Observation of progressive motion of ac-driven solitons

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Abstract

We report the first experimental observation of ac-driven phase-locked motion of a topological soliton at a nonzero average velocity in a periodically modulated lossy medium. The velocity is related by a resonant condition to the driving frequency. The observation is made in terms of the current-voltage, $I(V)$, characteristics for a fluxon trapped in an annular Josephson junction placed into dc magnetic field. Large constant-voltage steps, corresponding to the resonantly locked soliton motion at different orders of the resonance, are found on the $I(V)$ curves. An experimentally measured dependence of the size of the steps vs. the external magnetic field is in good agreement with predictions of an analytical model. The effect has a potential application as a low-frequency voltage standard.

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The important role played by collective nonlinear excitations in the form of solitons in various physical systems is commonly known. Experimental observation of dynamical effects produced by solitons is often difficult because real systems may be far from their idealized mathematical models which give rise to solitons. Among perturbations that destroy soliton effects, dissipation is the most important one. To compensate dissipative losses and thus make the soliton dynamics visible, one must apply an external force that is capable to support (in particular, to *drive*) a soliton[1].

Solitons of the simplest type are topological *kinks*, a well-known example of which is a magnetic flux quantum (fluxon) in long Josephson junctions (LJJs)[2,3]. A fluxon in LJJ can be easily driven by bias current applied to the junction. The motion of a fluxon gives rise to dc voltage V across the junction, which is proportional to the mean velocity of the fluxon. Varying the dc bias current I, one can obtain a dependence $V(I)$, which is the main dynamical characteristic of LJJ. An experimentally obtained $I(V)$ curve easily allows one to identify the presence of one or more fluxons trapped in LJJ[2].

Microwave field irradiating LJJ gives rise to an ac drive acting on the fluxon. In a spatially homogeneous lossy system, an ac drive may only support an oscillatory motion of a kink, which is hard to observe in LJJ due to the absence of dc voltage. However, it was theoretically predicted in Ref. 4 that an ac drive can support motion of a kink with a *nonzero* average velocity u in a system with a periodic spatial modulation. Indeed, a moving kink passes the modulation length (period) L during the time L/u. If this time is commensurate with the period $2\pi/\omega$ of the ac drive, i.e., $m(L/u) = 2\pi/\omega$ with an integer m, or

$$u = m(L\omega/2\pi), \tag{1}$$

one may expect a resonance (of order m) between the two periodicities. In other words, a moving kink may be phase-locked to the ac drive. This provides for permanent transfer of energy from the drive to the kink, making it possible to compensate the dissipation of energy[5]. The energy balance gives rise to a minimum (*threshold*) value Γ_{thr} of the ac drive's amplitude Γ, which can compensate the energy losses and support the motion of the ac-driven kink.

A more general case, when the system is simultaneously driven by the ac field and dc field I (dc bias current, in the case of LJJ), was considered in Ref. 6. It was predicted that the corresponding $V(I)$ characteristic has *steps* (constant-voltage segments) at the resonant velocities (1). The motion of the fluxon under the action of pure ac drive then corresponds to *zero-crossings*, when the steps cross the axis $I = 0$. In fact, the most straightforward way to observe the ac-driven motion of the soliton is through zero-crossings on the $V(I)$ characteristic.

A formally similar feature is known in *small* Josephson junctions as Shapiro steps[2]: ac drive applied to the junction gives rise to dc voltage across it (an inverse ac Josephson effect). However, a drastic difference of the effect sought for in this work from the Shapiro steps is that they are only possible at high frequencies exceeding the junctions's plasma frequency, while we will demonstrate below that the ac-driven motion of the fluxon can be supported by the ac drive with an arbitrarily low frequency. This circumstance also opens way for application to the design of voltage standards using easily accessible sources of low-frequency radiation, which are not usable with the usual voltage standards based on small Josephson junctions.

An objective of this paper is to report direct experimental observation of the ac-driven soliton motion in periodically modulated LJJs. Frequently, it is assumed that the necessary periodic spatial modulation along the junction can be induced by periodically changing the thickness of the dielectric barrier separating two superconductors. In the presence of the losses and drive, thus modulated LJJ is described by the perturbed sine-Gordon (sG) equation,

$$\phi_{tt} - \phi_{xx} + \left(1 + \varepsilon \sin \frac{2\pi x}{L}\right) \sin \phi = -\alpha \phi_t - \gamma - \Gamma \sin \omega t, \qquad (2)$$

where x and t are the length along the junction and time, measured, respectively, in units of the Josephson length λ_J and inverse plasma frequency ω_p^{-1}, ε is the normalized modulation amplitude, while $\gamma = j/j_c$ and $\Gamma = j_{ac}/j_c$ are the dc and ac bias current densities, respectively, normalized to the junction's critical current density j_c.

