Spectrum of linear modes in triangular arrays of Josephson junctions

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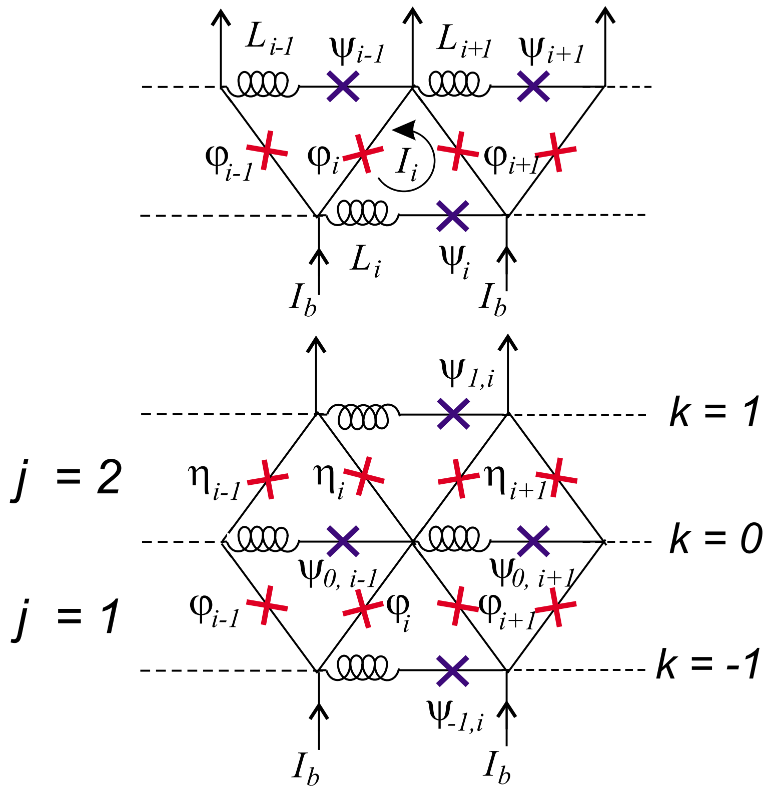
Abstract

Triangular arrays of small Josephson junctions in the presence of a magnetic field exhibit a number of resonances which depends on the number of the rows constituting the array. We present experiments performed in single and double row triangular arrays. The magnetic field dependencies of the voltage positions of the resonances are compared for different values of the cell inductance and different critical current densities. We have analytically obtained the spectrum of linear modes of electromagnetic waves propagating in the array, both for the single and double row cases. In the two row array the spectrum is determined by the behaviour of the horizontal Josephson junctions and corresponds to the ribbon or checkerboard state. The magnetic field dependence of the resonance voltages is mapped to this spectrum and a good agreement between the experiments and the model is observed.

The circuit model



$\varphi_{i,j}$ and $\varphi_{i+1,j}$ denote the vertical junctions of the cell i in the row j; $\psi_{i,k}$ denotes the horizontal junction of the cell i in the line k. L_i is the cell inductance; $V^v_{i,j}$ ($V^h_{i,k}$) is the voltage across the vertical (horizontal) junction. All junctions have capacitance C, resistance R and critical current I_c. I_i is the mesh current in the cell i, and I_b is the bias current.

The system equations

* Flux quantization in the cell:

$$\varphi_{i+1} - \varphi_i + \psi_i \;=\; -\frac{2\pi\Phi_i}{\Phi_0} \quad , \tag{1}$$

* Total flux:

$$\Phi_i \;=\; \Phi_{\mathrm{ext}} + L_i I_i \quad , \tag{2}$$

* Balance of the currents in the nodes (Kirchhoff's current low), and RCSJ model for the *vertical* and the *horizontal* junctions:

$$C\dot{V}_i^v + \frac{V_i^v}{R} + I_c \sin\varphi_i \;=\; I_b - I_i + I_{i-1} \quad , \tag{3}$$

$$C\dot{V}_i^h + \frac{V_i^h}{R} + I_c \sin\psi_i \;=\; I_i \quad . \tag{4}$$

Normalized equations for the one-row array:

$$\begin{aligned} \ddot{\varphi}_i + \alpha\dot{\varphi}_i + \sin\varphi_i &= \\ &= \gamma + \tfrac{1}{\beta_L}(\varphi_{i+1} - 2\varphi_i + \varphi_{i-1} + \psi_i + \psi_{i-1}) \\ \ddot{\psi}_i + \alpha\dot{\psi}_i + \sin\psi_i &= \\ &= \tfrac{1}{\beta_L}(\varphi_i - \varphi_{i+1} - \psi_i) - \tfrac{2\pi f}{\beta_L} \end{aligned} \tag{5}$$

