagnetic and paramagnetic-paramagnetic interaction can be included in E_0 or E_1 . This is modeled by the symbol ζ , which is related to mean mutual induction between loops. We assume that interaction energy E_1 will be proportional to the product $N_0 x N_1$ and to the product of magnetization of diamagnetic and paramagnetic loops:

$$E_{\rm I} = -\zeta N_0 N_1 m_0 m_1 \tag{5}$$

again we use ζ as coupling coefficient. In the following we assume ζ as a free parameter. By convention we assume interaction energy with a minus sign in front of Eq. (5), anyway the sign of ζ will be determined below.

To find the occupation numbers N_n we minimize the total energy for a given value of frustation f, for the simple case of two species of loops, diamagnetic and paramagnetic, this is sufficient to find a solution by adding the total loops number conditions $N=N_0+N_1$. To understand what happen for different values of ζ we can look at the Fig.3, here the total energy was written in function of two variables f and $x=N_1/N$, i.e., the fraction of paramagnetic loops. Contour plots in Fig.3a shown the energy on the unit square for $\zeta=0$, i.e., not interacting loops. The absolute minimum corresponds always to x=0 (x=1 for f>1/2) because paramagnetic (diamagnetic) loop energy is larger and so any paramagnetic (diamagnetic) loop introduced in the system will increase the total energy, so the effective distribution is the trivial one with x=0 below f=1/2 and x=1 above

f=1/2. We observe that positive ζ can only increase the interaction energy, so again the absolute minimum is always found at x=0 (x=1). This result is the simple consequence that a single paramagnetic (diamagnetic) loop in a diamagnetic (paramagnetic) sea have an energy cost for the system. In Fig.3b we plot the $\zeta=-0.1$ case, in this case the interaction energy reduce the total energy and paramagnetic states minimize energy in the array. We see that a line of local minima will form roughly along the straight line x=f. As we shown below by comparison with numerical simulations of field cooling process in the array, this case fits very well with field cooling simulated results. The preference in the field cooled JJA for paramagnetic loops is due to presence of diamagnetic currents at boundary of the sample in the field cooled JJA [9,11]. This geometrical property implies a negative coupling for the simple non-geometrical energy approach described here. Direct analytical expression of the fraction x(f) can be found deriving the energy with respect to x and setting it to zero. This function have the simple linear form: $x=f+(\xi/\zeta N)(f-1/2)$ where $\xi=(1/2)(\beta/\pi)(1+(\beta/4\pi))$. For $\zeta=-0.1$ the second term is practically negligible¹, thus implying that interaction energy is the dominant term. The above theory can be extended without effort to arbitrary uniform p junctions loop arrays as the coefficient A_n and B_n simply factor out and the only change is in the energy normalization.

In presence of a subset of π -junctions in the array the total energy will depends also from paramagnetic (or diamagnetic) π -junction fraction of total number of π -loops. Moreover there are four terms in the interaction energy: diamagnetic with paramagnetic, π -diamagnetic with π -paramagnetic and the mixed terms. There is no reason for using again a single parameter ζ for the coupling. Using a two parameters interaction energy we are able to find a minima by looking directly to the slices *f*=const.. We will show the result in sect. V. The energy analysis can be extended when we are in presence of a not uniform array with several species of loops,

¹ Another term quadratic in f came from direct interaction with the external field, but for small f this correction is also negligible. This term can be important in the case of large f.

i.e., loops with different *p*'s and possibly the presence of π -loops. The minimization will become multidimensional and the number of free parameters increases. We note however that this will be the case that better approach a real high-T_c material that is made by different species of loops. A discussion of this and others effects in high-T_c MCS will be continued in sect. V.

IV. JJA FULL MUTUAL INDUCTANCE MODEL

To compare exact results with the above simple theory we simulate a square array of $\sqrt{N} \times \sqrt{N}$ square loops each carrying four Josephson junctions. The equations describing the array in vector notation read as [11,12]:

$$\frac{\beta_L}{2\pi}\sin\vec{\varphi} + \sqrt{\frac{\beta_L}{\beta_C}}\,\,\vec{\phi} + \,\vec{\phi} = \hat{K}\hat{L}^{-1}\vec{m} \tag{6}$$

here $\vec{\varphi}$ represents a vector containing the phases of all the junctions in the array; $\beta_{C}=2\pi CI_0/G^2\Phi_0$ is the Stewart-McCumber parameter, with C the junction capacitance and G the quasi-particle conductance; \hat{K} is a matrix depending on array geometry and \hat{L} is the full mutual inductance matrix of the array, i.e., the matrix containing mutual inductances of any mesh of the array with all the other mesh. The vector \vec{m} represents the mesh magnetization and its expression is:

$$\vec{m} = \frac{\vec{\phi}_{_{tot}}}{\Phi_{_{0}}} - \frac{\vec{\phi}_{_{ext}}}{\Phi_{_{0}}} = \frac{1}{2\pi} \left(\hat{M}\vec{\phi} + 2\pi\vec{n} - 2\pi\vec{f} \right)$$
(7)

where \vec{M} is an integer matrix performing (oriented) sum of the phases of each loop; \vec{n} is an integer vector carrying the quantum numbers and \vec{f} is the normalized external flux containing the frustation f. Eq. (7) represents flux quantization with \vec{n} the quantum number. Here quantum numbers are also used to introduce noise in the system choosing them randomly distributed [11]. Details of the integration procedure to simulate field cooling can be found in [12] where also the expression of mutual inductance matrix is given. To compare our results with that in literature here we choose $\beta_{\rm C}$ =65 [10,12]. We use *N*=400 as in ref. [12,13] so the array is a square of 20x20 loops. Similar JJA's are treated in [11] with equivalent results in terms of diamagnetic/paramagnetic response. A simulated 2D-magnetization is reported in Fig.4, we see two magnetization states for a conventional arrays and four states for an unconventional one with a subset of π -loops (about 50%). In Fig.4b,d we report the magnetization histograms for the Fig.4a,c. The histograms are peaked at the values of single loop magnetizations with a small spread determined by the mutual inductance coupling. The above currents states can be easily related to the loop magnetization using $m_n=(1/2\pi)\beta\gamma_n$.

