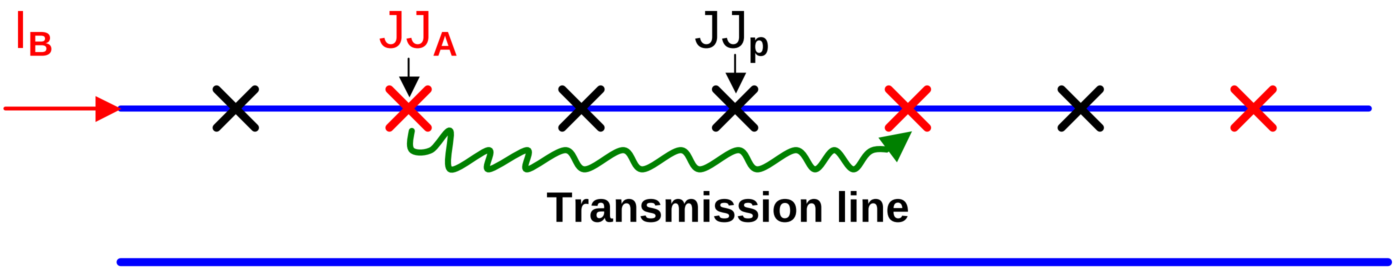
Electromagnetic wave propagation in Josephson junction arrays: low voltage resonances and microwave induced synchronization

Mikhail V. Fistul

Max-Planck Institut für Physik Komplexer Systeme



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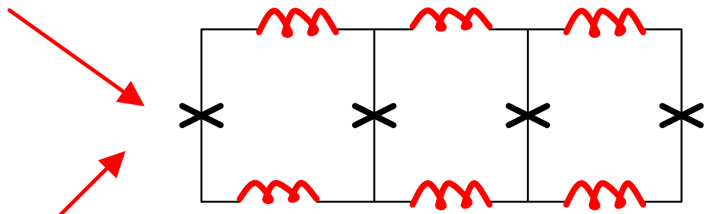
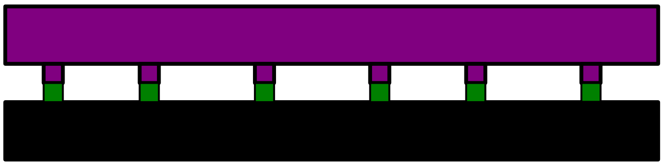
3. **Josephson series array with "ground plane":** **model and equations.**
4. **Branches of I-V curves:** **active and passive junctions.**
5. **Spectrum of linear modes:** **"ground plane" induced GAP in the ω(k).**
6. **Low-voltage resonances:** **the voltage position is determined by ω(0).**
7. **Amplitude and length of the JJ interaction:** **dependence on the parameters.**
8. **Disorder:** **critical current spread, random distances between active JJ.**
9. **Synchronization length:** **interaction vs disorder.**