$\dot{} = \partial/\partial t$; the unit of time is $\omega_{\mathrm{p}}^{-1} = \sqrt{\hbar C/(2eI_c)}$; $\alpha = 1/\sqrt{\beta_c}$ is the damping parameter; $\beta_L = 2\pi L_i I_c/\Phi_0$ is the discreteness parameter; $f = \Phi_{\mathrm{ext}}/\Phi_0$ is the frustration.

Assumptions: **whirling solution** along the vertical junctions and **small amplitude oscillations** of the horizontal junctions

$$\begin{aligned} \varphi_n &= \omega t + 2\pi f n + \varphi\, e^{i(\omega t + 2\pi q n)} \\ \psi_n &= \psi\, e^{i(\omega t + 2\pi q n)} \end{aligned} \tag{6}$$

where ω and q are, respectively, the angular frequency and the wave number of the linear modes.

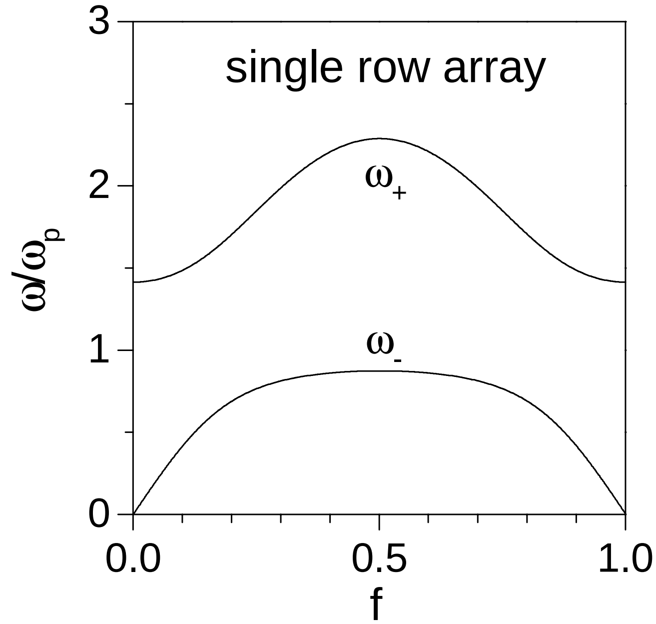
The spectrum of the electromagnetic waves propagating along the array contains two modes:

$$\omega_{\pm} = \omega_p \sqrt{\mathcal{F} \pm \sqrt{\mathcal{F}^2 - \mathcal{G}}} \qquad (7)$$

$$\mathcal{F} = 1/2 + 1/(2\beta_L) + (2/\beta_L)\sin^2(\pi q)$$
$$\mathcal{G} = (4/\beta_L)\sin^2(\pi q)$$

