Some properties of dynamical states in 2D underdamped Josephson junctions arrays

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Introduction

AMONG Superconducting devices proposed so far in these last years two-dimensional Josephson junction arrays (2D-JJA) have been one of the most promising for application mainly as local oscillators. The main features of 2D-JJA are the possibility of has a large number of Josephson junctions all coherently phase-locked together and so obtain a relevant power emission. Though in principle the power emitted from an array can be larger than that from a LJJ, only in these last years underdamped arrays have been fabricated and their efficiency proved [3,4].

The typical parameter for the measurement of array junctions coupling is the SQUID parameter $\beta_L = 2\pi I_0 L/\Phi_0$ where I_0 is the junction critical current, L the loop inductance, Φ_0 the flux quantum. In low β_L arrays (typically β_L <0.5) the network equations can simplified using some of next-neighbors approximation for the mutual inductance between array loops. Anyway most arrays have been realized with β_L >1, in this case only a model including the full mutual inductance matrix between array loops can give a realistic array model [5].

The most frequent geometry is the simple so-called horizontal square Josephson ladder (H-ladder) in which a single row of meshes is feed by bias current in any of the junction common to two adjacent meshes (cf. Fig.1a). Arrays formed by two or three of these rows have been also considered. Another geometry is the vertical ladder (V-ladder), in which a single *column* of meshes is feed (cf. Fig.1b). Two or more of such columns form arrays used in [4].

Coherent emission state is reached when the array is placed in a magnetic field perpendicular to the array plane. In this case the I-V characteristics of the array shown steps on which it is possible biasing the device in order to obtain the coherent signal. In presence of magnetic it is possible to make a simple cavity resonances theory based on the checkerboard pattern solutions [6,7]. It shows that the array behavior under the applied magnetic field is dominated by two singularities called V_+ and V_-. The largest frequency one, V_+, have to be considered the analogous of Eck step in LJJ.

In this work we use a full inductance model to simulate numerically the behavior of simplest geometry of underdamped 2D-JJA, i.e., the H- and V-ladder arrays within the range $0.1 < \beta_L < 2$. We study mainly the dynamics of V+ singularity, i.e., the Eck-like step, which would be the most promising for the applications.

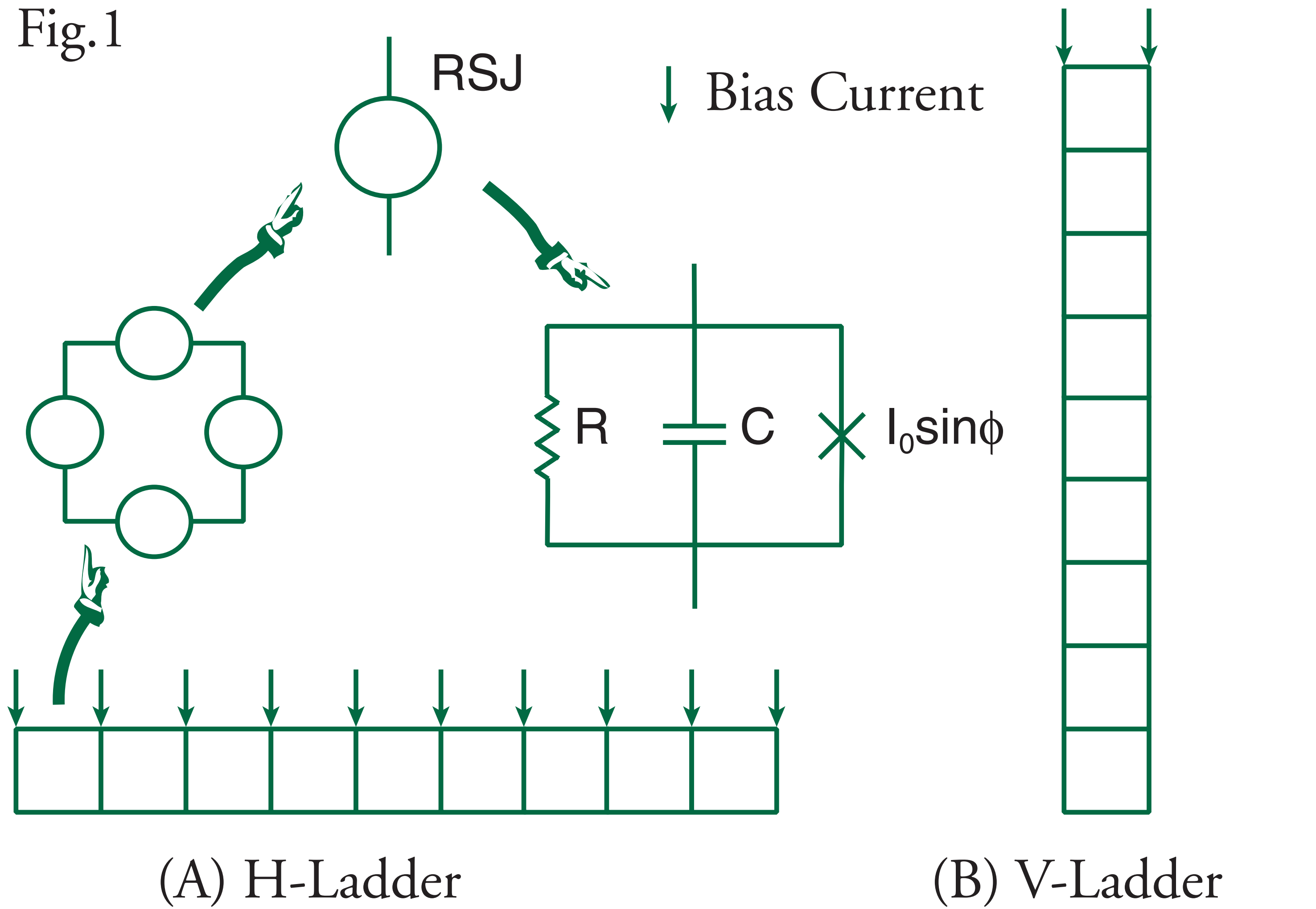


Fig. 1
RSJ
Bias Current
R
C
$I_0 \sin\phi$
(A) H-Ladder
(B) V-Ladder

The Model

We model a N_r x N_c meshes 2D-JJA following the procedure reported in [10]. Single junctions of the network are modeled as RSJ elements with a damping corresponding to $\alpha = (1/\beta_C)^{1/2}$. The coupling between the junctions of the array is introduced via standard fluxoid quantization rule and self-inductance of superconducting path between junctions of the network. In this model the self-inductance coupling is represented by a suitable value of the parameter β_L, whereas mutual inductance depends on the ratio R/a with R the mesh dimension and a the branch dimension in the array plane. An uniform orthogonal magnetic field is applied over the array via the frustation vector $\boldsymbol{f}$ which measure the external flux normalized to flux quantum, i.e., $\boldsymbol{f}=\Phi/\Phi_0$. The array equations will result:

$$\boldsymbol{\varphi}_{tt} + \alpha\boldsymbol{\varphi}_{t} + \sin\boldsymbol{\varphi} = \frac{1}{\beta_L}(\mathbf{U}\boldsymbol{\varphi} + \mathbf{V}\boldsymbol{f} + \mathbf{W}\boldsymbol{\gamma}) \qquad (1)$$

where $\boldsymbol{\varphi}$ is the phase vector of all junctions in the arrays, $\boldsymbol{\gamma}$ the bias current vector; **U** is the matrix expressing the coupling between all junctions, i.e., containing the full mutual inductance matrix of the whole array; **V** and **W** are the matrixes relating the external magnetic field flux and the bias current in each mesh of the array to the junctions. In (1) time is normalized to the inverse of plasma frequency ω_J of the junctions and bias currents to critical current of junctions. The system (1) are explicit equations that can be solved using standard integration routines for system of ODE equations. Finally, using R/a parameter, the normalized mutual inductance matrix have been chosen in order to have maximum coupling at β_L =4 [10].

Numerical Results: 1) Eck-like steps

We have solved (1) for some underdamped 2D-JJA ladder arrays.

In Fig.2 typical I-V characteristics for H-ladders and for V-ladders are shown. Here $\beta_L=1$ so step are comparable with that of [3,6]. Typical steps appears in I-V characteristics. The largest ones are the Eck-like steps: (i) (overlap) H-ladder Eck-like steps for a 10 and 100 meshes array; (j) V-ladder with 10 meshes; (k) V-ladder with 100 meshes. Parameters of simulations are: $\beta_L=1$, $\alpha=0.25$, $f=0.5$.