Various Josephson transmission lines: basic properties

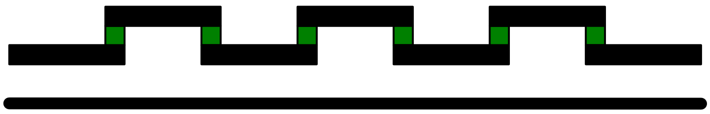
Long Josephson junctions

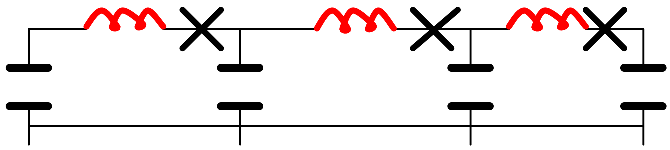


Strip-line parallel array



Josephson series array with “ground plane





Basic properties

Applied DC bias current: I_B

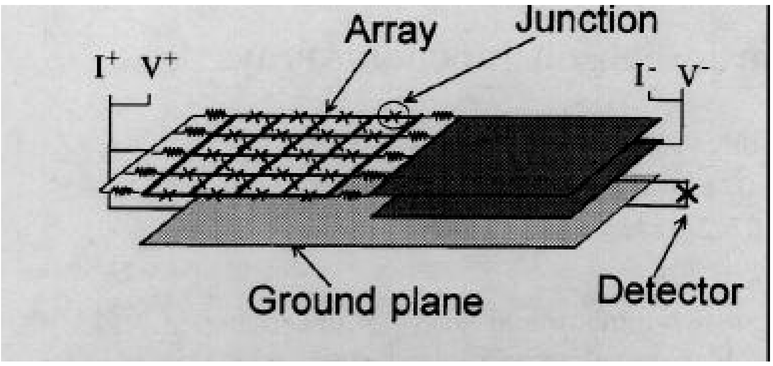
Josephson AC current excites EMW

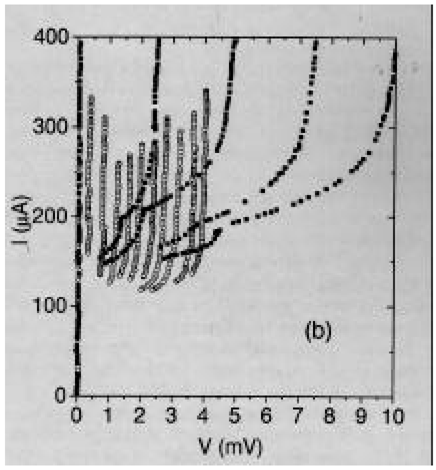
DC Voltage V corresponds to the frequency W of EMW