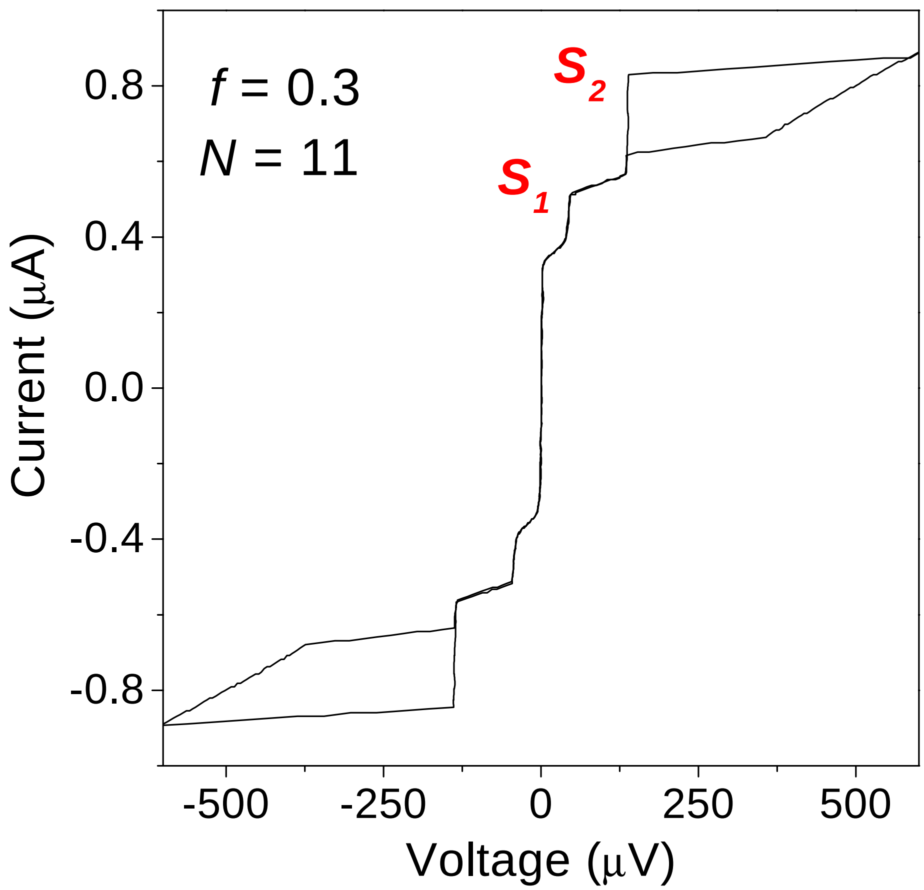
The resonances in the I-V characteristics are mapped to spectrum $\omega(q)$ by means of the relations:

