Some Properties of Eck-like Steps in 2D Underdamped Josephson junctions Arrays

Cinzia De Leo and Giacomo Rotoli

Abstract— **Two-dimensional (2D) Josephson Junctions arrays have been studied largely in the overdamped case. Only recently some experimental, numerical and theoretical results have been obtained in the underdamped case. The nature of dynamical states of an array when placed in a magnetic field is again not fully explained and only approximate (linear) model have been proposed for its explanation. On the other hand the comprehension of such dynamic is of great importance for the understanding of the behavior of the arrays especially in view of applications. In this work we study numerically some of the dynamical states in 2D arrays using a full inductance matrix in the array equations. The results show that the response of the array to magnetic field is governed at large by the parameter β_L. Moreover we study the underlying dynamics which is dominated by so-called checkerboard solutions. The properties of such arrays are investigated in view of applications.**

***Index Terms*—Superconducting Devices, Cryogenics Electronics, Active Circuits, Oscillators.**

I. Introduction

AMONG Superconducting devices proposed in recent years two-dimensional Josephson junction arrays (2D-JJA) have been one of the most promising for application in integrated mm and submm cryogenic circuits mainly as local oscillators. The main feature of 2D-JJA is the possibility for a large number of Josephson junctions to coherently phase-locke together and thus obtain a relevant power emission. At this moment Long Josephson junctions (LJJ) used as Flux-Flow oscillators (FFO) biased on the Eck step [1] are presently engineered as integrated component on the same chip to work as local oscillator for a SIS mixer [2]. Here the main advantages of FFO oscillators are the low dissipation (that implies a very narrow linewidth) and the easy biasing on well-known resonances in their I-V characteristic where power conversion from dc-bias to an ac signal occurs.

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In these respects the state-of-the-art of 2D-JJA appears to be less advanced. Though in principle the power emitted from an array can be larger than that from a LJJ, only in the last couple of years underdamped arrays have been fabricated and their operation demonstrated [3,4].

Perhaps one of the obstacles to the development of a reliable oscillator based on 2D-JJA was the complication introduced by the analysis of a discrete network of Josephson junctions. The parameter that measures the coupling of junctions in an array is the SQUID parameter $\beta_L = 2\pi I_0 L/\Phi_0$ where I_0 is the junction critical current, L the loop inductance, Φ_0 the flux quantum. In low β_L arrays (typically $\beta_L < 0.5$) the network be equations can simplified using some of the next-neighbors approximations for the mutual inductance between array loops. Anyway most arrays have been realized with $\beta_L > 1$, in this case only a model including the full mutual inductance matrix between array loops can give a realistic array description [5].

2D-JJAs have been fabricated in different geometries [3,4,5]. The most frequent is the simple so-called horizontal square Josephson ladder (H-ladder) in which a single row of meshes is fed by a bias current in all of the junctions common to two adjacent meshes (cf. Fig.1a). Arrays formed by two or three of these rows have been also considered. Another geometry is the vertical ladder (V-ladder), in which a single *column* of meshes is feed (cf. Fig.1b). Two or more of such columns form arrays used in [4]. Similar geometry can be fabricated also for the triangular mesh case [3].

In the recent experiments on underdamped arrays coherent emission state is reached when the array is placed in a magnetic field perpendicular to the array plane. In this case the I-V characteristics of the array shows steps on which it is possible to bias the device and obtain a coherent signal. In the case when there is an applied magnetic field it is possible to make a simple cavity resonances theory based on the checkerboard pattern solutions [6,7]. It shows that the array behavior under the applied magnetic field is dominated by two singularities V_+ and V_-. The higher frequency one, V_+, is considered the analogous to the Eck step in LJJ (an Eck-like step was first identified in discrete 1D arrays by Orlando and Van der Zant [8]). On the other end some multi-row arrays seem show a richer structure in their I-V characteristics [9].

In this paper we use a full inductance model to numerically simulate the behavior of of underdamped 2D-JJA with the simplest geometry, i.e., the H- and V-ladder arrays within the range $0.1 < \beta_L < 2$. We study mainly the dynamics of the V_+ singularity, i.e., the Eck-like step, which would be the most promising for applications.