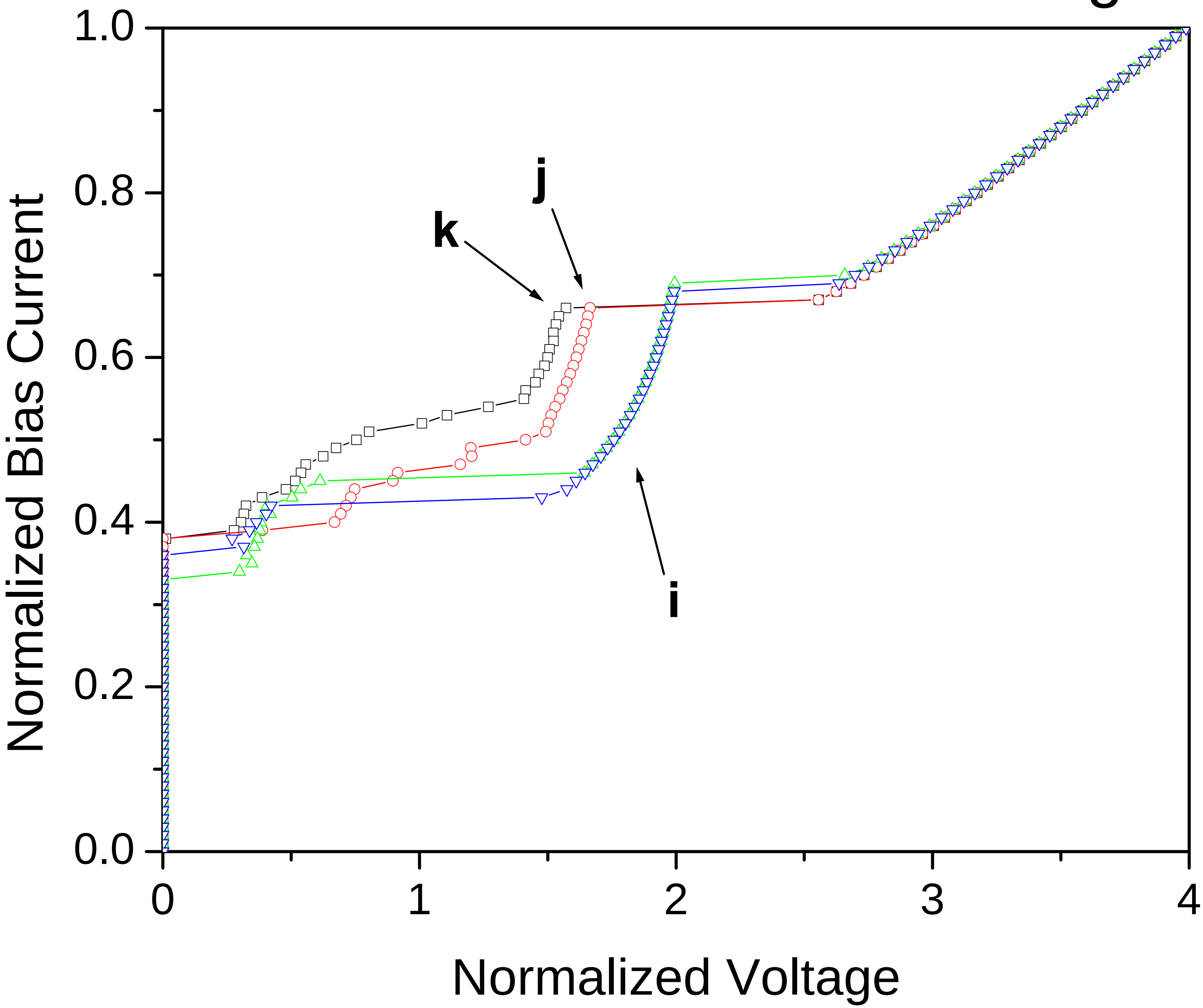
Resonances of V_+ and V_- type are visible. All simulated arrays shown these resonances, but we note that the V-ladder shows a lower voltage (more than 15%) just from relatively small arrays (6x1) and smaller amplitudes.

Moreover the voltage of Eck-like step depends on the array length in V-ladders case (this is much more evident for small arrays with $N_r<5$), while in the H-ladders case the Eck-like steps for 10 and 100 meshes overlap almost completely.

The V-ladders show also a more rich structure in resonances below the Eck-like step, whereas the H-ladders show only V_- (we refers to Eck-like step in V-ladder as the topmost frequency resonance).

All simulated Eck-like steps attain their maximum amplitude at $f=0.5$, where also the maximum voltage is attained in most cases (see below).

Fig.2



1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
Normalized Bias Current
Normalized Voltage
k
j
i

Numerical results: 2) Dependence on β_L

Changing the parameter β_L two families of Eck-like steps are generated on the I-V characteristics according to array type.

In Fig.3a we report the H-ladder case 1x10 for typical values α=0.25, f=0.5 of parameters and for the optimal frustation f=0.5. Eck-like steps appear at lower voltage for higher values of β_L. For the H-ladder a comparison with resonances theory given in Refs. [6,7] is shown in Fig.4, where mutual inductance effects have been accounted using an "effective"

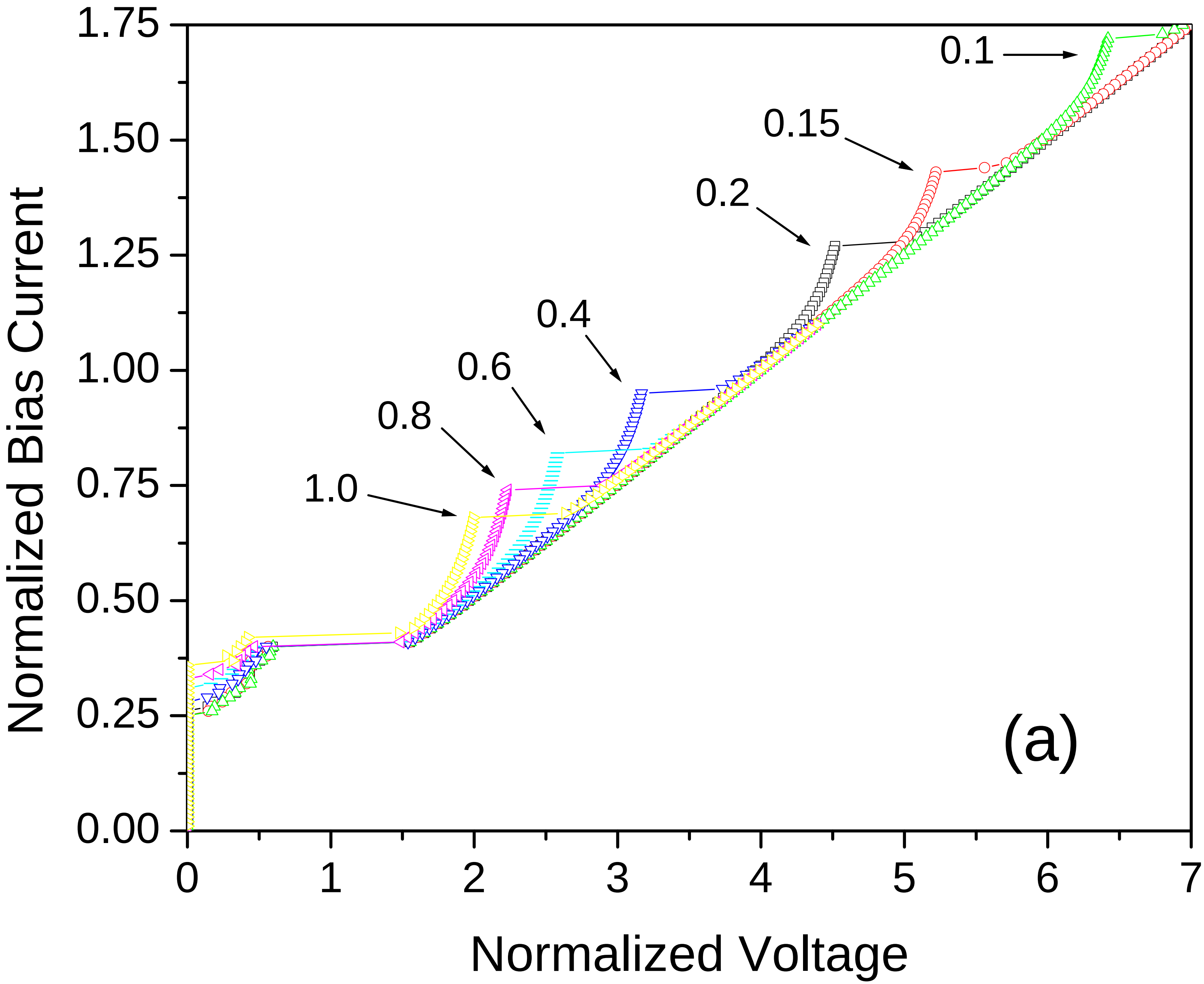
$$\beta_{Leff} \sim 1.5\beta_L$$

(corresponding roughly to $L_{eff} \sim L+2M$). We note that agreement is very good also for large values of β_L where long range mutual inductance effects are not accounted by "effective" β_L correction.