Current-Voltage resonances are determined by the spectrum of EMW

EXPERIMENTS:
JJs embedded in the transmission line

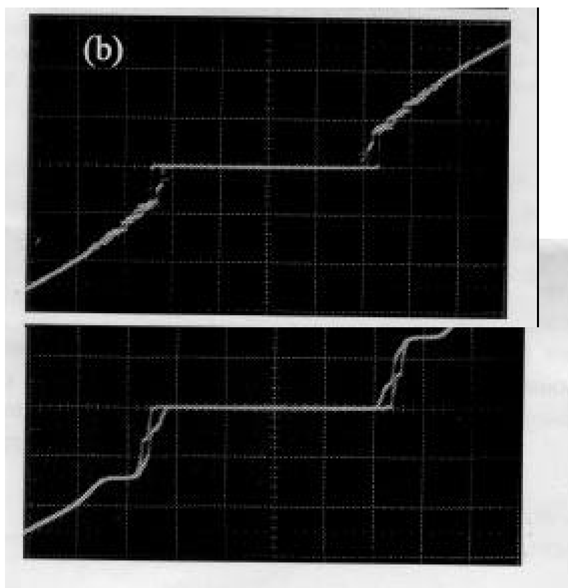
Underdamped Josephson junction arrays



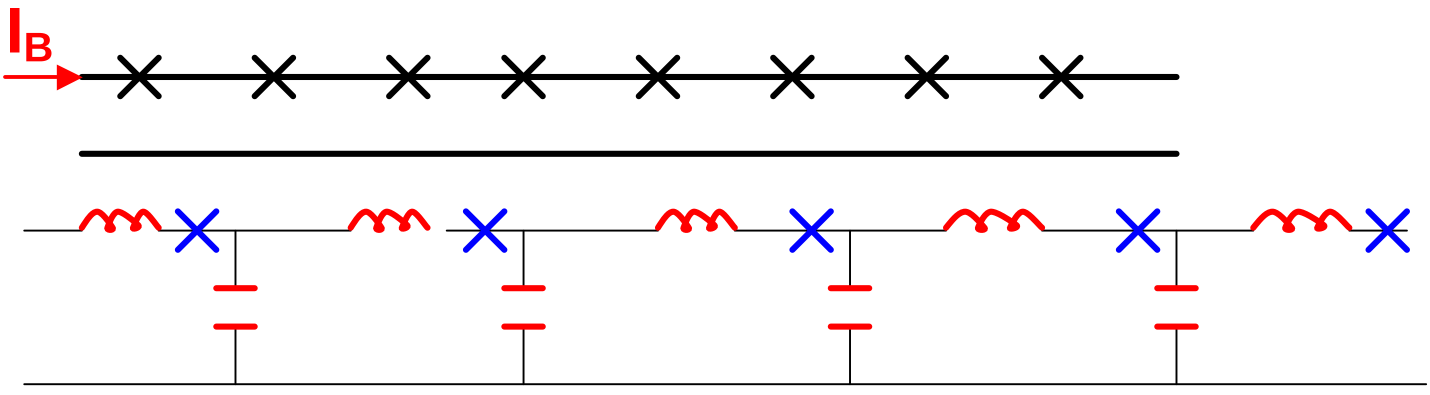


Low voltage resonances (row switching);
Radiation from Josephson junction array as $N>N_{cr}$

Overdamped Josephson junction series array



Josephson series array with “ground plane”: model and equations



Josephson junctions: ***RSJ model, I_C, R, C***

Transmission line: **L_0, C_0**

Equations:

I_n is the alternating current in the n-th cell

V_n is the voltage at the end of the n-th cell

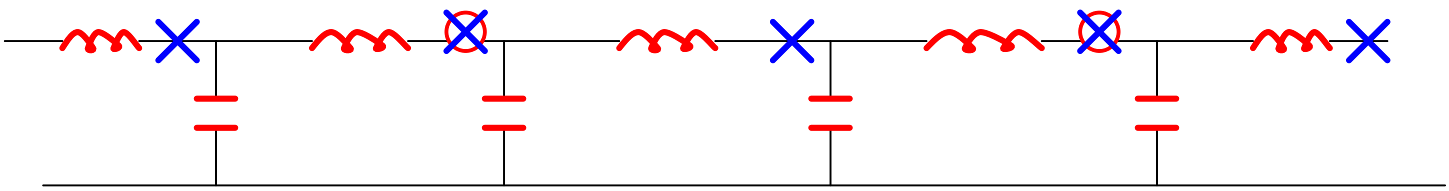
j_n is the Josephson phase of the n-th JJ.

$$V_{n+1} - V_n = L_0 \frac{dI_n}{dt} + \frac{\hbar}{2e} \frac{dj_n}{dt}$$

$$I_n - I_{n-1} = C_0 \frac{dV_n}{dt}$$

$$I_B + I_n = C \frac{\hbar}{2e} \frac{d^2 j_n}{dt^2} + \frac{\hbar}{2eR} \frac{dj_n}{dt} + I_c \sin(j_n)$$

Branches of I-V curves:
active and passive junctions



JJs that are in the voltage (rotation) state: active junctions

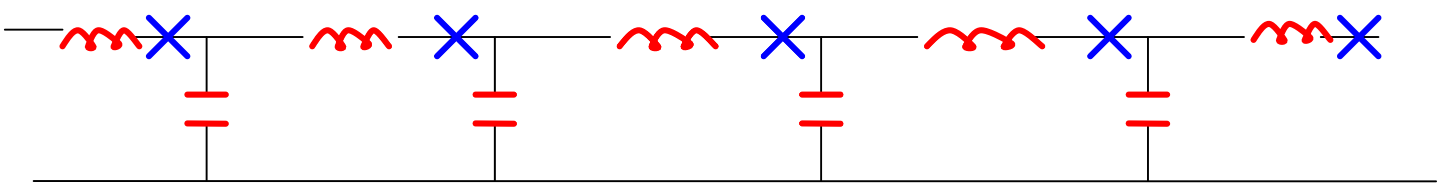
$$j_A = wt + y_A + j_{A,1} e^{iwt}$$

JJs that are in the superconducting (libration) state: passive junctions

$$j_P = j_0 + j_{P,1} e^{iwt}$$

Spectrum of linear EMW:

“ground plane” induced GAP in the ω(k)