$$2eV = \hbar\omega(q) \quad \& \quad q = f \qquad (8)$$

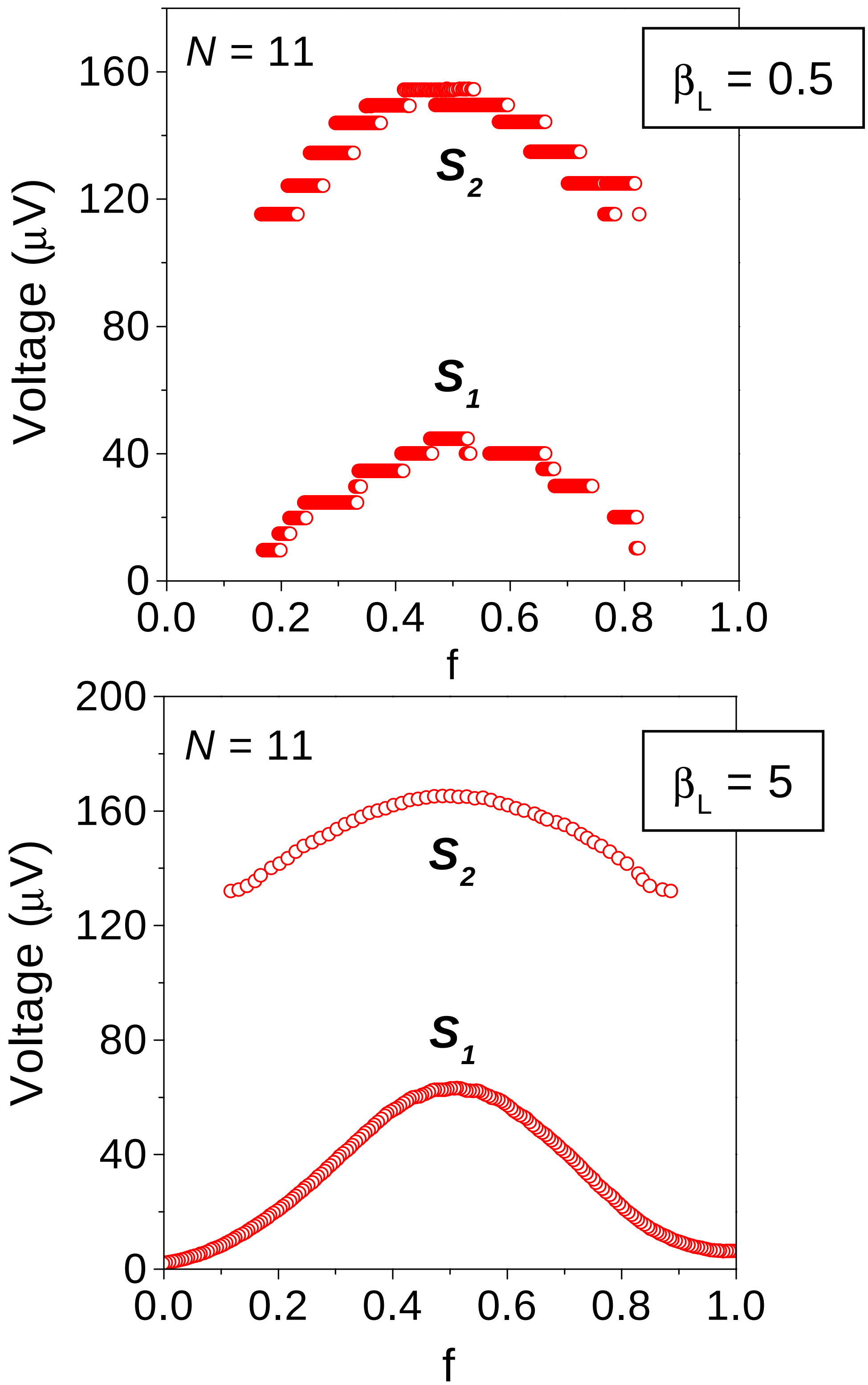


One-row array: observation of two resonances in the I-V characteristics

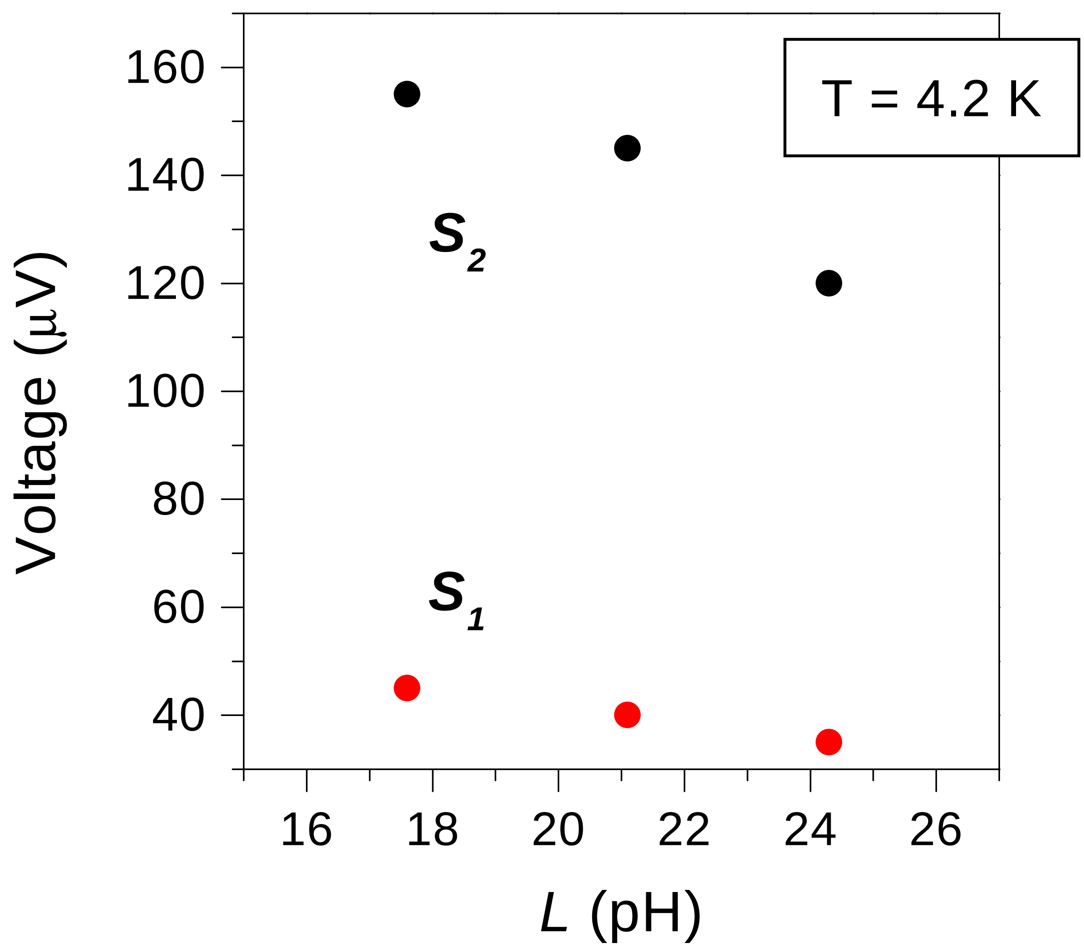
Experimental results on single row arrays with discreteness parameter $\beta_L \approx 0.5$