In Fig.3b the same I-V characteristics are reported for V-ladder 10x1 case. Comparison with the resonance theory is reported again in Fig.4. The behavior appears similar to H-ladder case but resonance voltage is generally lower than linear cavity mode prediction [6], which give only small differences with H-ladder case.

Moreover the steps show a resonant fine structure especially for lower β_L. About voltage we note that it is impossible to use an "effective" uniform β_L. In fact the observed difference tends to be small for low β_L values indicating that a different mechanism is involved in producing the observed voltages

Fig.3a



Normalized Bias Current
Normalized Voltage
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
0
1
2
3
4
5
6
7
0.1
0.15
0.2
0.4
0.6
0.8
1.0
(a)

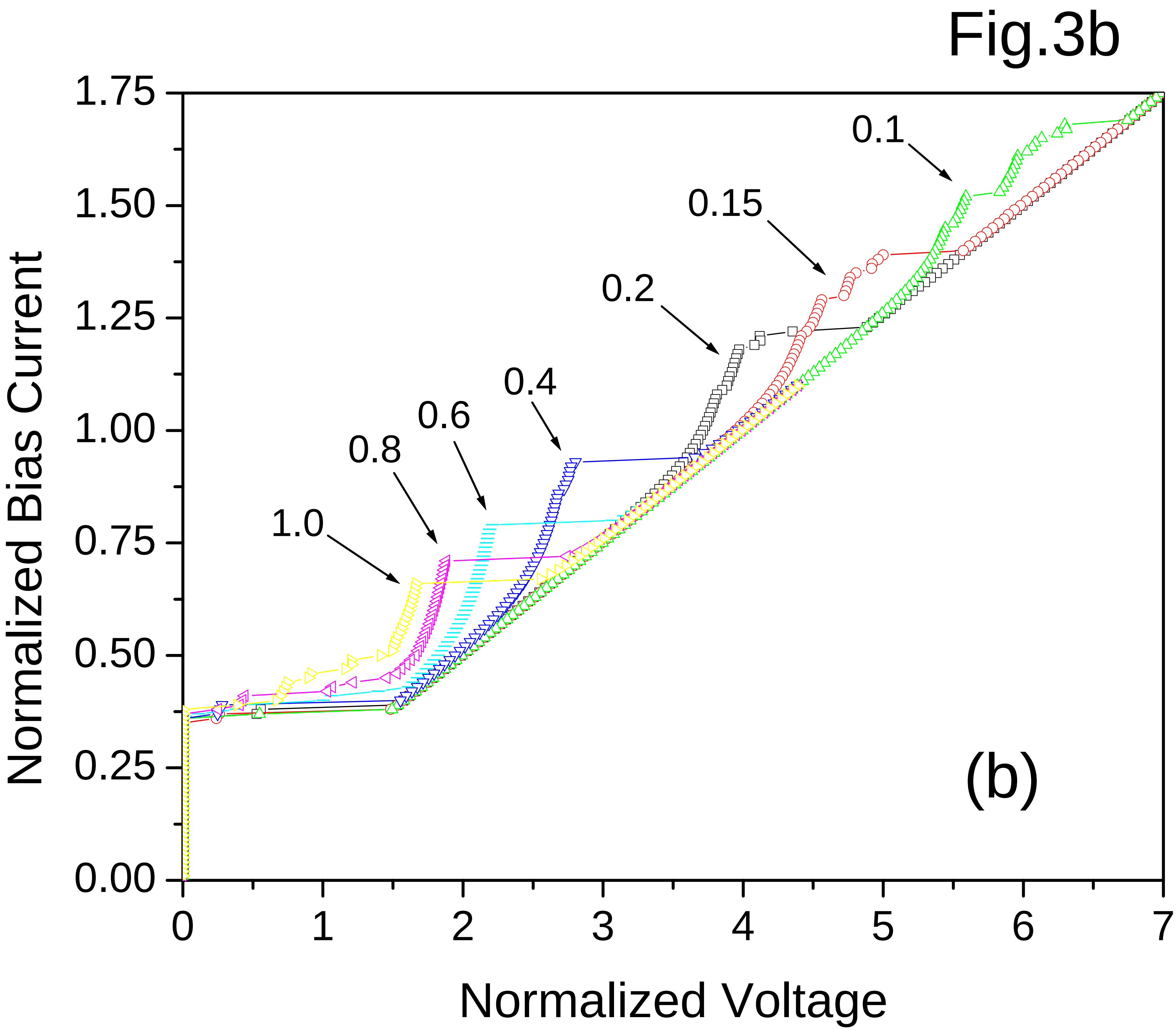
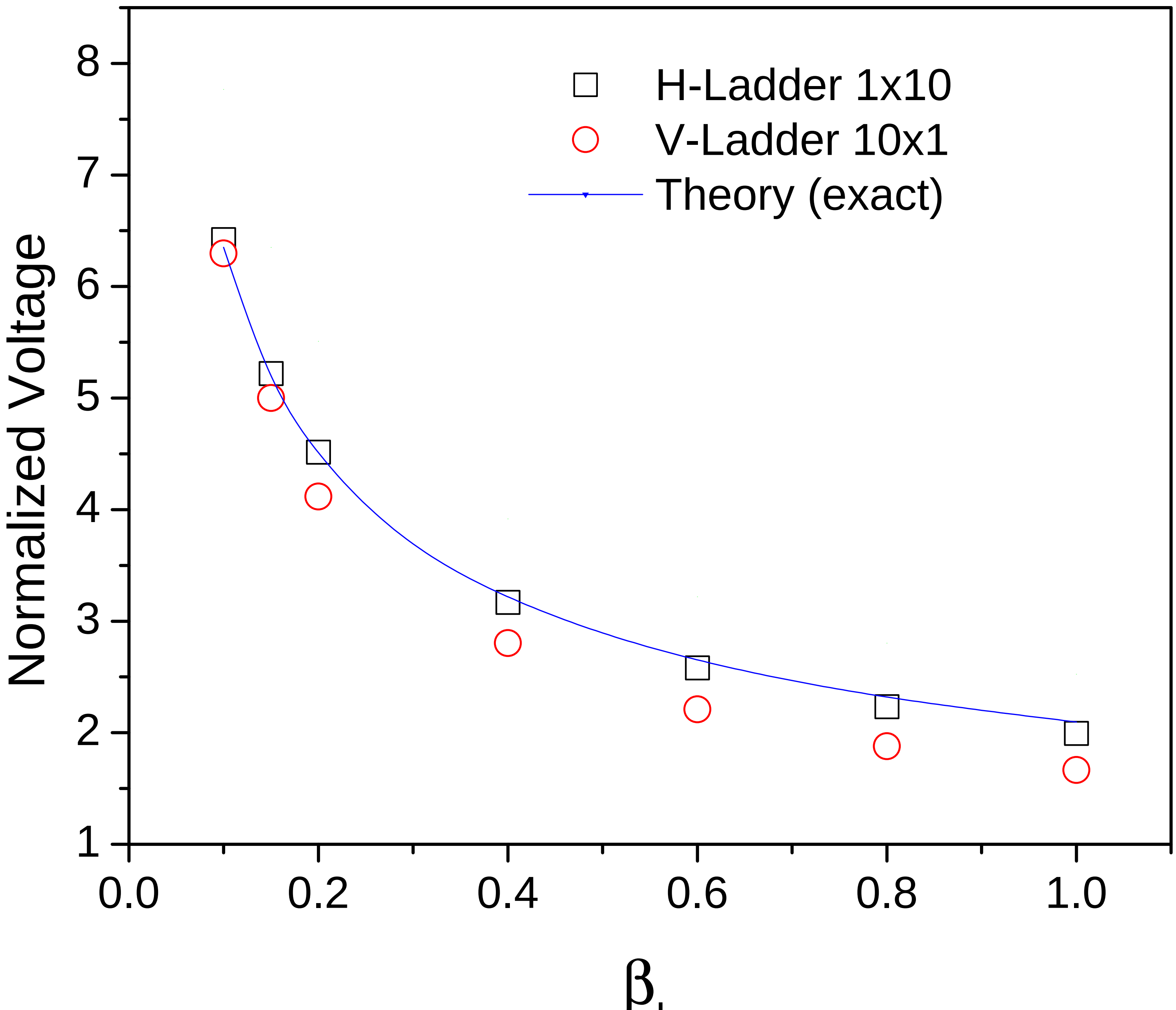