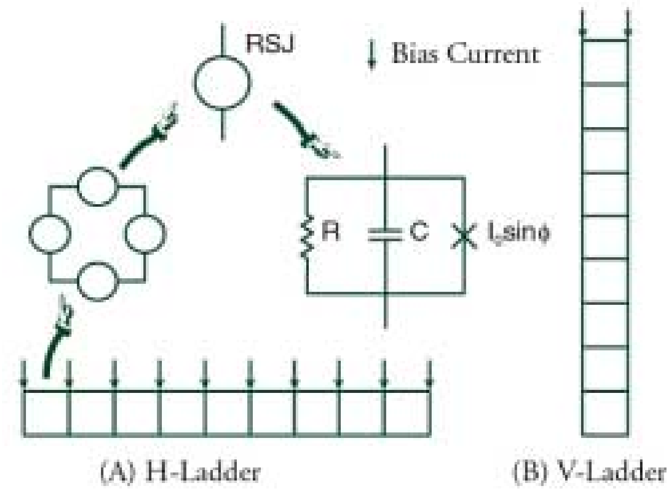

Fig. 1. Typical geometry for a 2D-JJA: (a) the Josephson H-ladder; (b) the Josephson V-ladder. Each branch corresponds to the RCSJ model as shown above.

II. THE MODEL

We model a N_r x N_c mesh 2D-JJA following the procedure reported in [10]. Single junctions of the network are modeled as RCSJ elements (here without a spread in the critical currents or electrical parameters and no asymmetry between horizontal and vertical junctions) with a damping corresponding to $\alpha = (1/\beta_C)^{1/2}$. The coupling between the junctions of the array is introduced via the standard fluxoid quantization rule and the self-inductance of the superconducting loop between junctions of the network. In this model the self-inductance coupling is represented by a suitable value of the parameter β_L, whereas mutual inductance depends on the ratio R/a, with R being the mesh dimension, and a the branch dimension in the array plane. A uniform orthogonal magnetic field is applied over the array via the frustation vector $\boldsymbol{f}$ which measures the external flux normalized to the flux quantum, i.e., $\boldsymbol{f}=\boldsymbol{\Phi}/\Phi_0$. The resulting array equations are:

$$\boldsymbol{\varphi}_{tt} + \alpha\boldsymbol{\varphi}_t + \sin\boldsymbol{\varphi} = \frac{1}{\beta_L}(\mathbf{U}\boldsymbol{\varphi} + \mathbf{V}\boldsymbol{f} + \mathbf{W}\boldsymbol{\gamma}) \qquad (1)$$

where $\boldsymbol{\varphi}$ is the phase vector of all junctions in the arrays, $\boldsymbol{\gamma}$ the bias current vector; **U** is the matrix expressing the coupling between all junctions, i.e., containing the full mutual inductance matrix of the whole array; **V** is the matrix relating the external magnetic field flux in each mesh of the array to the junctions, and finally **W** is the matrix carrying the bias current feed to the array. We note that both **V** and **W** also depends on the full mutual inductance matrix due to the fact that a current in some mesh of the array couples via the full inductance matrix to all other meshes of the array. In (1) time is normalized to the inverse of the plasma frequency ω_J of the junctions and the bias currents are normalized to the critical current of the junctions. The system (1) are explicit equations that can be solved using standard integration routines for a system of ODE. The bias current is introduced at the top nodes of the array and subtracted at the bottom nodes, this corresponds to a choice similar to that of Ref. [5] appendix C (the H-ladder differs from the V-ladder simply in the number of rows N_r and columns N_c). Finally, using the R/a parameter, the normalized mutual inductance matrix has been chosen to have maximum coupling at β_L =4 [10].