The above model assumes the modulation of the local magnitude of the maximum Josephson current to be harmonic. The latter condition is very hard to realize in an experiment due to the exponential dependence of the critical current on the thickness of the dielectric barrier. A much easier and fully controllable way to induce a strictly harmonic periodic modulation can be realized in an *annular* (ring- shaped) LJJ, to which uniform dc magnetic field H is applied in its plane[7]. As it was demonstrated experimentally, fluxons can be readily trapped in such a LJJ[8,9]. In this case, the sG model takes the form

$$\phi_{tt} - \phi_{xx} + \sin\phi + h\sin\frac{x}{R} = -\alpha\phi_t - \gamma - \Gamma\sin\omega t, \tag{3}$$

where h is a renormalized magnetic field, and R is the radius of the ring. Periodic boundary conditions supplementing Eq. (3) in the case of the annular junction are $\phi_x(x+2\pi R) \equiv \phi_x(x)$ and $\phi(x+2\pi R) \equiv \phi(x) + 2\pi N$, N being the number of the trapped fluxons (in this work, $N = 1$). Comparison with the experiment shows that, unlike the model (2), the one (3) is, virtually, exact[10].

We assume the spatial size of the fluxon, which is ~ 1 in the present notation, to be much smaller than the circumference $L \equiv 2\pi R$ of the junction. The large value of L imposes an upper limit on the driving frequency ω which can support the ac-driven motion: as the fluxon's velocity cannot exceed the maximum (Swihart) group velocity of the electromagnetic waves in LJJ, $\equiv 1$ in our notation, Eq. (1) demands that $\omega \lesssim 1/L$.

A different type of the ac drive for fluxons in circular LJJs was proposed in Ref. 7, viz., ac magnetic field. In this case, the terms $h\sin(x/R)$ and $\Gamma\sin(\omega t)$ are replaced by a single one, $h\sin(x/R)\sin(\omega t)$, which may be naturally decomposed into two waves traveling in opposite directions, $(1/2)[\cos(x/R - \omega t) - \cos(x/R + \omega t)]$. As it was shown analytically and numerically[7], either traveling wave may capture a fluxon, dragging it at the wave's phase velocity $\pm\omega R$. Another model belonging to the same type was proposed in Ref.[11], in which the fluxon is dragged by *rotating* magnetic field. A difference of our model (that corresponds to real experiments reported below) is the separation between the fields that induce the spatial modulation and ac force, which admits to control the dynamics in a more flexible way.

It is straightforward to derive an equation of motion for the fluxon in the adiabatic approximation, following the lines of Refs. 4, 6 ($\dot{\xi} \equiv d\xi/dt$):

$$\frac{d}{dt}\left(\frac{\dot{\xi}}{\sqrt{1-\dot{\xi}^2}}\right) = \frac{\pi h}{4\sqrt{1-\dot{\xi}^2}}\cos\frac{\xi}{R} - \frac{\alpha\,\dot{\xi}}{\sqrt{1-\dot{\xi}^2}} + \frac{\pi}{4}\left[\gamma + \Gamma\sin(\omega t)\right] . \tag{4}$$

For further analysis, one may assume (as it was done in Refs. 7, 11) that, in the lowest approximation, the fluxon is moving at a constant velocity $\dot{\xi}_0 \equiv u$ belonging to the resonant spectrum (1), so that $\xi(t) = ut + R\delta$, where δ is an arbitrary constant. Then, the first correction to the instantaneous fluxon's velocity, generated by the spatial modulation, can be easily found from Eq. (4),

$$\dot{\xi}_1 = (\pi Rh/4u)\left(1-u^2\right)\sin\left[(u/R)t+\delta\right] . \tag{5}$$

The approximation applies provided that the correction (5) is much smaller than the unperturbed velocity u, which amounts to a condition $Rh \ll u^2/\left(1-u^2\right)$.