Measured dependences of the maximum voltage of the steps as a function of frustration in one-row arrays

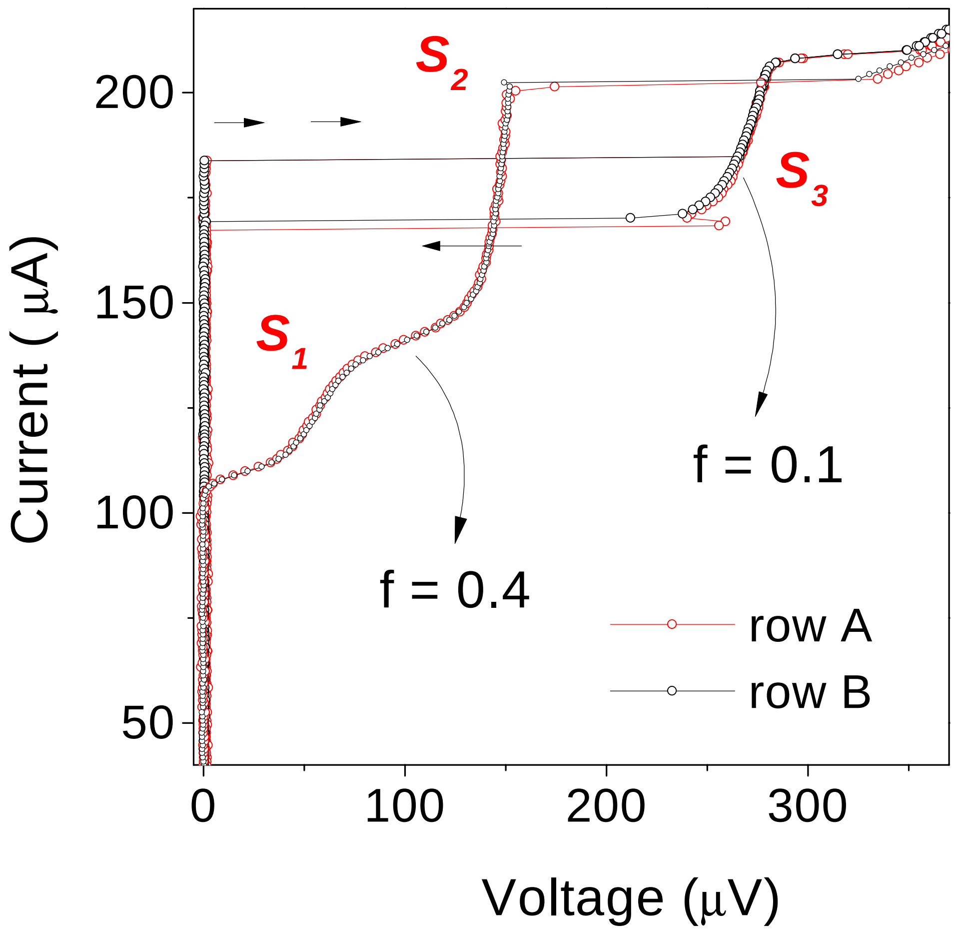


Maximum voltage of the steps S_1 and S_2 @ $f = 0.5$ in arrays having 3 different values of cell inductance L. All arrays have critical current density $J_c \approx 50\mathrm{A/cm}^2$