III. NUMERICAL RESULTS

A. *Eck-like resonances.*

We have solved (1) for some underdamped 2D-JJA ladder arrays. In Fig.2 typical I-V characteristics for H-ladders (1x10 and 1x100), and for V-ladders (10x1 and 100x1) are shown. Here β_L=1 so the step are comparable with that of [3,6]. We note that resonances of both the V_+ and V_- type are visible. The V-ladders show also a richer structure in resonances below the Eck-like step, whereas the H-ladders show only V_- (so here we refer to the Eck-like step in a V-ladder as the topmost frequency resonance, generally the largest one). All simulated arrays show these resonances, but we note that the Eck-like step in V-ladder shows a lower voltage (more than 15%) and smaller amplitudes. Moreover the voltage of the Eck-like step depends on the array length in the V-ladder case (this is much more evident for small arrays with N_r<5), while in the H-ladders case the Eck-like steps for 10 and 100 meshes overlap almost completely. All simulated Eck-like steps attain their maximum amplitude at f=0.5, where in most cases the maximum voltage is also attained (see below).

B. *Dependence on* β_L

By changing the parameter β_L two families of Eck-like steps are generated on the I-V characteristics, depending on array type. Fig.3a shows the H-ladder case for typical values of parameters and for the optimal frustation f=0.5. Eck-like steps appear at lower voltage for higher values of β_L. For the H-ladder a comparison with resonances theory given in [6,7] is shown in Fig.4, where mutual inductance effects have been accounted using an "effective" β_{Leff} ~1.5β_L (corresponding roughly to L_{eff} ~$L+2M$). We note that agreement is very good also for large values of β_L where

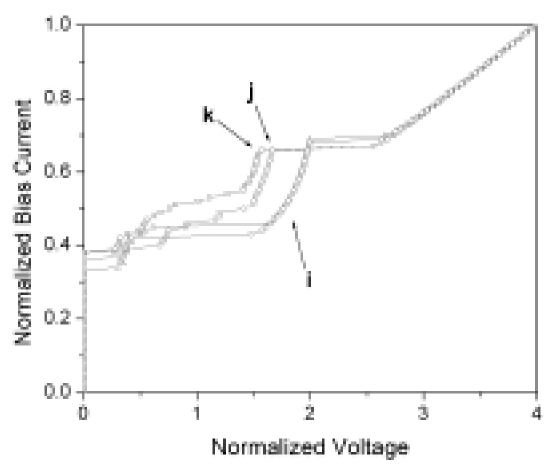

Fig. 2. Typical steps in the I-V characteristic of some 2D-JJA ladder arrays. The largest ones are the Eck-like steps: (i) (overlap) H-ladder Eck-like steps for a 10 and 100 meshes array; (j) V-ladder with 10 meshes; (k) V-ladder with 100 meshes. Parameters of the simulations are: β_L=1, α=0.25, f=0.5.

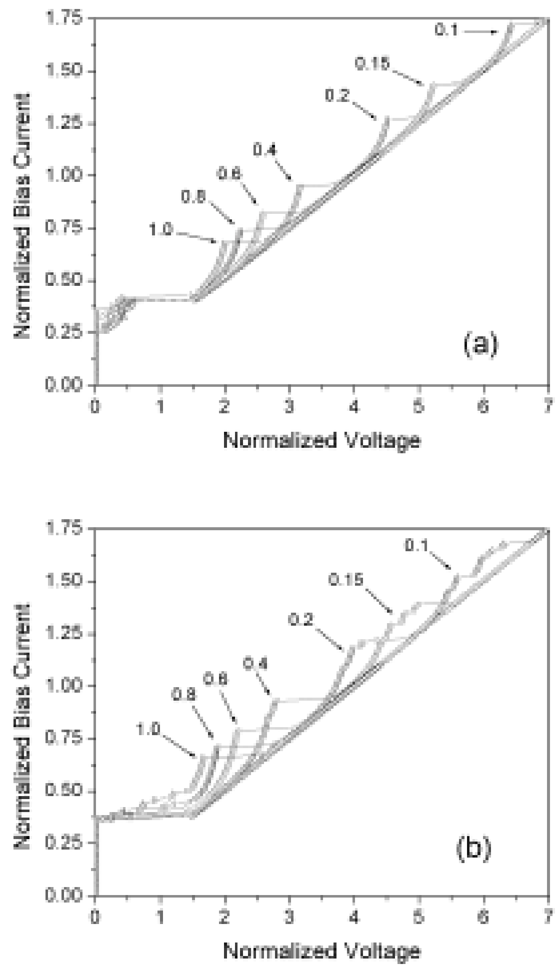

Fig.3 Dependence of the Eck-like step on the parameter β_L . (a) H-ladder (1x10); (b) V-ladder (10x1). Parameters of simulations are α=0.25, *f*=0.5. Numbers indicate the values of β_L.