A key ingredient in the dynamical analysis of this problem is the *energy-balance* equation[7]. In the model (3) it is based on the correction (5) to the velocity[4,6], while in the above-mentioned models with ac magnetic fields[7,11] the approximation $\dot{\xi} = u$ was sufficient. In the case of the fundamental resonance ($m = 1$ in Eq. (1)), i.e., $u = R\omega$, the energy balance yields, after a straightforward algebra,

$$\gamma = \frac{4\alpha u}{\pi}\left(1-u^2\right)^{-1/2} - \frac{\pi Rh\Gamma}{8u^2}\left(1-u^2\right)\cos\delta. \tag{6}$$

Setting $|\cos\delta| = 1$ and $\gamma = 0$ in Eq. (6) gives a minimum (*threshold*) amplitude of the ac drive which can support the fluxon's motion in the absence of the dc bias current, $\Gamma_{\rm thr} = (32/\pi^2)\,(\alpha/Rh)u^3\left(1-u^2\right)^{-3/2}$. For the comparison with experimental results, the most important consequence of Eq. (6) is an interval of γ in which the phase-locked ac-driven motion of the fluxon is expected. This is produced by varying $\cos\delta$ between -1 and $+1$:

$$\gamma_1^- < \gamma < \gamma_1^+;\ \gamma_1^{\pm} \equiv \frac{4\alpha u}{\pi\sqrt{1-u^2}} \pm \frac{\pi R \Gamma\,(1-u^2)}{8u^2} h. \tag{7}$$

Note that the size of the interval, $\gamma_1^+ - \gamma_1^-$, strongly depends, via the relation $u = R\omega$, on the driving frequency, while in the model with the ac magnetic field[7] it does not depend on ω at all, provided that $2\pi R \gg 1$.

Experiments have been performed with Nb/Al-AlO_x/Nb Josephson annular junction with the mean diameter $2R = 95\,\mu$m and the annulus width $5\,\mu$m, applying the bias current I and measuring the dc voltage V across the junction. The distribution of the bias current was uniform, which was concluded measuring the critical current I_c in the state without trapped fluxons at $H = 0$. I_c was found to be about 0.9 of its value for the small junction. The annular LJJ had the Josephson length $\lambda_J \approx 30\,\mu$m and plasma frequency $f_p \equiv \omega_p/2\pi \approx$ 50 GHz. Note that these parameters imply the ratio ~ 10 of the junction's length $2\pi R$ to the fluxon's size, which is $\sim \lambda_J$, i.e., the junction may indeed be regarded as a long one. The measurements were done at $T = 4.2$ K, using a shielded low- noise measurement setup. The ac driving current with $f = \omega/2\pi$ between 5 and 26 GHz was supplied by means of a coaxial cable ending with a small antenna inductively coupled to the junction. The power levels mentioned below pertain to the input at the top of the cryostat.

Following Ref. 10, trapping of a fluxon in the junction was achieved by cooling the sample below the critical temperature $T_c \approx 9.2$ K for the transition of Nb into the superconductive state, while a small bias current was applied to the junction. At $H = 0$, the fluxon depinning current $I_{\rm dep}$ was found to be very small, less than 1% of the Josephson critical current I_c measured without the trapped fluxon. As a fluxon can only be trapped by junction's local inhomogeneities in the absence of the magnetic field, this indicates at fairly high uniformity of the junction. At low values of the field H, linear increase of $I_{\rm dep}$ with H was observed, which is well described by the theoretical model based on Eqs. (3) and (4): the zero-voltage state exists as long as the maximum fluxon's pinning force exerted by the field-induced potential remains larger than the driving force induced by dc bias current, which is satisfied at $|\gamma| < h$. In the low-field range, fluxon depinning and re-trapping in the external magnetic field have been already studied experimentally and analytically in Ref. 10.

An evidence for the progressive motion of the fluxon under the ac drive is presented in Fig. 1a. This $I(V)$ characteristic was measured at $H = 0.35\,\mathrm{Oe}$ and $f = 18.1\,\mathrm{GHz}$. Its salient feature is two large symmetric constant-voltage steps. The points where they intersect the zero-current axis correspond to the fluxon moving around the junction with a nonzero average velocity at zero dc driving force. Another remarkable feature is the absence of any step at the zero voltage, i.e., in the present case the fluxon cannot be trapped by the effective potential, even when the dc bias current is small. For comparison, in Fig. 1b we show the $I(V)$ curve measured at the same power and frequency of the drive, but with $H = 0$. In this case, a substantial zero-voltage step is seen, extending up to the current $I_0 \approx \pm 0.1$ mA. In the absence of the ac drive, the critical current I_0 is much smaller, less than $20\,\mu\mathrm{A}$ (that residual I_0 may be explained by small inhomogeneities of LJJ, see above).