Fig.3b
Normalized Bias Current
Normalized Voltage
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
0
1
2
3
4
5
6
7
0.1
0.15
0.2
0.4
0.6
0.8
1.0
(b)

Fig.4



H-Ladder 1x10
V-Ladder 10x1
Theory (exact)
Normalized Voltage
β_L
1
2
3
4
5
6
7
8
0.0
0.2
0.4
0.6
0.8
1.0

Numerical results: 3) Stability

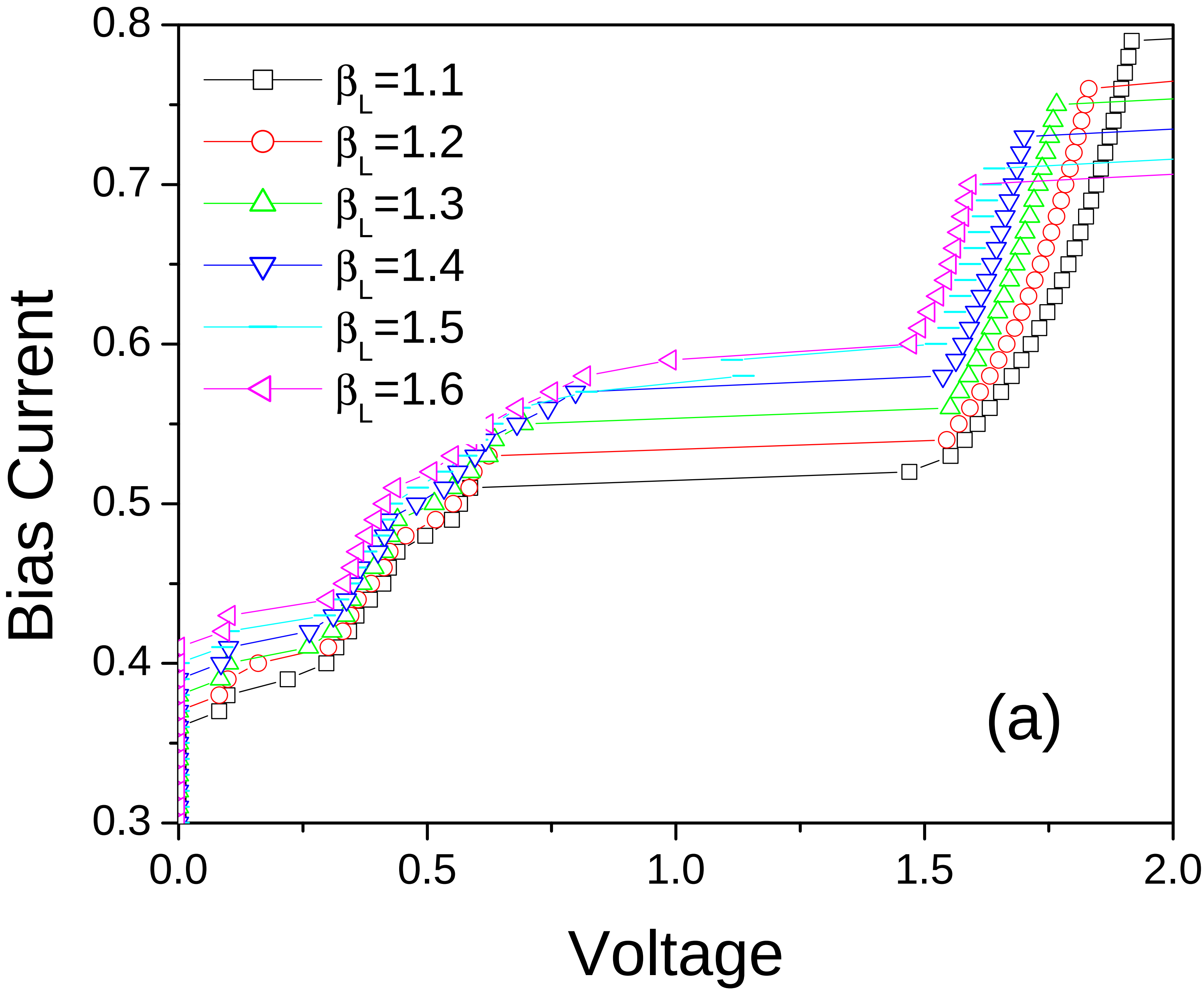
After reaching the maximum amplitude at β_L~1, steps rapidly decrease in amplitude and at a critical β_L^c disappears abruptly. This behavior is reported in Fig.5a. In [10] the maximum step depletion for rf-induced steps occurs near maximum mutual coupling β_L depending on array geometry (here near β_L=4): a similar mechanism seems occur also here.

Increasing the dissipation parameter α steps can be found also at higher β_L values. Thus, as higher dissipation has the effect to increase the stability of the system, steps could be unstable over β_L^c. In Fig.5b we report the stable and unstable regions over the α-β_L plane, separated by $\beta_L^c(\alpha)$ curves, for H-ladder 1x10 and V-ladder 10x1.

Numerical results: 4) Magnetic field behavior

In Fig.6 typical field behavior (frustation) is reported for a H-ladder, 1x10 with β_L =0.5: the step voltage is maximum for f=0.5 that is the typical behavior of Eck-like steps predicted also by linear theory [6,7]. The percentual variation of voltage is about 4% in this case. Increasing to β_L =1 have the effect to flatten the curve giving a voltage practically constant being the maximum variation of less than 0.5%. In Fig.6b others simulations are reported for H-ladder.

The behavior of V-ladder 10x1 is substantially different: always in Fig.6 in both cases, β_L =0.5 and 1, f=0.5 corresponds to minimum value of voltage, with a variation of about 13% and 17% respectively. This behavior is analogue in some sense of multi-row arrays in which linear resonances theory can give this results and some experiments seems confirm it (cf. Ref. [9]). It is worth to note that anyway maximum screening currents are always found at f=0.5.



Bias Current
Voltage
0.3
0.4
0.5
0.6
0.7
0.8
0.0
0.5
1.0
1.5
2.0
β_L=1.1
β_L=1.2
β_L=1.3
β_L=1.4
β_L=1.5
β_L=1.6
(a)