long range mutual inductance effects are not accounted by the "effective" β_L correction. In Fig.3b the same I-V characteristics are reported for the V-ladder case. Comparison with the resonance theory is given in Fig.4. The behavior appears similar to the H-ladder case but the resonance voltage is generally lower than the linear cavity mode prediction [6], which gives only small differences with H-ladder case. Moreover the steps show a resonant fine structure especially for lower β_L. We note that it is impossible to use an "effective" uniform β_L. In fact the observed difference tends to be small for low β_L values, indicating that a different mechanism is involved in producing the observed voltages.

C. Stability

As shown in [10] the maximum step depletion for rf-induced steps occurs near the maximum mutual coupling β_L depending on array geometry (here near β_L=4). A similar mechanism seems to occur also in the case of Eck-like steps. After reaching the maximum amplitude at β_L~1, steps rapidly decrease in amplitude and at a critical β_L^c disappear abruptly. By increasing the dissipation parameter α, steps can also be found at higher β_L values. Since higher dissipation increases the stability of the system, steps could become stable for β_L greater than β_L^c. In Fig.5 we report the stable and unstable regions in the α-β_L plane, separated by $\beta_L^c(\alpha)$ curves, for a 1x10 H-ladder and 10x1V-ladder. Linear theory [7] predicts that step amplitudes decrease for large β_L, though this decrease is much slower and not abrupt due to the absence of mutual inductance coupling.

D. Magnetic field behavior

In Fig.6 typical field behavior (frustation) is reported for H- and V-ladder. As can be see for an H-ladder, 1x10 with β_L =0.5, the step voltage is maximum for *f*=0.5, which is the typical behavior of Eck-like steps predicted by linear theory [6,7]. The percentual variation of voltage is about 4% in this case. Increasing to β_L to 1 flattens the curve, giving a voltage practically constant wiht a maximum variation of less than 0.5%. Compared to classical Eck step in LJJ or in1D arrays [1,8] this percentile variation of step voltage is small or

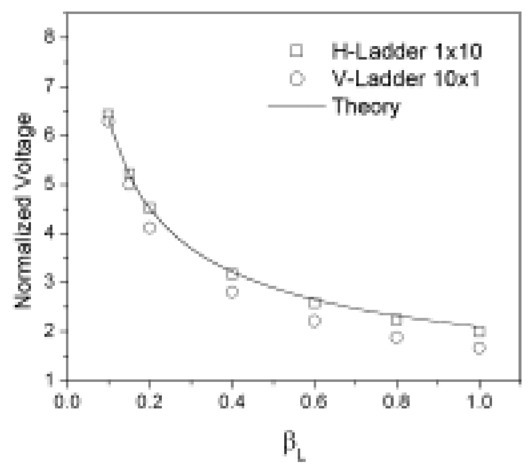

Fig.4 Maximum voltage of Eck-like step in H-ladder (1x10) and V-ladder (10x1) arrays plotted vs. β_L. Solid line is the cavity resonances theoretical prediction with β_{Leff} =1.5β_L. Parameters of simulations are α=0.25, *f*=0.5.