Linear modes in JJ transmission line:

$$I_{n+1}+I_{n-1}-2I_n\omega^2\left\{C_0L_0+\frac{\hbar C_0}{2e}\,\frac{1}{I_c\cos(\varphi_0)-\frac{C\hbar}{2e}\omega^2+\frac{\hbar i\omega}{2eR}}\right\}=0$$

Spectrum of linear modes:

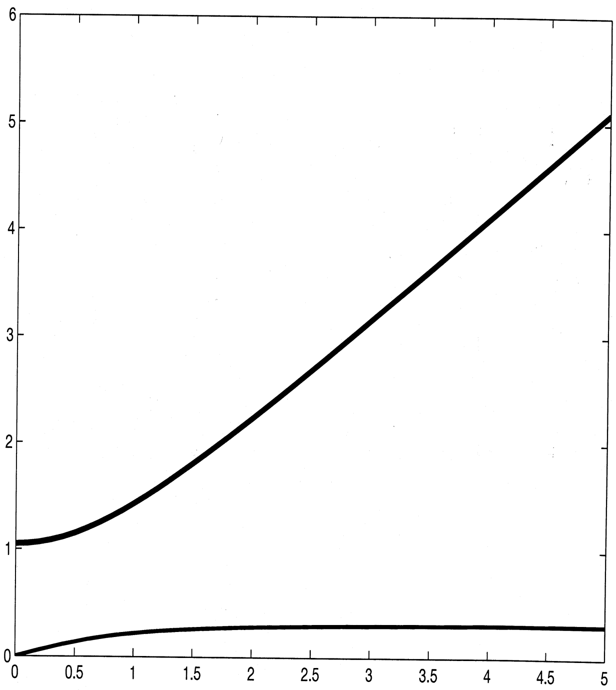
1. **C=0, ω(k)~k (“standard transmission line”)**
2. **C>0, two branches, GAP**

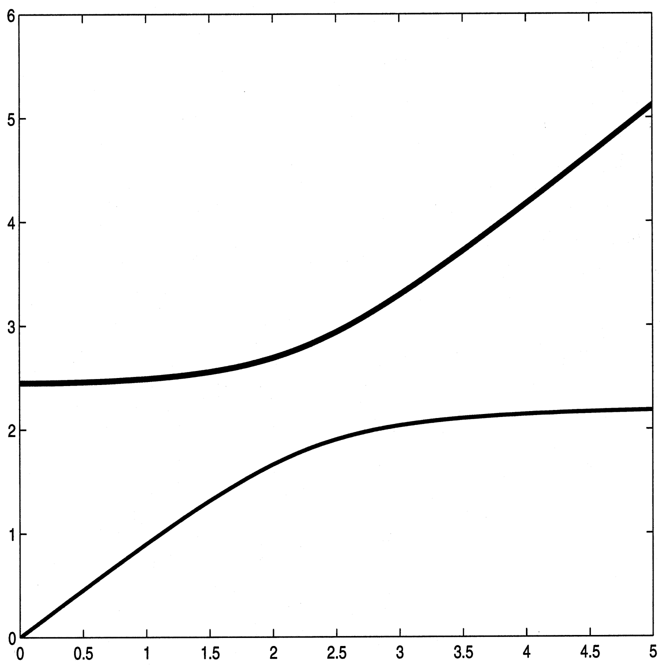
Dependence on

$$\beta=\frac{2eI_cL_0}{\hbar}$$

GAP=

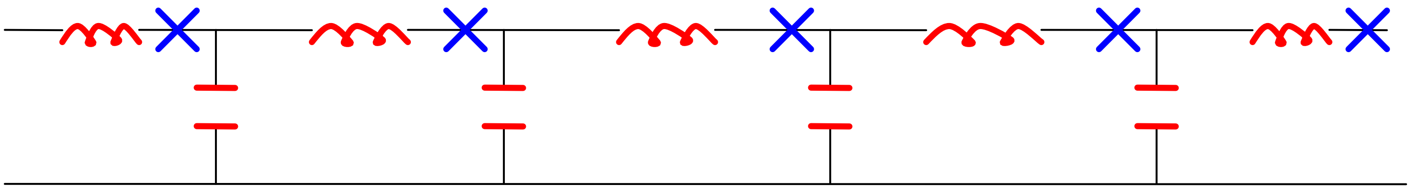
$$\begin{cases} \dfrac{1}{\sqrt{L_0 C}}, & b \ll 1 \\ b\cos(j_0)\dfrac{1}{\sqrt{L C_0}}, & b \gg 1 \end{cases}$$