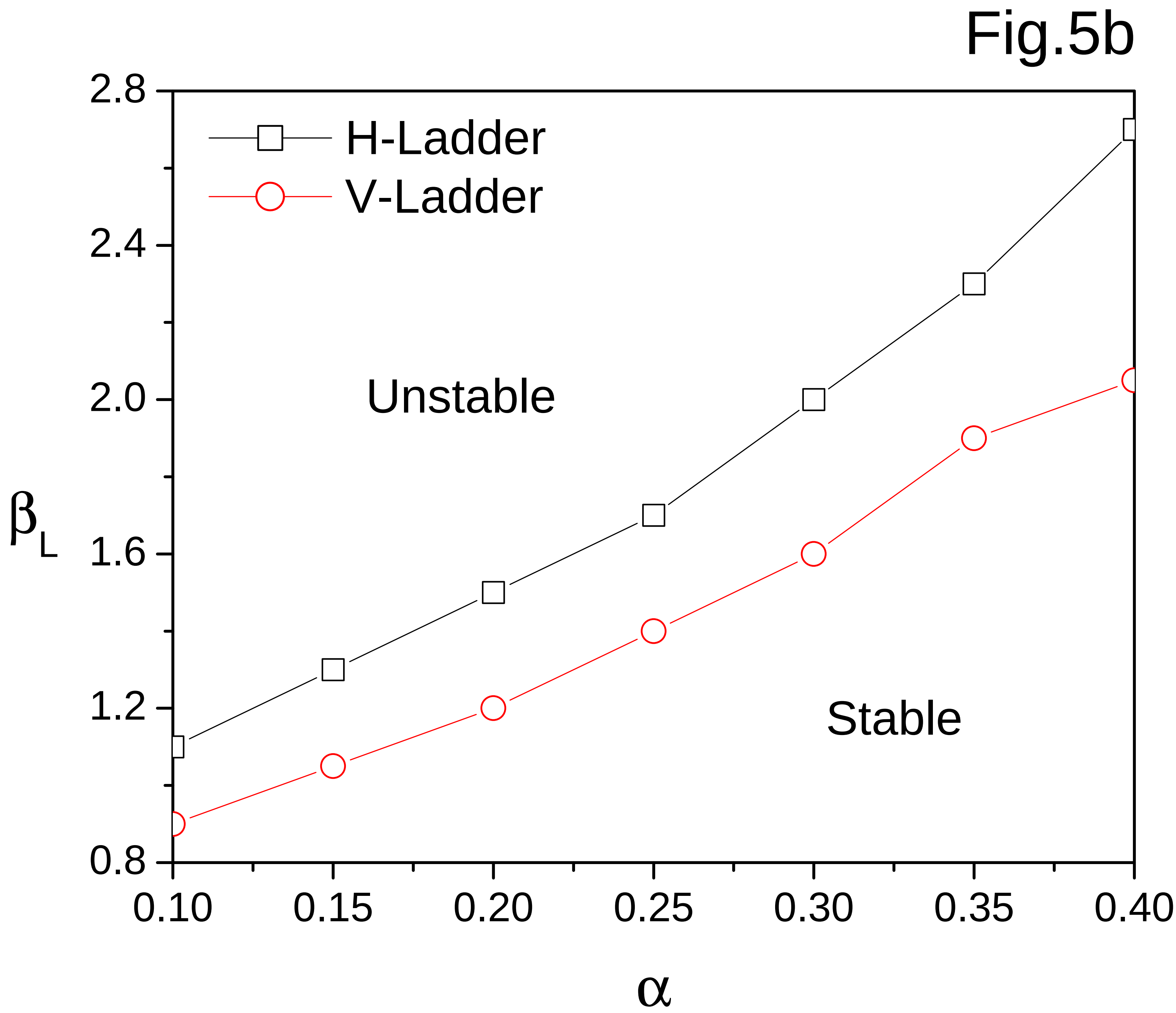
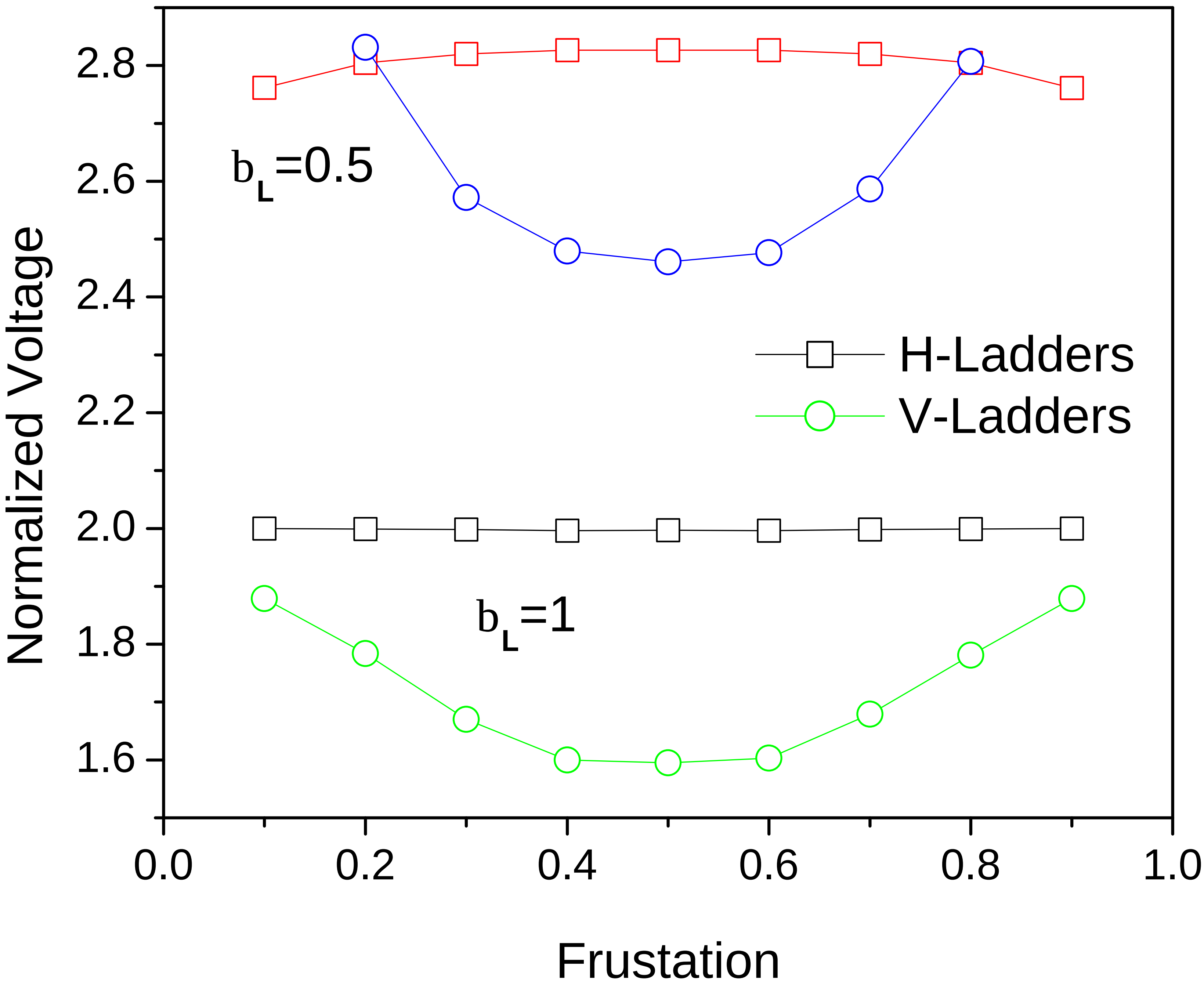


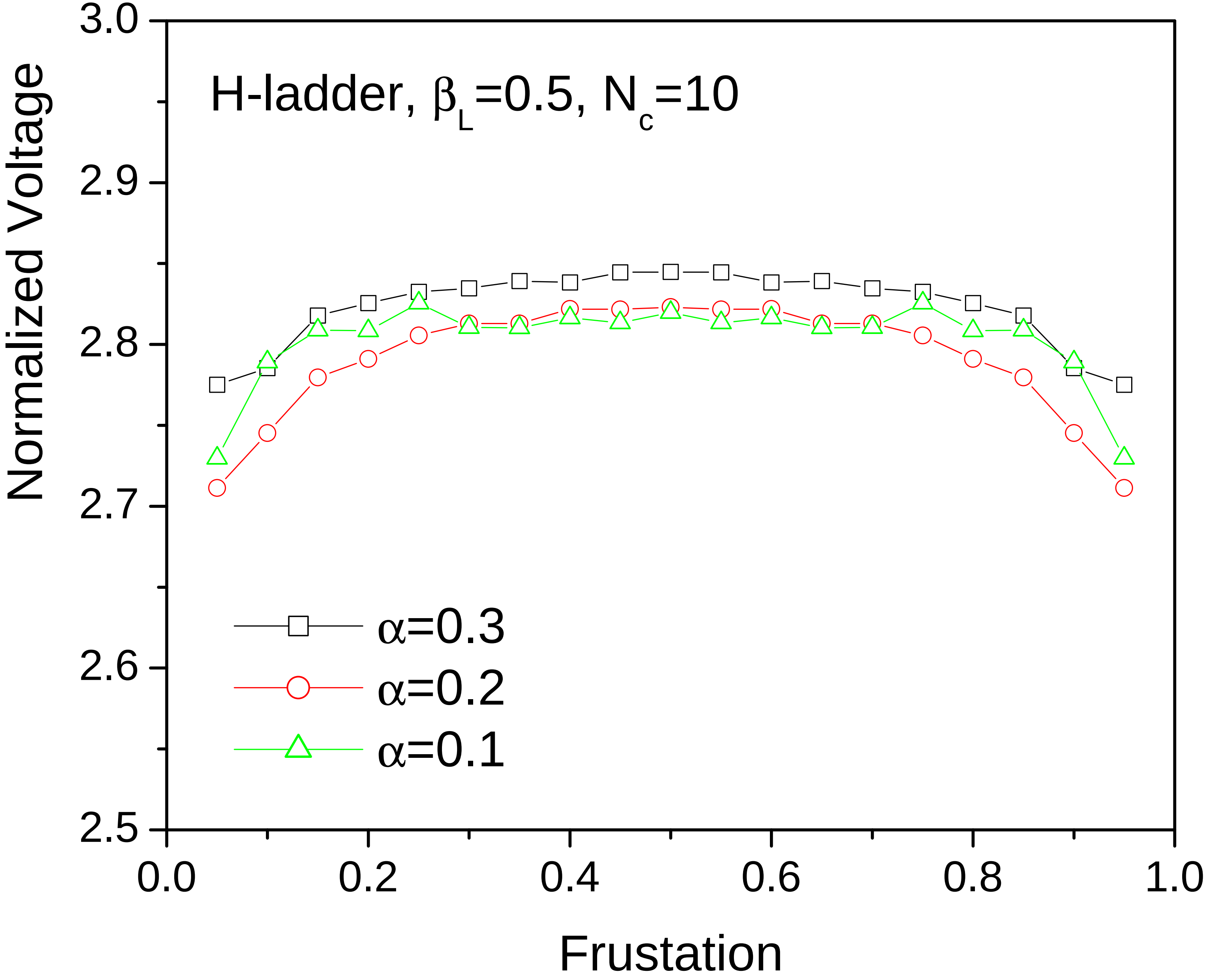
Fig.5b
2.8
2.4
2.0
1.6
1.2
0.8
H-Ladder
V-Ladder
Unstable
Stable
β
L
0.10
0.15
0.20
0.25
0.30
0.35
0.40
α

Fig.6



2.8
2.6
2.4
2.2
2.0
1.8
1.6
b_L=0.5
b_L=1
H-Ladders
V-Ladders
Normalized Voltage
0.0
0.2
0.4
0.6
0.8
1.0
Frustation

Fig.6b



H-ladder, β_L=0.5, N_c=10
Normalized Voltage
Frustation
3.0
2.9
2.8
2.7
2.6
2.5
0.0
0.2
0.4
0.6
0.8
1.0
α=0.3
α=0.2
α=0.1

Internal Dynamics

The differences between Eck-like steps of H- and V-ladder noted above can be explained looking at the array internal dynamics.