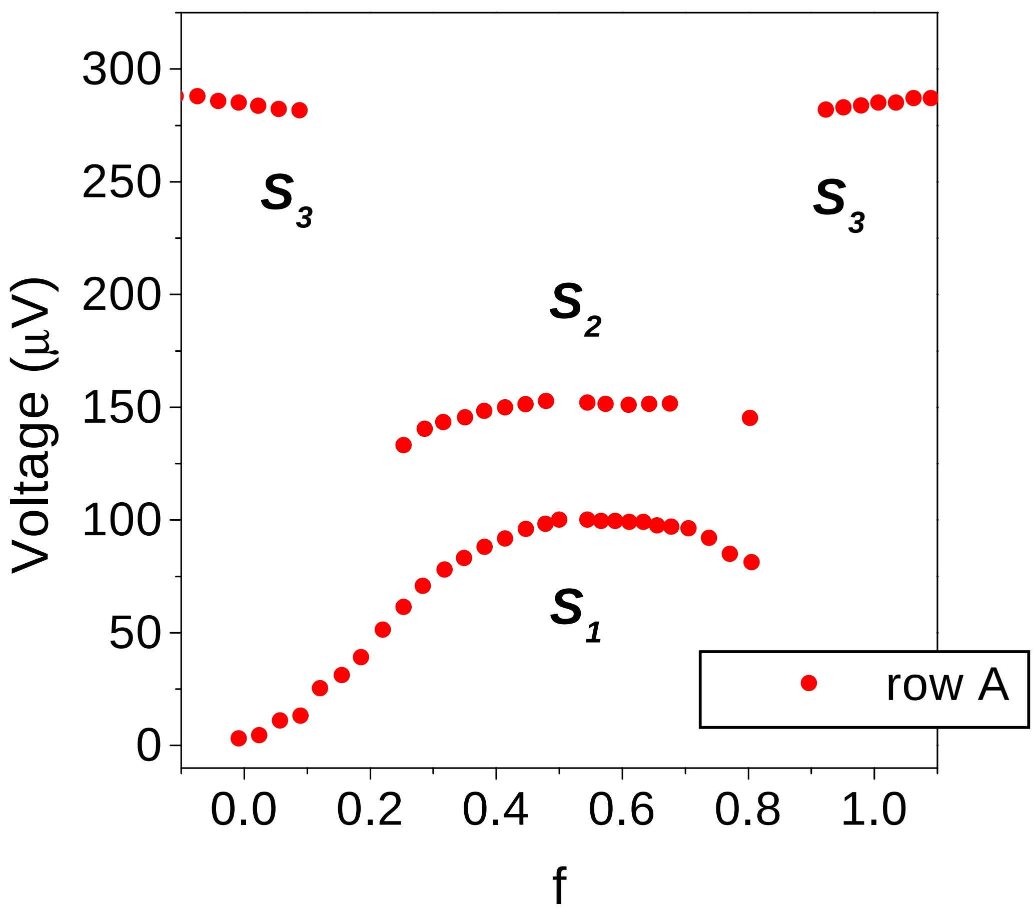


Two-row array: observation of three resonances in the I-V characteristics

Experimental results on double row arrays with discreteness parameter $\beta_L = 5$



Measured dependences of the maximum voltage of the steps as a function of frustration in two-row arrays