The conspicuous zero-voltage step in Fig. 1b may be explained by the fact that the magnetic component of the ac drive creates its own modulated potential. This argument also helps to explain two symmetric constant-voltage steps at $V \approx 37\,\mu\mathrm{V}$ in Fig. 1b as resonant steps supported by the ac-drive-induced modulation. Note, however, that the latter steps do *not* feature zero-crossing. All the data collected in the experiments show that the zero crossing is possible solely on the resonant steps that occur in the presence of dc magnetic field. In other words, the ac-driven motion of the fluxons is not possible without a stationary spatially periodic potential.

Coming back to the resonant steps induced by the dc field H, which is the main subject of the work, we have also measured their size vs. H, see Fig. 2. The result is that both edge values I_1^+ and I_1^- indicated in Fig. 1 vary nearly linearly with H, up to $H \approx 0.37\,\mathrm{Oe}$. At still larger fields, the phase- locked ac-driven motion of the fluxon gets interrupted in some current range (the perturbation theory does not apply to so strong fields). These findings are in good agreement with the theoretical prediction given by Eq. (7), as concerns both the upward shift of the lines $I_1^{\pm}(H)$ (recall that I and H are proportional to γ and h, respectively) and their linear change with the magnetic field. The residual nonzero value of

$I_1^+ - I_1^-$ at $H = 0$ matches the small non-zero-crossing step in Fig. 1b. It is noteworthy too that the current range of the zero-voltage state, $I_0^- < I < I_0^+$ (see Fig. 1b), decreases nearly linearly with H, disappearing at $H \approx 0.09\,\mathrm{Oe}$.

Equation (7) also predicts a dependence of the step's size on the ac-drive's amplitude. Comparison with experimental data shows an agreement in a certain power range. As for the dependence on the ac-drive's frequency at a fixed value of the amplitude, it is hard to measure it, as variation of the frequency inevitably entails a change in the ac power coupled to the junction.

All the above results pertained to the fundamental resonance, with $m = 1$ in Eq. (1). It is also easy to observe zero-crossings corresponding to higher-orders resonances. This is illustrated in Fig. 3, showing $V(I)$ curves with the resonant steps generated by both the fundamental and second-order (i.e., corresponding to $m = 2$ in Eq. (1)) resonances.

The effects described above, i.e., the zero-crossing steps at finite voltages and disappearance of the zero-voltage state, have been observed in a broad range of the ac frequencies, starting from about 5 GHz and up. On the other hand, as it was mentioned above, the condition that the moving fluxon cannot exceed the Swihart velocity $\bar{c}$ sets a natural upper cutoff for the frequency that can support the phase-locked motion of fluxons. In our junctions, $\bar{c}$ corresponds to the dc voltage $V = \bar{c}\Phi_0/(2\pi R) \approx 80\,\mu\mathrm{V}$, which translates, via Eq. (1), into the cutoff frequency $\sim 40\,\mathrm{GHz}$ for the case of the fundamental resonance.

In conclusion, we have reported the first observation of ac-driven motion of a soliton topological soliton in a periodically modulated lossy medium. The observation was made in an annular uniform Josephson junction placed into constant magnetic field. Experimentally measured data, such as the size of the constant-voltage step, are in good agreement with predictions of the analytical model. The effect may take place in a broad class of nonlinear systems and, in terms of the Josephson junctions, it may have a potential application as a low-frequency voltage standard.

