Paramagnetism in 1d mixed π/conventional Josephson arrays

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Abstract

One-dimensional mixed π/conventional arrays of Josephson junctions admit paramagnetic solutions besides the standard diamagnetic ones. We analyze the arrays magnetization properties when they show paramagnetism varying the parameters controlling the self-field and the length of the array. We show that paramagnetic solutions are found in all cases independently on the effect of self-field and, moreover, for small length and negligible self-field the paramagnetic solution is the lowest energy stable solution. These arrays can be used to model long Grain Boundaries (GB) in artificial or intrinsic high-T_c junctions made by unconventional *d*-wave superconductors.

Keywords: Josephson, Paramagnetism, Grain Boundaries, Arrays.

Paramagnetism in high-T_c compounds was discovered in the first half of nineties [1]. Then many studies have been dedicated to understand its relationship with *d*-wave symmetry [2], its, soon discovered, low-T_c counterpart [3] and experiments on low-T_c Josephson junction arrays (JJA) [4,5]. Recent results [6,7] indicate that mesoscopic Josephson junctions in Grain Boundaries (GB) can show paramagnetic effects due to formation of π-loops at the GB interfaces. A π-loop is formed when at least two junctions, one conventional and the other one a not conventional π-junction break a superconducting loop. π-junctions are consequence of *d*-wave pairing [8].

A Josephson (multi)-junction loop may have several magnetization states when magnetic field is applied. These can be represented as a normalized current flowing in the loop $\gamma_{n,k}$. Here *n* is the quantum number in the flux quantization expression, *k* is an index, which is 1 if an odd number of π-junctions is present in the loop. For a loop of *p* identical junctions the current $\gamma_{n,k}$ is a solution of:

$$\gamma_{n,k} = \sin\left(\frac{1}{p}\left(2\pi\, n - \pi\, k - 2\pi\, f - \beta\gamma_{n,k}\right)\right) \qquad (1)$$

here β is the SQUID parameter $2\pi I_0 L/\Phi_0$ of the loop, $f=\Phi_e/\Phi_0$ is the frustation [9]. Varying the quantum number between n=0 and p-1 gives different independent solutions within a 2π phase change. A study of Eq.(1) for large β was made in [9]. Here we study the small β case, which is interesting when mesoscopic π-loops can occur between small junctions lining along a GB. Consider for example p=2, i.e., the simple π-SQUID. Currents are shown in Fig.1a for two values of β. For f=0 the Eq.(1) has two solutions corresponding to two spontaneous currents. For nonzero magnetic field $0<f<1/2$ the positive solution is paramagnetic and negative is the diamagnetic. The current absolute value is lower for paramagnetism giving a lower energy for paramagnetism. Symbols in Fig.1a have been calculated using a direct numerical integration of π-SQUID equations without solving Eq.(1) with a root finder (see Eq. (2) below). As is seen in the Fig.1a the diamagnetic solution is soon left for very small f

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and the lower energy paramagnetic solution is setting on (the left lower corner of Fig.1a is shown in Fig.1b). The instability occurs when the energy of paramagnetic solution is just equal to minimal energy of diamagnetic solution. The system relaxes on the minimum energy branch. This is characteristic of multi-junction loops and does not occurs in single junction loop where there is a single state for $\beta<1$.

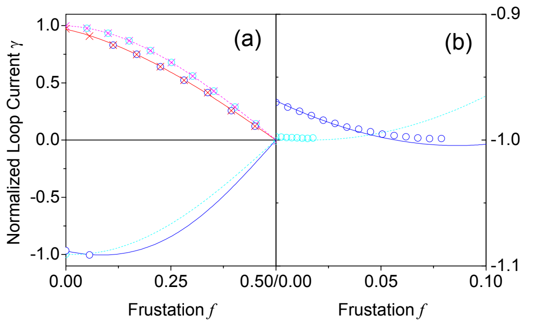

Fig.1 Normalized Loop Current γ as function of frustation f for a π-loop with p=2, dashed curves show the case β=0.1 and full curves β=0.5. Lines represents the (approximate) solution of Eq.(1) within o(β^3), crosses and circles the same solution evaluated by means of direct solution of Eq.(2). Crosses (circles) start on paramagnetic (diamagnetic) branch . The lower left corner of (a) is show in (b).

To extend the above theory to arrays we will describe the GB as an 1d array of N+1 Josephson junctions placed along it. In the simplest extension of π-SQUID loop the array is divided into two pieces made respectively of a conventional junction and a π-junction [10,11]. The magnetization can be found using the Discrete Sine-Gordon equation [7]:

$$(-1)^{k(j)}\sigma_i \sin\varphi_i = \frac{1}{\beta}\Delta^2\varphi_i \qquad (2)$$

where φ_i is the phase of the i-th junction in the array, the index k(i) will be 0 for conventional junctions and 1 for π-junctions, Δ is the difference operator and σ_i is a Gaussian variable with mean 1. To include boundaries we set $\varphi_0=\varphi_1+2\pi f$ and $\varphi_{N+1}=\varphi_{N+2}+2\pi f$. Can be shown that Eq.(2) for N=1 implies Eq.(1). Details on the solution technique for Eq.(2) are given in [7]. Typical solutions of Eq. (2) are shown in term of local magnetization $m_i=\Delta\varphi_i/2\pi-f$ in Fig.2a for N=16. Zero field solutions appear symmetric respect to x-axis corresponding to two opposite spontaneous currents. Non-zero field solutions develop diamagnetic screening currents at the boundary. These currents actually add to spontaneous currents shifting the solution toward diamagnetism. Fig.2b the mean magnetization for N=4 and 16 is reported. The mean magnetization follow a decreasing behavior for longer arrays. There is a limiting field η_l where paramagnetic solution becomes diamagnetic. This occurs when diamagnetic screening currents overdue the spontaneous magnetization of the system. For N=4 we observe the same instability of diamagnetic solution found in the single loop. This disappears for N=16 indicating that in longer arrays both diamagnetic and paramagnetic solutions are stable. The relation of this phenomenon with “flat solution” instability [11] should be analyzed, but there is no apparent connection between them.

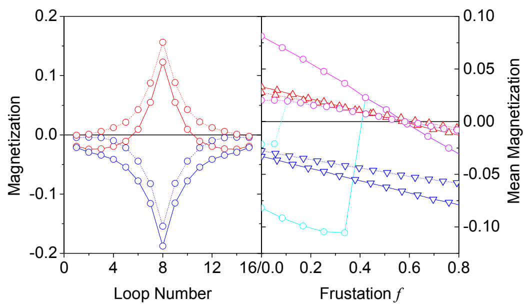

Fig.2 Local and mean magnetization calculated by means of Eq.(2) for N=4 and 16. (a) N=16 local magnetization with β=0.1, dashed curves refer to f=0 and full curves to f=0.4. (b) N=4 (circles) and N=16 (triangles) mean magnetization, dashed curves refer to β=0.1 and full to β=0.5.

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