Arrays of stacked long Josephson junctions

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Abstract

We consider a structure consisting of two parallel arrays of long Josephson junctions sharing a common electrode that allows inductive coupling between the arrays. In the long dimensin this structure supports either the oscillation of coherent fluxon strings in one of the arrays or the oscillation of coherent fluxon-antifluxon strings in both the arrays. Applying an external magnetic field, cavity modes are excited that exhibit synchronization in both the dimensions. The experimental results are explained in terms of analytical and numerical investigation of a model based on coupled Sine-Gordon Equations.

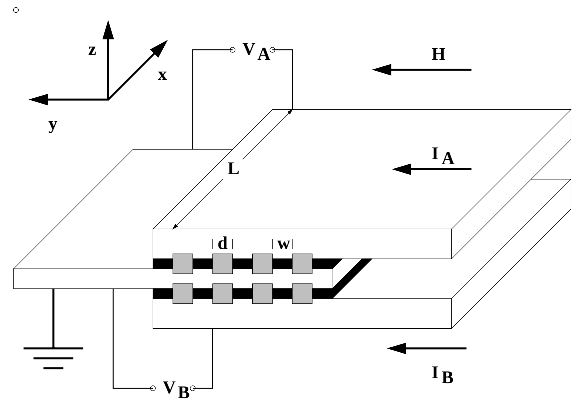
Experimental features of the stacked array

➢ Nb electrodes, Al_2O_3 barriers, SiO_2 insulation

➢ The middle electrode is $< \lambda_L$ $(\approx 100\mathrm{nm})$

➢ $L = 600\,\mu\mathrm{m}$, $w = 20\,\mu\mathrm{m}$, $d = 20\,\mu\mathrm{m}$, $< \lambda_J \approx (60\,\mu\mathrm{m})$

➢ $\sum w < \lambda_j$



- Magnetic field perpendicular to the long dimension

1. Two families of Fiske Steps
2. Whichever the polarity of the bias of each array, they can be locked to the same voltage (absolute value) on FS's belonging to the c^- family
3. Locking range is quite large
4. Voltage locking is also observed on FS's belonging to the c^+ family

- Magnetic field parallel to the long dimension

1. Nonevenly spaced current steps in one array (using the bias in the other to generate the field)

- Zero external magnetic field

1. Zero Field Steps in one array (the other is at $V = 0$ or in the McCumber state)
2. Phase-locking of both the arrays on ZFS's (c^-)
3. The asymptotic voltage of ZFS's in the single array sometimes is $> c^-$

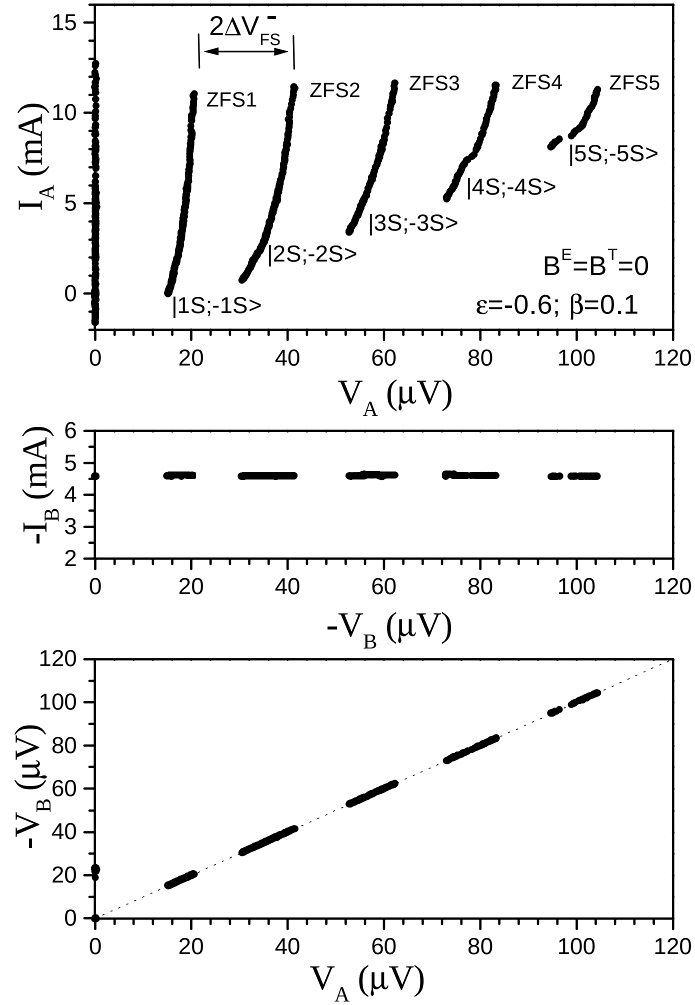
Experimental features of the stacked array /2