The model equations for the two-row array

Similar to the one row case, we derive the set of equations for two row array. The spectrum of linear modes is now obtained from the system of seven equations:

$$\left\{ \begin{array}{rcl} -\omega^2\varphi_1 & = & \varphi_2(1+e^{-iq}) - 2\varphi_1 + \psi_0 + \psi_1 e^{-iq} \\ -\omega^2\varphi_2 & = & \varphi_1(1+e^{iq}) - 2\varphi_2 - \psi_1 - \psi_{-0} \\ -\omega^2\eta_1 & = & \eta_2(1+e^{-iq}) - 2\eta_1 + \psi_0 + \psi_1^{-iq} \\ -\omega^2\eta_2 & = & \eta_1(1+e^{iq}) - 2\eta_2 - \psi_0 - \psi_{-1} \\ (-\omega^2+1)\psi_{-1} & = & (\varphi_2 - \varphi_1 e^{iq} - \psi_{-1})\frac{1}{\beta_L} \\ (-\omega^2+1)\psi_1 & = & (-\eta_1 e^{iq} + \eta_2 - \psi_1)\frac{1}{\beta_L} \\ (-\omega^2+1+\frac{1}{\beta_L})(\psi_1+\psi_{-1}) & = & \psi_0\frac{(1-\omega^2)(e^{iq}-1)}{\beta_L\omega^2} \end{array} \right. \quad (9)$$

The solution of system (9) gives seven branches in the $\omega(k)$ dependence. Three branches are determined by the equation:

$$\omega^6 - (\frac{5}{\beta_L} + 1)\omega^4 + (\frac{4}{\beta_L} + \frac{6}{\beta_L^2} - \frac{4}{\beta_L^2}\cos^2\frac{q}{2})\omega^2 - \frac{4}{\beta_L^2}\sin^2\frac{q}{2} = 0 \tag{10}$$

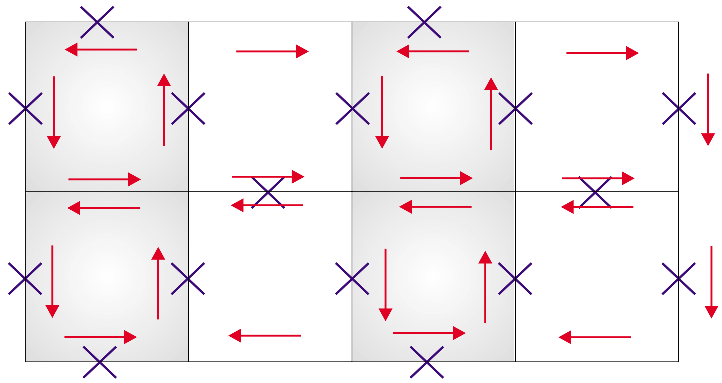
These branches correspond to a "ribbon state", as the Josephson junctions in the middle row are non active.

The other four branches are determined by the equation:

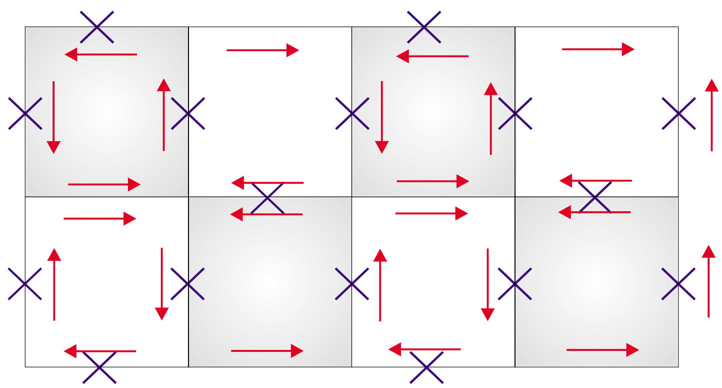
$$[(-\omega^2 + \frac{2}{\beta_L})\beta_L^2(1 - \omega^2 + \frac{1}{\beta_L}) - 2][-2\omega^2 + (\frac{2}{\beta_L} - \omega^2)(1 - \omega^2)] = 2(1 - \omega^2)^2\cos^2\frac{q}{2} \tag{11}$$

They depend on the properties of the junctions in the middle row and, therefore, correspond to a "checkerboard state".