b**<<1** b**>>1**

Low-voltage resonances

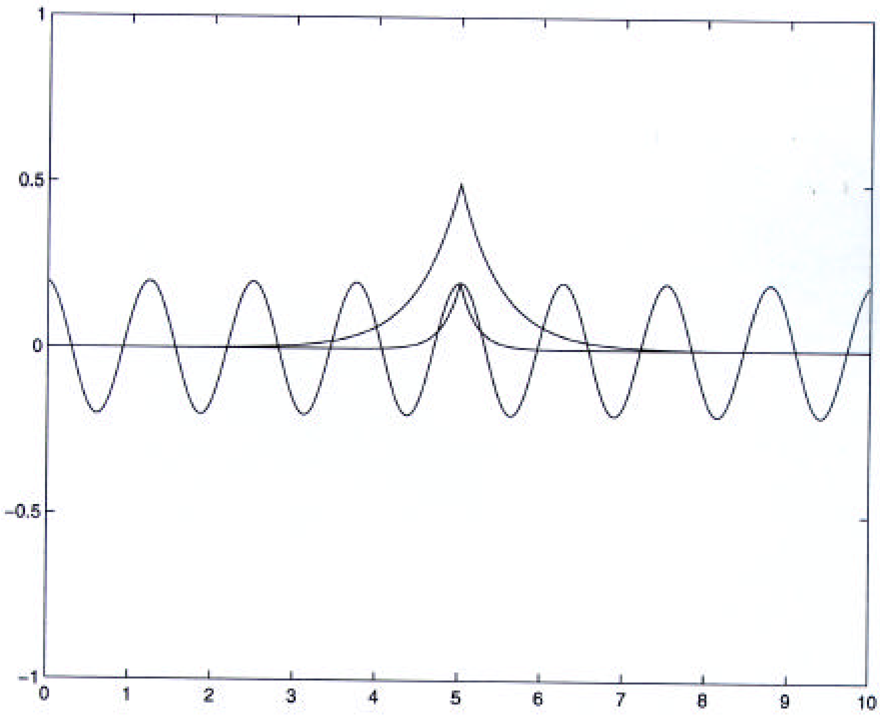


The AC Josephson phase distribution along the transmission line in the presence of active junctions

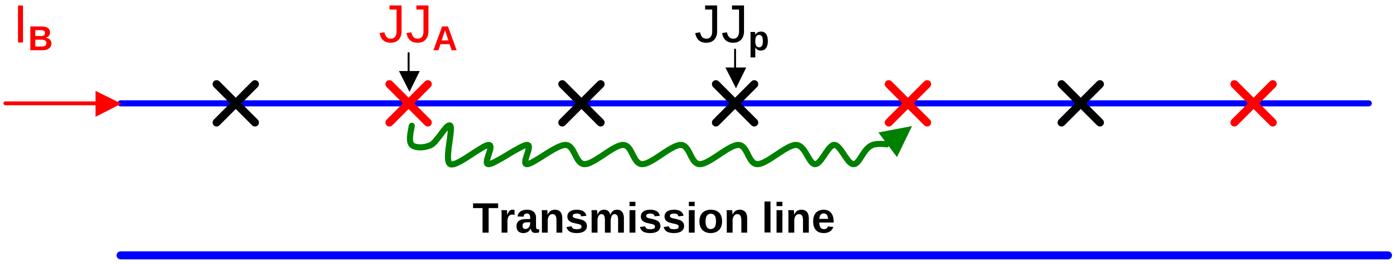
$$\frac{\partial^2 I(x)}{\partial x^2} + \frac{I(x)}{\omega_{tr}^2}\left[\omega^2 - \omega_G^2 + i\alpha\omega_G\right] = \frac{C_0}{C}\sum_n e^{i\psi_n}\delta(x - x_n)$$

$$\omega_{tr} = \frac{1}{\sqrt{L_0 C_0}}, \qquad \alpha = (RC)^{-1}$$

The resonance voltage positions are determined by $\omega(0)=\omega_G$!