From top to the bottom:

ZFS's recorded from the top array of a two-stack while the bottom array is biased with a negative constant current

voltage of the bottom array while the bias current of the top array is swept on the ZFS's

voltage of the bottom array as a function of the voltage of the top array.



The model equations

To model the 2-stack array in Fig. (b) [next page], we start from the description of the 2D double overlap stack in Fig.(a) which is modelled as in G. Carapella *et al.*, Phys. Rev. B **58** (1998).

$$\varphi_{xx} + \varphi_{yy} - \varphi_{tt} = \sin\varphi + \alpha\varphi_t + \varepsilon\left(\psi_{xx} + \psi_{yy}\right), \tag{1}$$

$$\psi_{xx} + \psi_{yy} - \psi_{tt} = \sin\psi + \alpha\psi_t + \varepsilon\left(\varphi_{xx} + \varphi_{yy}\right), \tag{2}$$

$$\varphi_x(0) = \varphi_x(l) = (1+\varepsilon)\eta^E, \tag{3}$$

$$\psi_x(0) = \psi_x(l) = (1+\varepsilon)\eta^E, \tag{4}$$

$$\varphi_y(0) = -(1+\varepsilon)\eta^T - \tfrac{w}{(1+\varepsilon)}\tfrac{\gamma_A - \gamma_B}{2}, \tag{5}$$

$$\varphi_y(w) = -(1+\varepsilon)\eta^T + \tfrac{w}{(1+\varepsilon)}\tfrac{\gamma_A + \gamma_B}{2}, \tag{6}$$

$$\psi_y(0) = -(1+\varepsilon)\eta^T - \tfrac{w}{(1+\varepsilon)}\tfrac{\gamma_B - \gamma_A}{2}, \tag{7}$$

$$\psi_y(w) = -(1+\varepsilon)\eta^T + \tfrac{w}{(1+\varepsilon)}\tfrac{\gamma_A + \gamma_B}{2}, \tag{8}$$

Assuming the junctions in the parallel arrays to be nearly one-dimensional ($\Delta \ll \lambda_J$), the structure in Fig. (b) becomes the continuous system of Fig. (a) in the limit $p \to \infty,\ \Delta y \to 0;\ p\Delta y = W$. For a finite separation between the junctions Δy and for a finite number of junctions p, the system in Fig. (b) is the y-discretized version of the continuous system in Fig. (a). Thus, the model for the two-stack of parallel arrays of long Josephson junctions can be obtained by discretizing the 2D model in the y direction.

Basic behavior $\longrightarrow$ *Perturbed Sine Gordon Equation*

Vertical coupling $\longrightarrow$ *Mutual inductance* $(-1 < \varepsilon < 0)$

Horizontal coupling $\longrightarrow$ *SQUID-like* $(\beta \equiv \Delta y^2/\lambda_J^2)$

Cross coupling $\longrightarrow$ *Mixed* (ε/β)

Boundary $\longrightarrow$ $\mathbf{H} \equiv (H^T, H^E, 0)$ $(\eta^{T,E} = H^{T,E}/H_c)$

The model equations /2

For the internal junctions:

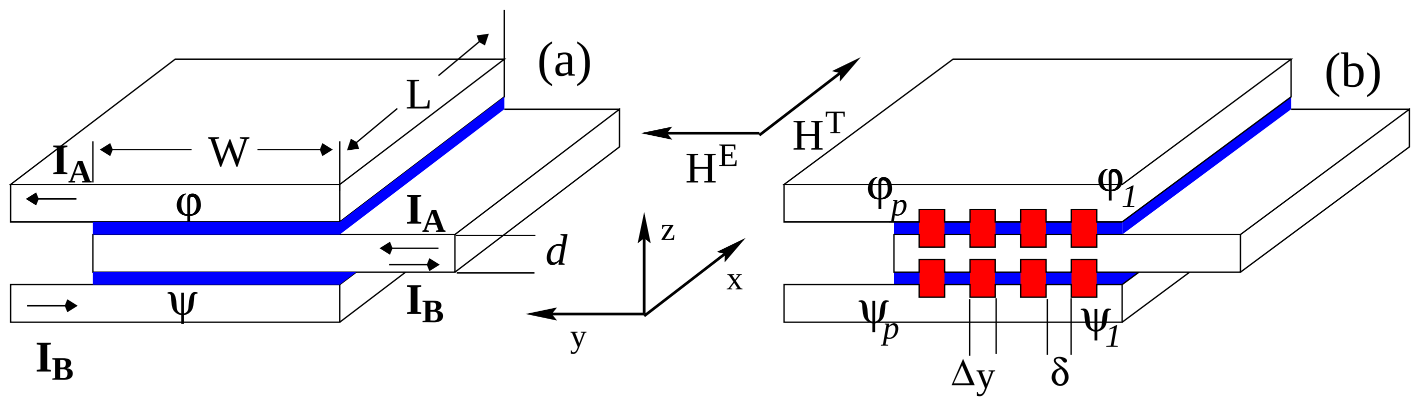
$$\varphi_{ntt} = \varphi_{nxx} - \sin\varphi_n - \alpha\varphi_{nt} - \varepsilon\psi_{nxx} + \frac{1}{\beta}(\varphi_{n+1} - 2\varphi_n + \varphi_{n-1}) - \frac{\varepsilon}{\beta}(\psi_{n+1} - 2\psi_n + \psi_{n-1})$$
$$\psi_{ntt} = \psi_{nxx} - \sin\psi_n - \alpha\psi_{nt} - \varepsilon\varphi_{nxx} + \frac{1}{\beta}(\psi_{n+1} - 2\psi_n + \psi_{n-1}) - \frac{\varepsilon}{\beta}(\varphi_{n+1} - 2\varphi_n + \varphi_{n-1})$$

For the first (1) and the last (p) junctions:

$$\varphi_{(1,p)tt} = \varphi_{(1,p)xx} - \sin\varphi_{(1,p)} - \alpha\varphi_{(1,p)t} - \varepsilon\psi_{(1,p)xx}$$
$$+\frac{2}{\beta}\left(\varphi_{(2,p-1)} - \varphi_{(1,p)}\right) - \frac{2\varepsilon}{\beta}\left(\psi_{(2,p-1)} - \psi_{(1,p)}\right) + \frac{2}{\sqrt{\beta}}\left(1-\varepsilon^2\right)\eta^T + (p-1)(\gamma_A \mp \gamma_B)$$
$$\psi_{(1,p)tt} = \psi_{(1,p)xx} - \sin\psi_{(1,p)} - \alpha\psi_{(1,p)t} - \varepsilon\varphi_{(1,p)xx}$$
$$+\frac{2}{\beta}\left(\psi_{(2,p-1)} - \psi_{(1,p)}\right) - \frac{2\varepsilon}{\beta}\left(\varphi_{(2,p-1)} - \varphi_{(1,p)}\right) + \frac{2}{\sqrt{\beta}}\left(1-\varepsilon^2\right)\eta^T + (p-1)(\gamma_B \mp \gamma_A)$$

Boundary conditions:

$$\varphi_{nx}(0) = \varphi_{nx}(l) = (1+\varepsilon)\eta^E$$
$$\psi_{nx}(0) = \psi_{nx}(l) = (1+\varepsilon)\eta^E$$