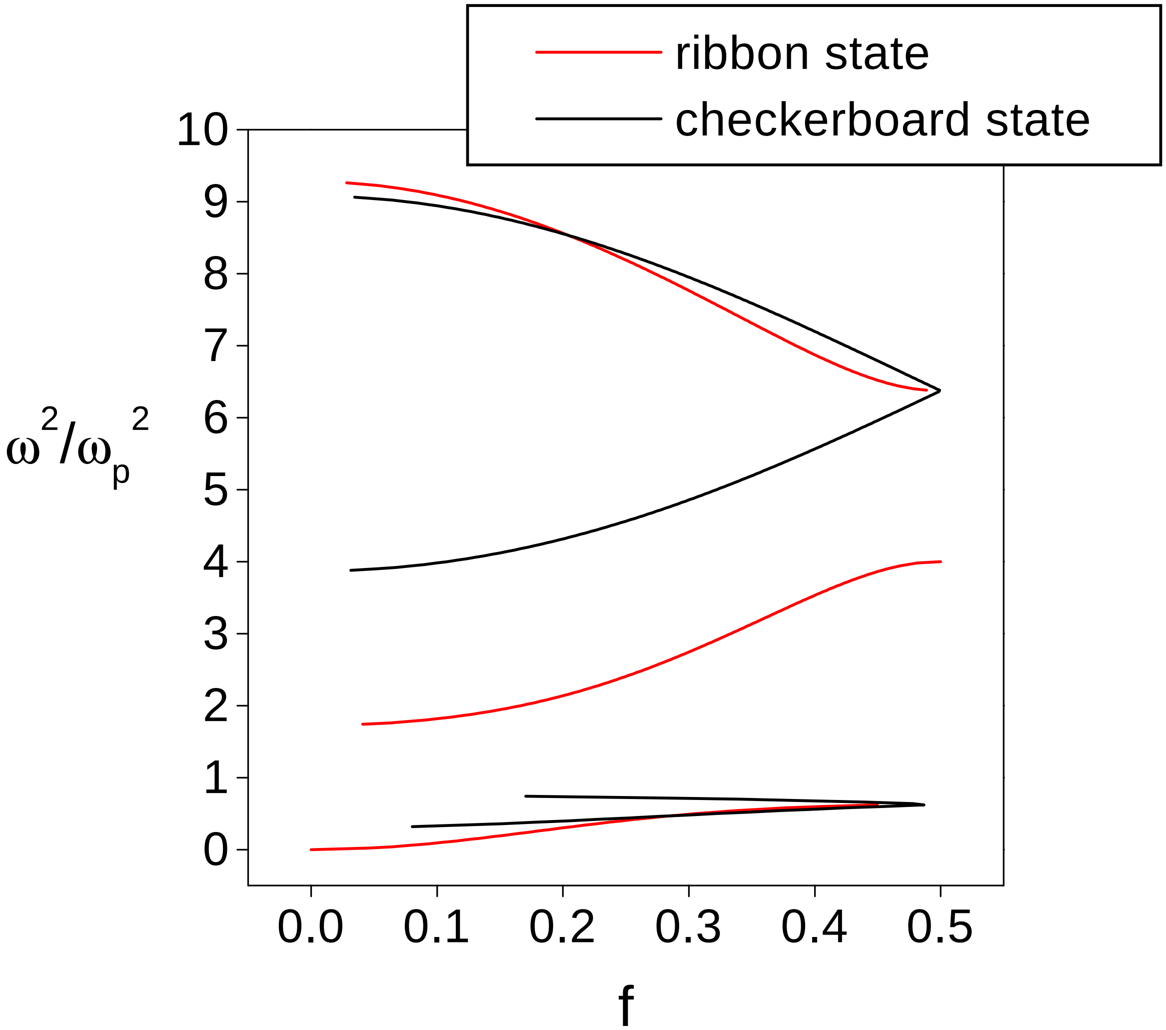
Ribbon state



Checkerboard state



Calculated spectrum of the linear modes for the two-row array ($\beta_L = 0.5$)



Conclusion

$*$ The number of resonances observed in the current-voltage characteristics of triangular arrays depends on the number of rows of the array, $i.e.$ it depends on the degrees of freedom of the system.

$*$ In single row arrays the two observed resonances have a field dependence which has a maximum at $f = 0.5$.

$*$ Double row arrays show three resonances: two of them have the same field dependence as in the single row array, while the third resonance displays an out of phase behaviour.

$*$ Our analytical model allows to obtain the spectrum of linear modes both for single and double row arrays. The voltage position of the observed resonances is mapped to this spectrum and a good agreement between experiments and theory is observed.

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