“Attenuation” length in JJ transmission lines: interaction **length between active junctions**



$$I(x) = \int_0 dx_1 G(x, x_1) \left[\frac{C_0}{C} e^{i\psi(x_1)} \sum_n \delta(x_1 - x_n) \right]$$

$$G(x, x_1) = -\frac{1}{2k} e^{-k|x - x_1|}, \qquad k = \frac{1}{\omega_{tr}} \sqrt{\omega^2 - \omega_G^2 + i\alpha\omega_G}$$

The interaction length:

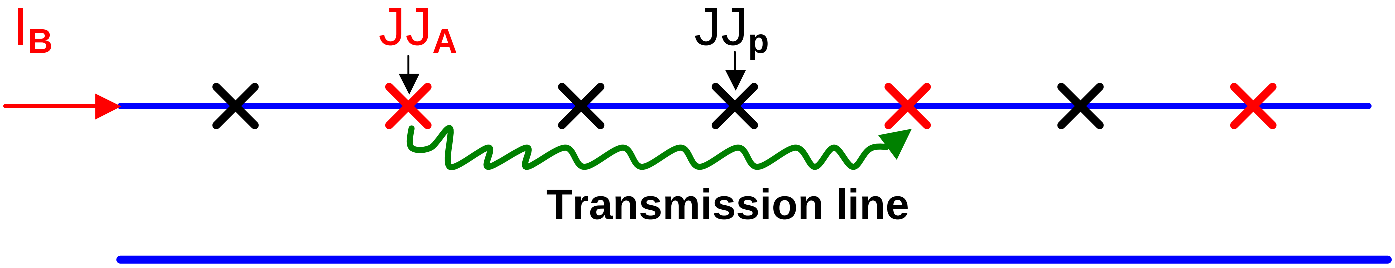
$$L_{in} = (\mathrm{Re}\, k)^{-1} = \frac{\omega_{tr}}{\sqrt{\alpha\omega_G}}$$

(~30-40 cells, depends on the damping α**)**

The interaction strength:

$$\chi \approx \frac{C_0}{C} \frac{1}{(\mathrm{Re}\, k)^2}$$

Interaction vs Disorder:
synchronization equation



Disorder: **critical current spread ($\Delta I_c/I_c \sim 5\%$);**
Random distances between active junctions

Synchronization equation

$$\mathrm{Im} \sum_j G(x_i, x_j)\left(e^{i[\psi(x_i) - \psi(x_j)]} - 1\right) = 0$$

Synchronization length:

mapping to “Anderson localization”

Synchronization equation

(the Schrodinger Equation with a random potential)

$$\frac{d^2 y(x)}{dx^2} - \frac{1}{c}\frac{dn(x)}{n_0} y(x) = 0$$

$$\left(\frac{d^2 y(x)}{dx^2} + U(x) y(x) = 0), \qquad < U(x)U(y) >= D d(x - y) \qquad \right)$$

Sychronization length (Localization length)

$$L_S \text{ » } c^{2/3} n_0^{1/3} \qquad (L_L \text{ » } D^{-1/3})$$

$$n_0 > \frac{L^3}{c^2} \text{ ® } \quad SYNCHRONIZATION$$

Summary

Spectrum of linear modes: **“ground plane” induced GAP in the w(k).**

Low-voltage resonances: **the voltage position is determined by w(0).**

Amplitude and length of the JJ interaction: **dependence on the parameters.**

Disorder: **critical current spread, random distances between active JJ.**

Synchronization length: **interaction vs disorder.**