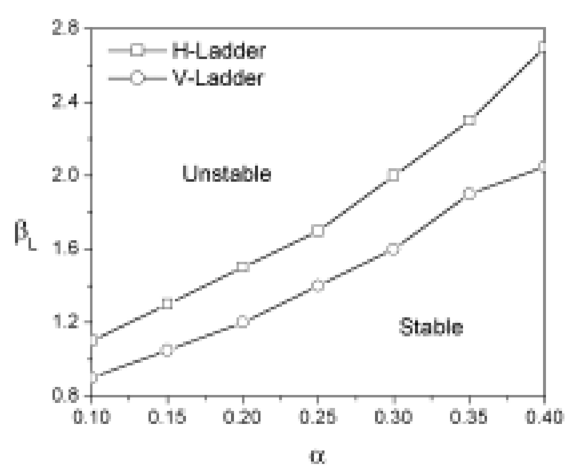

Fig.5 Stability of Eck-like step in H-ladder 1x10 and V-ladder 10x1 arrays: typical stable and unstable regions in the plane α-β_L (here *f*=0.5).

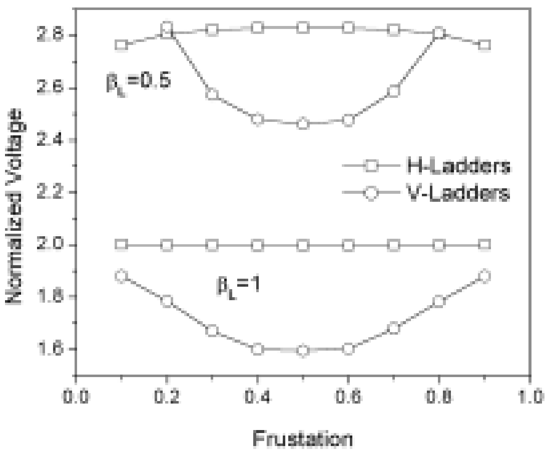

Fig.6 Resonance voltages vs. frustation *f* for H-ladder 1x10 and V-ladder 10x1. Parameters of simulations are: β_L=1 & 0.5, α=0.25, *f*=0.5.

almost absent in some cases. The behavior of a 10x1V-ladder is substantially different: in both cases, β_L =0.5 and β_L =1, *f*=0.5 corresponds to the minimum value of the voltage, with a variation of about 13% and 17% respectively. This behavior is analogous to multi-row arrays in which linear resonances theory gives this results which also seems to be confirmed by some experiments seems (cf. [9]). It is worth noting that the maximum screening currents are, in general, always found at *f*=0.5.

IV. INTERNAL DYNAMICS

The differences between Eck-like steps of H- and V-ladder arrays noted above can be explained looking at the array internal dynamics (for sake of brevity we limit ourselves to few comments, more details are in [11]). H-ladders show clearly checkerboard (ribbon) dynamics of order ½ in the whole interval from *f*=1/3 to 1/2. V-ladder dynamics is again checkerboard-like, but the coherence length of the checkerboard pattern is subject to a phase-shift along the array so that distant meshes cannot be mapped exactly one to one. V-ladder behavior is much more reminiscent of a multi-row array in which other modes appears depending on the number of rows. Another indication of this is the fine structure observed in the V-ladder Eck-like steps. The observed lower voltage of the resonance is explained by the fact that resonance theory has to take into the fact that the system consist of multiple rows (cf. [9]). Actually “rows” in a V-ladder are single meshes. Flux propagation in such “short” structures is replaced by “beating solutions” [12], i.e., biased junctions working out of phase simulating the propagation. In these terms a V-ladder is much easier to understand as a collection of coupled lumped phase-shifted “flux” oscillators rather than “flux-flow”. This can help explain why a frequency selection element, such as a ground plain used in [4], is necessary to obtain large power emission from V-ladder type arrays.

V. CONCLUSION

The main consequences for applications of the above properties of Eck-like steps can be summarized as follows: 1) an optimal β_L range exists, where steps attain their maximum amplitude giving rise to the highest output power: for the mutual inductance model used here the optimal β_L is between 0.4 and 0.8; 2) low dissipation α<0.1 can reduce stability and tunability; 3) step fine structure should be avoided in order to have a smooth tunable oscillator. The analysis of two different geometries, H or V-ladder, shows that the H-ladder is a true flux-flow device with characteristics comparable to LJJs, even the poor tunability can be a problem. On the other hand a V-ladder is a different type of device working much more as a collection of coupled lumped oscillators.

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