H-ladder shows clearly checkerboard (ribbon) dynamics of order ½ in the whole interval from f=1/3 to 1/2.

V-ladder dynamics is again checkerboard-like, but coherent length of checkerboard pattern is subject to a phase-shift along the array so that distant meshes cannot be mapped exactly one to one. This is reported in Fig.7 where phases of current feed junction are reported for an array 20x1, f=0.5, β_L=1, α=0.25.

Also power spectrum of V-ladder, reported in Fig.8b, shows more sidebands structures with respect to H-ladder spectrum in Fig.8a. Parameters here are the same of above except for the array dimension which is 10x1.

So V-ladder behavior is much more reminiscent of a multi-rows array in which other modes appears depending on the number of rows. Actually "rows" in a V-ladder are single meshes. As flux propagation in such "short" structures is replaced by "beating solutions" [11] V-ladder is much easier to understand as a collection of coupled lumped phase-shifted "flux" oscillators rather than "flux-flow". This can help to explain why a frequency selection element, as ground plain used in Ref. [4], is necessary to obtain large power emission from V-ladder type arrays.

Fig.7a

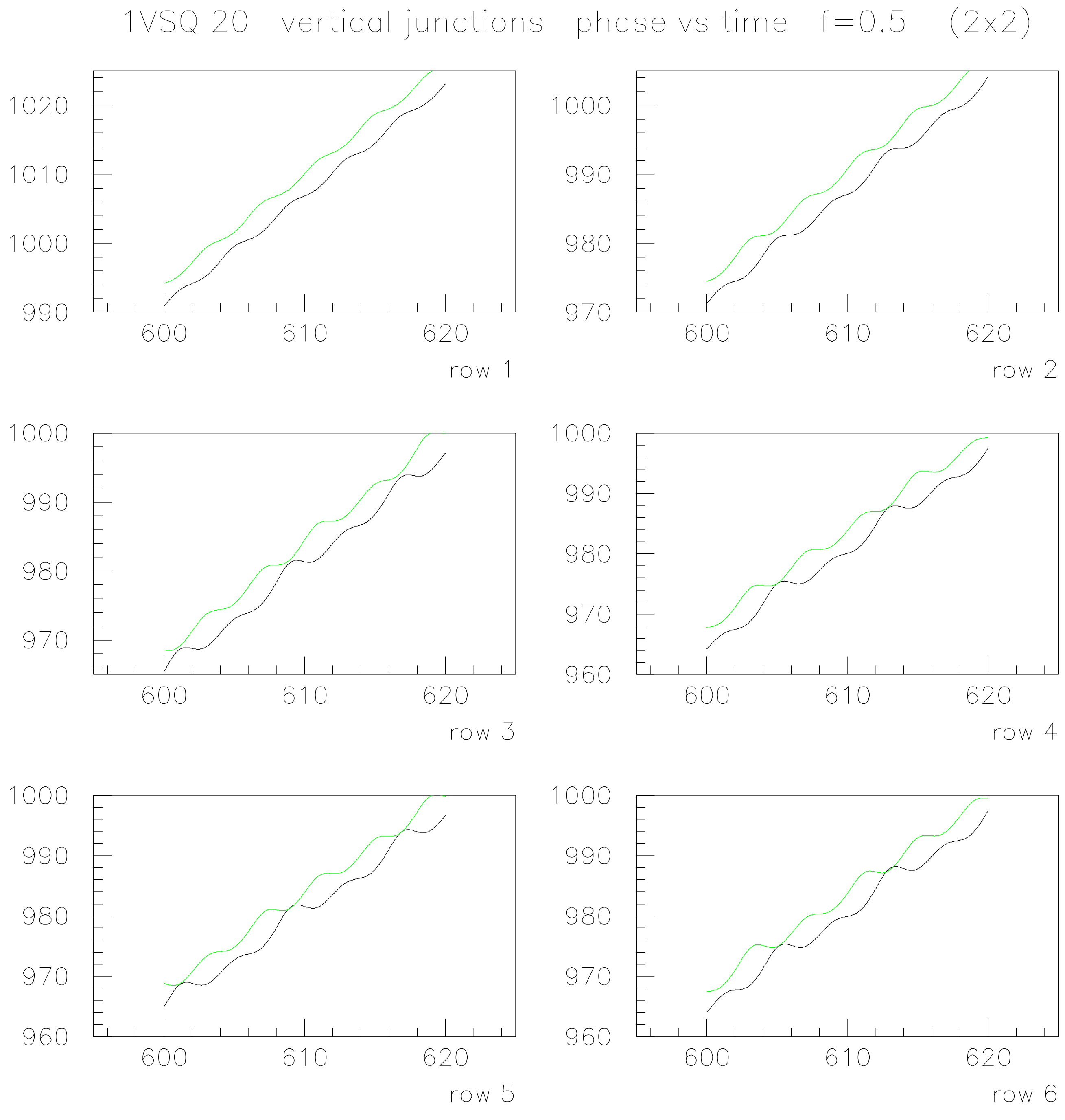
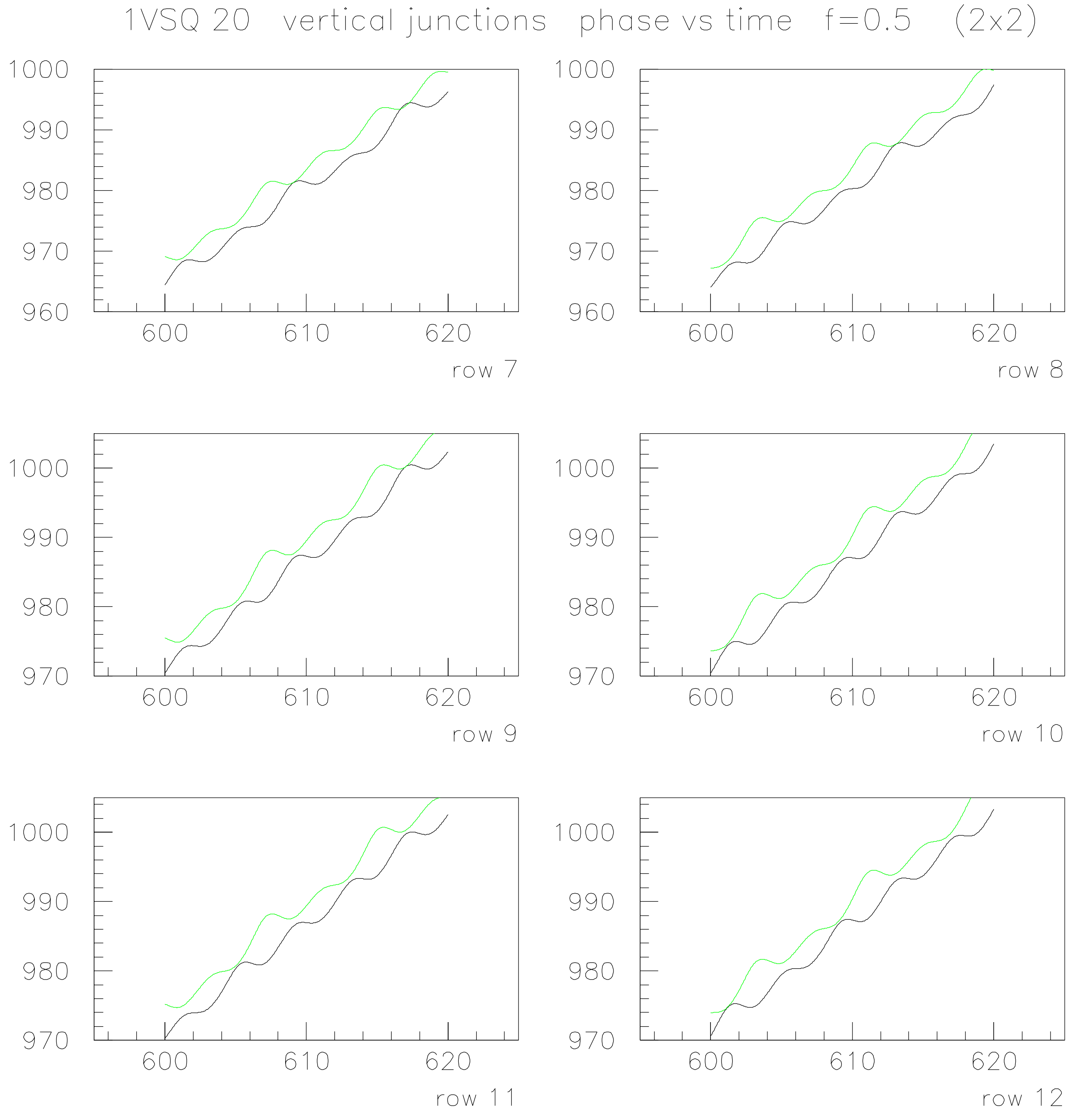