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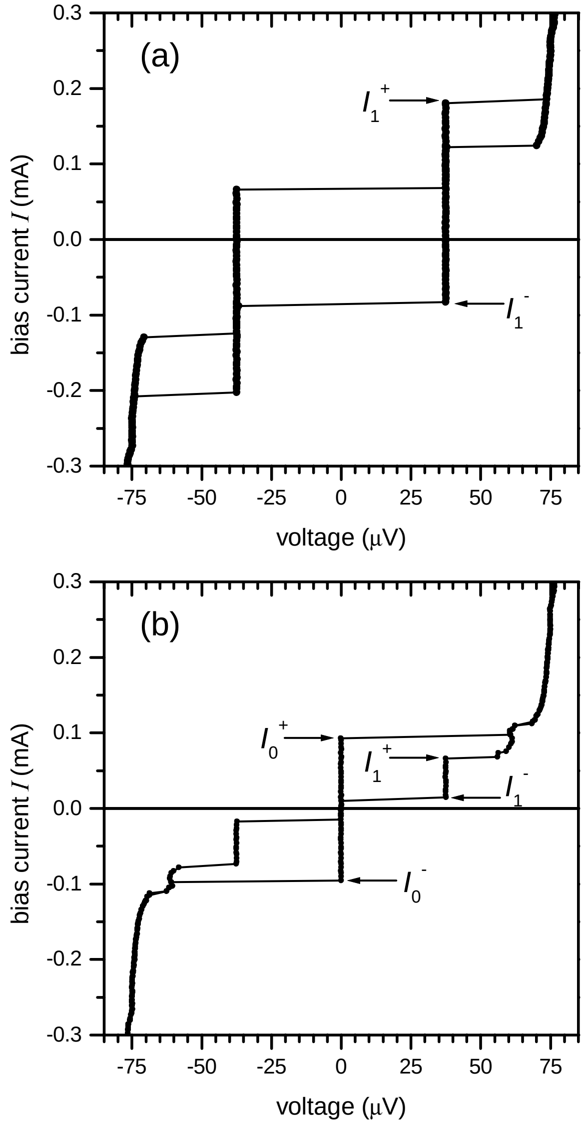

FIG. 1. Current-voltage characteristics for a fluxon trapped in the annular Josephson junction irradiated by the ac signal with the frequency 18.1 GHz and power $P_{\rm ac} = -8\,{\rm dBm}$. The dc magnetic field is (a) $H = 0.35\,{\rm Oe}$ and (b) $H = 0$.

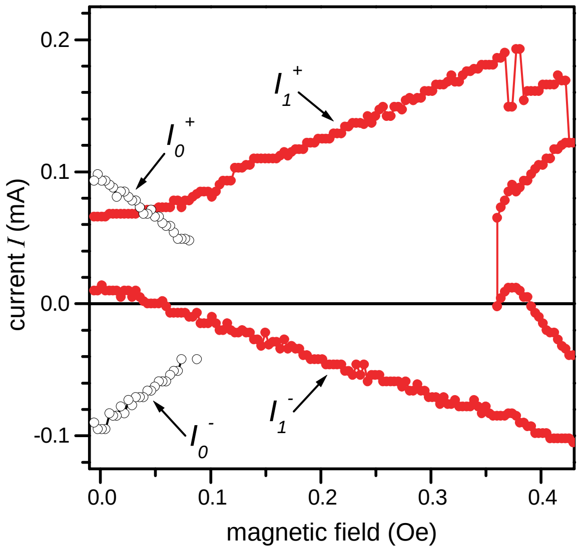

FIG. 2. The critical values $I_0^{\pm}$ and $I_1^{\pm}$ of the dc bias current, indicated in Fig. 1, vs. the external dc magnetic field.

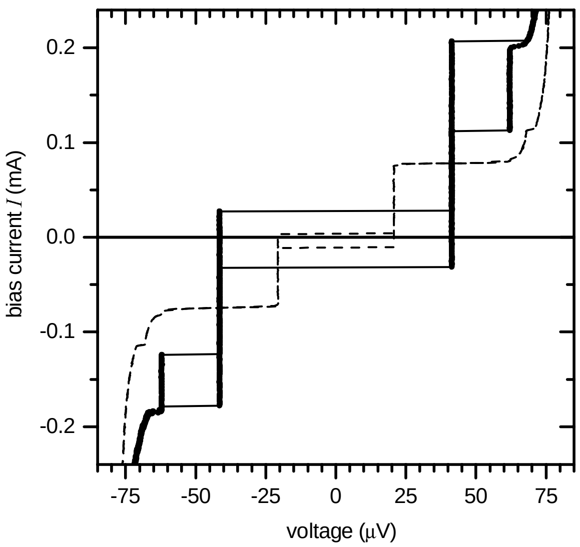

FIG. 3. Current-voltage characteristics for a fluxon in the annular Josephson junction at $H = 0.40\,\mathrm{Oe}$, irradiated by the ac signal with the frequency 10.0 GHz. The signal's power $P_{\rm ac}$ is $-3.4\,\mathrm{dBm}$ (solid line) and $-12.4\,\mathrm{dBm}$ (dashed line). The constant-voltage steps on the two lines correspond, respectively, to the second-order and fundamental resonance in Eq. (1).