Linear wave analysis

Neglecting perturbations and tunneling $\longrightarrow$ Coupled transmission lines
Substituting plane wave solutions $\longrightarrow$ Dispersion relation

$$\begin{pmatrix} \varphi_n(x) \\ \psi_n(x) \end{pmatrix} = \begin{pmatrix} A \\ B \end{pmatrix} e^{i\left[k_x x + (n-1)\sqrt{\beta} k_y - \omega t\right]} \quad \Longrightarrow \quad \omega^{\pm} = u^{\pm} \sqrt{k_x^2 + \frac{4}{\beta} \sin^2 \frac{k_y \sqrt{\beta}}{2}}$$

$u^{+} = \sqrt{1 + |\varepsilon|} \longrightarrow$ velocity of the in-phase ($A = B$) mode
$u^{-} = \sqrt{1 - |\varepsilon|} \longrightarrow$ velocity of the out-of-phase ($A = -B$) mode

With open circuit b.c., cavity mode resonances at

$$\omega_{j,m}^{\pm} = u^{\pm} \sqrt{\left(\frac{j\pi}{l}\right)^2 + \frac{4}{\beta} \sin^2 \frac{m\pi}{2(p-1)}} \qquad j, m \text{ integers} \qquad l \equiv L/\lambda_J$$

Four families of current steps in the $I - V$ characteristic:

- evenly spaced with asymptotic voltages

$$V_j^{\pm} = \frac{j\Phi_0 \overline{c}^{\pm}}{2L} = \frac{j\Phi_0 \overline{c}}{2L} \sqrt{1 \pm |\varepsilon|} = j\Delta V_{FS}^{\pm}$$

- not evenly spaced with asymptotic voltages

$$V_m^{\pm} = \overline{c}^{\pm} \frac{\Phi_0}{\Delta y} \left| \sin \frac{m\pi}{2(p-1)} \right|$$

and upper-limited by the maximum voltages

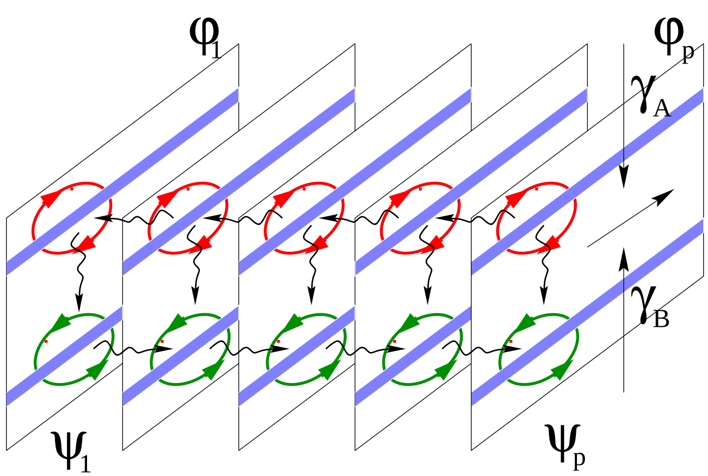
$$V_{\max}^{\pm} = \frac{\overline{c}^{\pm} \Phi_0}{\Delta y}$$

$\overline{c}$ is the Swihart velocity in the stack

Nonlinear wave analysis

To investigate the stability of the solitonic states we shall go through the following steps:

- Consider the unperturbed system in the limit of infinite-length junctions
- Take the basic solitonic solution $\phi = 4\tan^{-1}\left\{\exp\left[\gamma\left(\frac{u}{\sqrt{1-\sigma\varepsilon}}\right)\frac{x-ut}{\sqrt{1-\sigma\varepsilon}}\right]\right\}$
- Build up the ansatz for F-F and for F-A pairs (misaligned) in the V plane
- Build up F-F or F-A strings, misaligned
- Write the Hamiltonian density of the system ...
- ... and the interaction energy between pairs (nearest-neighbours)
- Calculate numerically the energy ...
- ... or get analytical approximation for small misalignment



To calculate the I-V characteristic:

- Assume a stable state
- Apply power balance

The stable fluxon states:

strings (rows) of fluxons in the H planes... ➢ ...bound to strings of antifluxons (fluxons) in the V planes... ➢ ...oscillating with velocity u^- (u^+).