1VSQ 20 vertical junctions phase vs time f=0.5 (2x2)
1020
1010
1000
990
600
610
620
row 1
1000
990
980
970
600
610
620
row 2
1000
990
980
970
600
610
620
row 3
1000
990
980
970
960
600
610
620
row 4
1000
990
980
970
960
600
610
620
row 5
1000
990
980
970
960
600
610
620
row 6

Fig.7b



1VSQ 20 vertical junctions phase vs time f=0.5 (2x2)
1000
990
980
970
960
600
610
620
row 7
row 8
row 9
row 10
row 11
row 12

Fig.7c

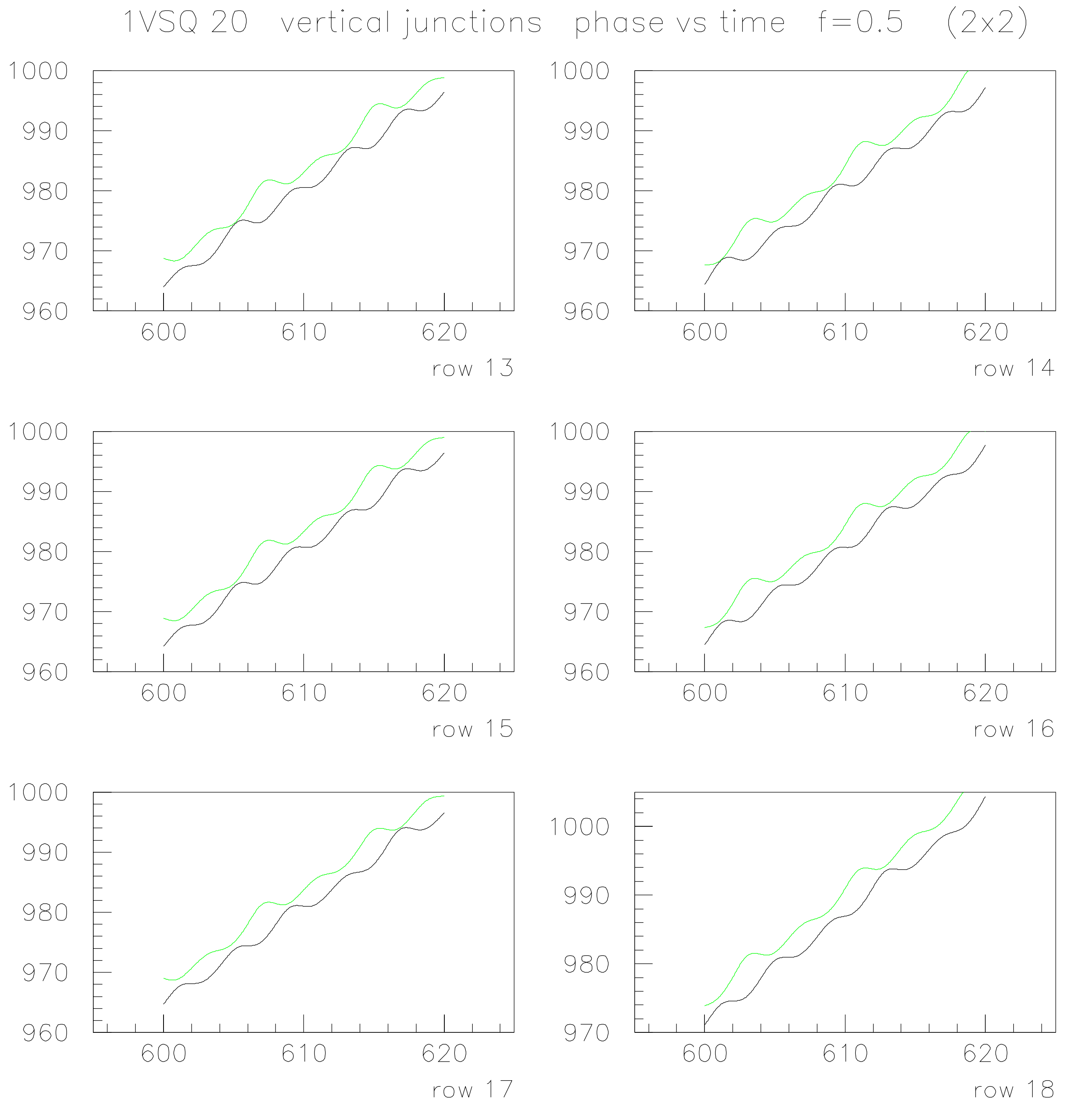
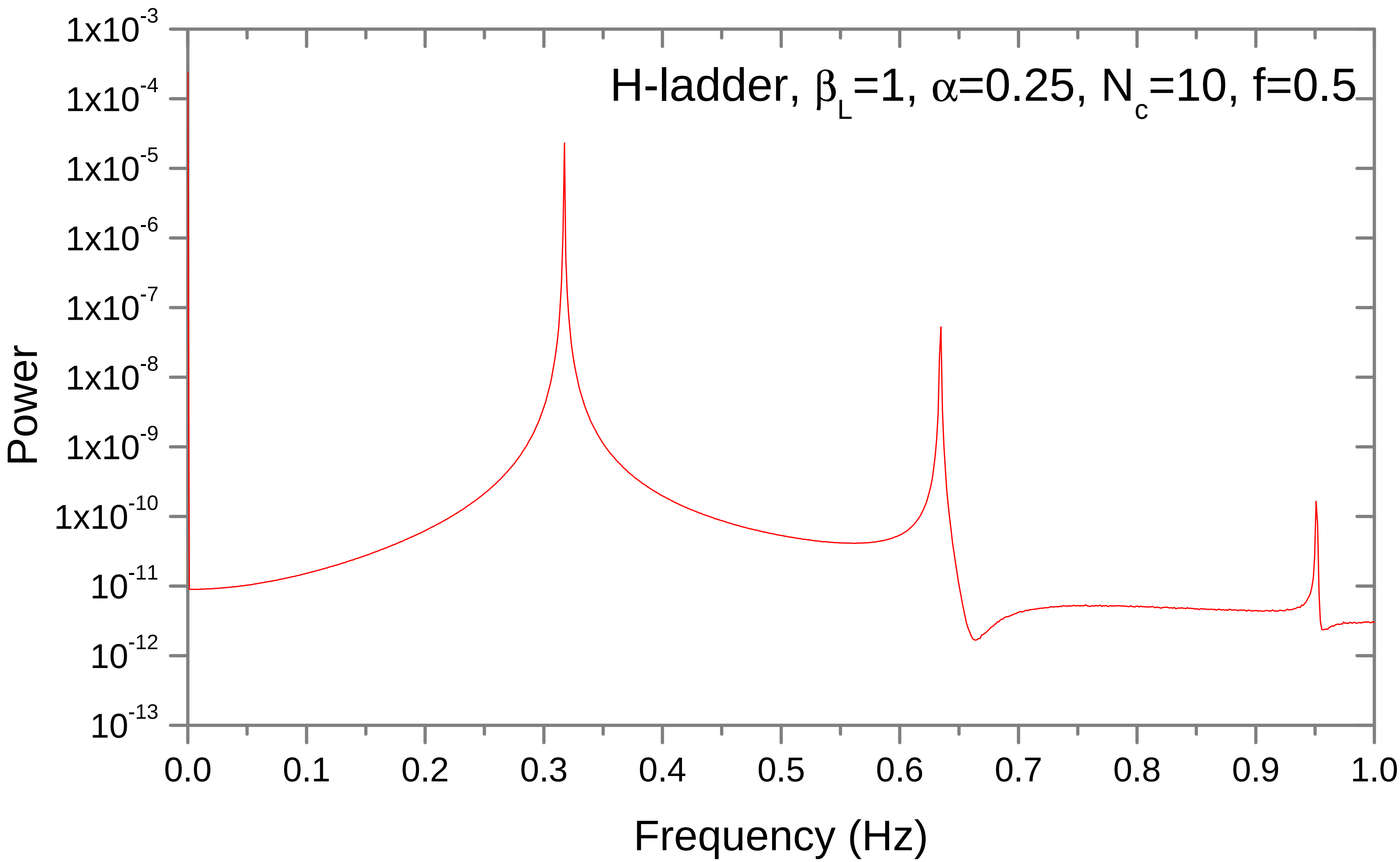
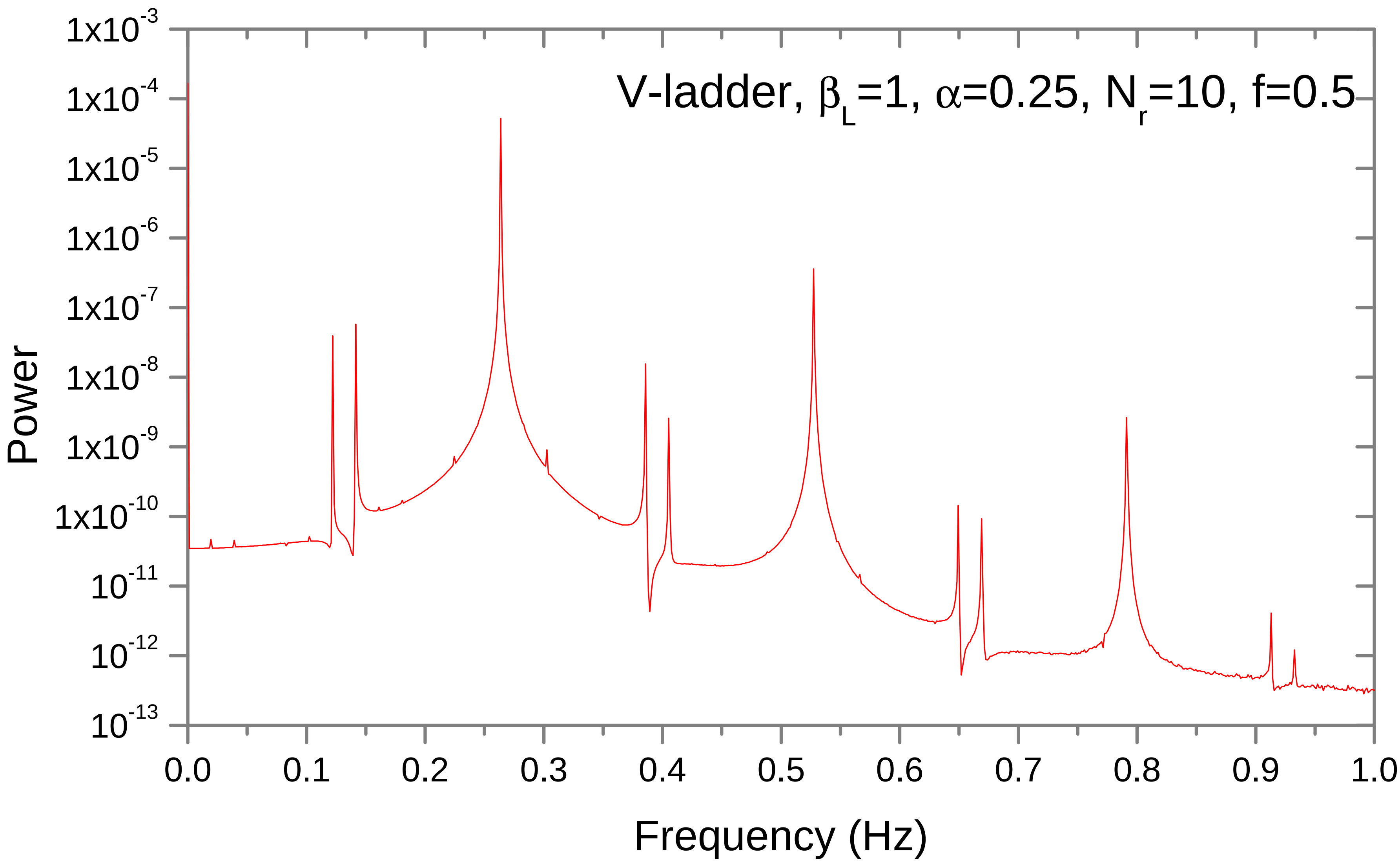