V. RESULTS AND DISCUSSION

In Fig. 5a we report the fitting of the theory given in sect. III with the exact distribution evaluated by numerical simulations of Eq. (6) for the square array of Fig. 4a. The exact distribution appears to be linear implying a large contribution from interaction energy. The fitting parameter ζ is found to be -0.116 using a least square linear fit. In Fig. 5b we report the fitting of the theory for four junctions loops with the adding of a subset of π -loops (see Fig. 4b). The two-parameter fit is made by using again ζ for the diamagnetic with paramagnetic and π -diamagnetic with π -paramagnetic interaction, and another parameter ζ_m for the mixed loops interaction. The number of π -loops was set to N/2. The least square fit will give ζ =-0.0225, and ζ_m =-0.0045. These figures are lower with respect to conventional array implying that similar loops are now mixed with different species of loops. Compared with Fig. 5a the fitting in Fig. 5b is poor especially for the π -loops, also if the general behavior is well reproduced (also a slightly asymmetry around *f*=1/2 appears in the

fit). In presence of π -loops standard deviations due to statistical quantum number realization are typically larger, 10% to 15%, due to the increased number of states with respect to conventional arrays where standard deviations are from 2% to 5% [12]. In conclusion mean field approach shows how interaction between loops is the dominant mechanism to explain the observed magnetization histograms for small magnetic field (*f*<1). An exact prediction of histograms is difficult for the mixed π /conventional array, but the qualitative behavior is easily recovered.

Finally we discuss some implications for the high- T_c ceramics. The loop junction number p can be seen as one of factors that will play a role in the modeling of a high- T_c ceramics. It is reasonable that disordered high- T_c ceramic materials can be described as made of random p junction loops both conventional and π . As shown above each loop (conventional or π) have different magnetization states making possible for the system have a lot of loop states in the magnetization histogram plot. Anyway for p not larger than 8-10 states are again relatively well separated (cf. Fig.2b). This it is evident especially for f=0 or f=0.5. In the first case spontaneous currents are present in the sample in zero field. The second case is the dual state: π -loops collapse to zero magnetization, whereas conventional loop play the part of symmetric "spontaneous currents". A non-zero field experiment intended to evidence the second situation will be a natural counterpart of the first permitting a quantitative estimate of the number of π -loops in the sample [13].

However loops in disordered materials have also in general different critical currents for the junctions making the loop and different loop area. We have shown that weak disorder in critical current do not alter the discrete nature of loop states until a spread of 20%. Moreover larger spread in currents imply a large spread in the magnetization states, but for f=0.5 a symmetric histogram is again found (see Ref. [13]). The second effect is more complex to analyze because it will imply that frustation is different for different loops in the array changing not only β , but also the frustation f experienced by the loop in the Eq.(6). This modifies also

the periodicity in f of single loop solutions because now the period cannot be equal to one for all loops. For a large spread of areas (>20%) this further destroy the symmetry of the single loops solutions giving a larger interval of values of magnetization. This suggests that the off sample far field magnetic image of the array [17] would results in a single Gaussian histogram as observed by Braunisch et al. [7].

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FIGURE CAPTIONS

Fig.1. Two-dimensional square four junction per loop Josephson junction array with a random distribution of π -junctions. The gray loops represent the π -loops, i.e., loops with an odd number of π -junctions.

Fig.2. Single loop magnetization states: (a) conventional loop full line p=4, π -loop dashed line $p=3+1\pi$; (b) other loops full lines conventional loops with p=3, p=6, p=12, p=24, dashed lines π -loops with p=3, p=6, p=12, p=24. The slope of the straight lines decreases for larger p according to Eq. (2b).

Fig.3. Contour plots of the total magnetic energy on the plane (*f*,*x*) for *n*=4 loop: (a) ζ =0; (b) ζ =-0.1.

Fig.4. Simulated 2D-magnetization via Eq. (6): (a) magnetic image of a conventional 20x20 array with f=0.35; (b) magnetization histogram of a conventional 20x20 array with f=0.35; (c) magnetic image of a 20x20 array with f=0.35 and 380 π -junctions; (d) magnetization histogram of a 20x20 array with f=0.35 and 380 π -junctions;

Fig.5. Concentration of paramagnetic loops in a 20x20 array: (a) conventional array p=4, k=0 circles, full line is the fit made using the model of sect. III with $\zeta=-0.116$, dashed line represents the same model with $\zeta=-0.011$; (b) mixed π /conventional array p=4 and k=0,1 with about 50% of π -loops, boxes represents the concentration of conventional paramagnetic loops, circles the concentration of paramagnetic π -loops, full lines is the fit made using the model of sect. III with $\zeta=-0.0225$ and $\zeta_m=-0.0045$.



Fig.1 De Leo & Rotoli



Fig.2



Fig.3a



Fig.3b



Fig.4a,b



Fig.4c,d



Fig.5b