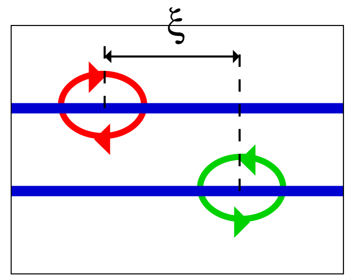
Nonlinear wave analysis /2

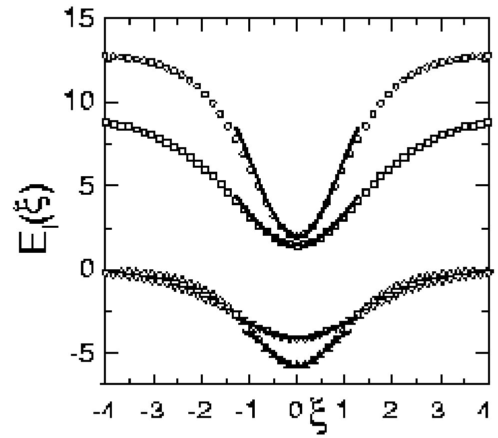
F A interaction in the V-plane

$$E_{\varepsilon}(\xi) = -8\Lambda\sigma\varepsilon\frac{(\Lambda\xi)}{\sinh(\Lambda\xi)}$$

$$\sigma = \pm 1$$

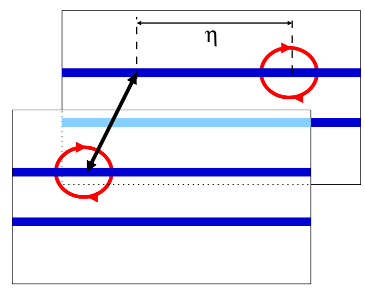
$$\Lambda = \gamma\left(\frac{u}{\sqrt{1-\sigma\varepsilon}}\right)\frac{1}{\sqrt{1-\sigma\varepsilon}}.$$

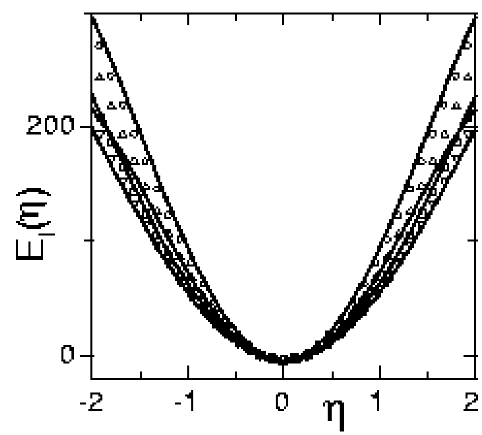




F A (F) interaction in the H-plane

$$E_I(\xi,\eta) \approx -\sigma 8\Lambda\varepsilon + \frac{4}{3}\Lambda^3\sigma\varepsilon\xi^2 + \frac{8}{\beta}\Lambda(1-\sigma\varepsilon)\eta^2 - \frac{7}{45}\sigma\varepsilon\Lambda^5\xi^4 + \frac{4}{3}\Lambda^3\frac{\sigma\varepsilon}{\beta}\xi^2\eta^2 - \frac{2}{9}\frac{\Lambda^3}{\beta}(1-\sigma\varepsilon)\eta^4$$





Work in progress

We have recorded, and are going to compare to the predictions of the model, the following features:

1. The stability range of the static state $(V = 0)$ at $\mathbf{H} = 0$
2. The stability range of the static state at $\mathbf{H} \neq 0$
3. The stability range of the single fluxon string in magnetic field
4. The stability range of the fluxon-antifluxon strings in magnetic field
5. The stability range of the single fluxon string varying the current in the other junction (static)
6. Resonances in the I-V characteristics that might correspond to complex states consisting of fluxons superimposed to other dynamical states (uniform rotation or cavity modes)

To be done: experiments with magnetic field parallel to the long direction

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