1VSQ 20 vertical junctions phase vs time f=0.5 (2x2)
1000
990
980
970
960
600
610
620
row 13
row 14
row 15
row 16
row 17
row 18

Fig.8a



H-ladder, β_L=1, α=0.25, N_c=10, f=0.5
Power
Frequency (Hz)
1x10^-3
1x10^-4
1x10^-5
1x10^-6
1x10^-7
1x10^-8
1x10^-9
1x10^-10
10^-11
10^-12
10^-13
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Fig.8b



V-ladder, βL=1, α=0.25, Nr=10, f=0.5
Power
Frequency (Hz)
1x10-3
1x10-4
1x10-5
1x10-6
1x10-7
1x10-8
1x10-9
1x10-10
10-11
10-12
10-13
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Conclusion

The main consequences for applications of above properties of Eck-like steps can be resumed as following: 1) exist an optimal β_L range where steps attains their maximum amplitude giving arise to the best output power, for the mutual inductance model used here is between 0.4 and 0.8; 2) low dissipation $\alpha<0.1$ can reduce stability and tunability; 3) step fine structure would be avoided in order to have a smooth tunable oscillator.

The analysis of two different geometry, H- or V-ladder shows that the H-ladder is a true fluxflow device with characteristics comparable to LJJs, also if poor tunability can be a problem. On the other hand V-ladder is a different type of device working much more as a collection of coupled lumped oscillators.

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REFERENCES

[1] M.Cirillo, N.Grombech-Jensen, M.Samuelsen, M.Salerno, G.Verona-Rinati, «Fiske modes and Eck steps in long Josephson junctions: theory and experiments», Phys.Rev.B, vol.58, pp.12377-12384, 1998.

[2] V.P.Koshelets et al.., «Flux flow oscillators for sub-mm wave integrated receivers», IEEE Trans. on Appl. Supercond., Vol.9, pp. 4133-4136, 1999.

[3] P.Caputo, N.C.H.Lin, S.P.Yukon, A.V.Ustinov, «Radiation emission from triangular arrays of Josephson junctions», IEEE Trans. on Appl.Supercond., vol.9, pp.4538-4541, 1999.

[4] P.Barbara, A.B.Cawthorne, S.V.Shitov and C.J..Lobb, «Stimulated emission and amplification in Josephson junction arrays», Phys.Rev.Letters, vol.60, pp.7575-7578, 1999.

[5] J.R.Phillips, H.S.J.van der Zant, J.White and T.P.Orlando, «Influence of induced magnetic fields on Shapiro steps in Josephson junction arrays», Phys.Rev.B, vol.50, pp.9387-9396, 1994; D.Dominguez and V.Josè, «Magnetic and transport dc properties of inductive Josephson junction arrays», Phys.Rev.B, vol.53, pp.11692-11713, 1996.

[6] M.Barahona et al., «Resonances of dynamical checkerboard states in Josephson arrays with self-inductance», Phys.Rev.B, vol.55, pp.R11989-R11992, 1997.

[7] P.Caputo, M.Fistul, A.V.Ustinov, B.Malomed and S.Flach, «Cavity resonances in Josephson ladders», Phys.Rev.B, vol..59, pp.14050-14053, 1999.

[8] H.S.J.van der Zant and T.P.Orlando, «Eck peak in underdamped discrete supeconducting vortex flow devices», J.App.Phys., vol.76, pp.7606-7612, 1994.

[9] P.Caputo, M.Fistul and A.V.Ustinov, «Spectrum of linear modes in triangular arrays of Josephson junctions», web-proceedings of Euroconference Future Perspectives of Superconducting Josephson Devices, Acquafredda di Maratea, Italy, 1-6 July 2000, http://www.ing.univaq.it/energeti/research/Fisica/proc1.htm.

[10] G.Filatrella, A.Petraglia and G.Rotoli, «Mutual inductance effects in driven planar Josephson junctions arrays», Eur.J.Phys.B, vol.12, pp.23-30, 1999.

[11] A.Petraglia, G.Filatrella and G.Rotoli, «Screening Currents effects in Josephson junctions arrays», , Phys.Rev.B, vol.53, pp.2732-2737, 1996.