Today we will discuss:
1) McCabe-Thiele graphical construction
2) Determination of N and $X_B$
3) Minimum number of stages $N$
4) Minimum reflux
5) Example
6) Subcooled Reflux
7) Multiple Feeds
8) Side stream products
9) Open steam
10) Non-ideal distillation: Murphree efficiency
Construction Lines for McCabe-Thiele Method

Rectifying Section:
Operating line
Slope = \( \frac{L}{V} = \frac{R}{R+1} < 1 \)

\[
y = \frac{L}{V} x + \frac{D}{V} x_D
\]

Stripping Section:
Operating line
Slope = \( \frac{L}{V} = \frac{(V_B+1)}{V_B} \)

\[
y = \frac{L}{V} x - \frac{B}{V} x_B
\]

Distillation

Overhead vapor
Reflux drum
Reflex
Distillate
Rectifying section stages
Feed
Feed Stage
Boilup
Partial reboiler
Bottoms
Stripping section stages

Lecture 13: McCabe-Theile
Construction for the McCabe-Thiele Method

1. Plot the equilibrium curve and the 45° line.

2. Plot the given compositions (F, B, and D).

3. Draw the q-line from L_F and V_F.

4. Determine R_{min} from the intersection of the rectifying section OL and the equilibrium curve.

5. Determine R from R/R_{min}.


7. Draw OL for the Stripping section.

Step 1: Plot equilibrium curve and 45 degree line.
Step 2: Plot given compositions (F, B, and D).
Step 3: Draw q-line from L_F and V_F.
Step 4: Determine R_{min} from intersection of the rectifying section OL and the equilibrium curve.
Step 5: Determine R from R/R_{min}.
Step 7: Draw OL for Stripping section.
Feed Location for the McCabe-Thiele Method

Equilibrium curve

Feed stage located one tray too low.

Feed stage located one tray too high.
Optimum Feed Location for McCabe-Thiele

Optimum feed stage location.
Determination of N and $x_B$ for McCabe-Thiele

**Construction:**
Step 1: Plot *equilibrium curve* and 45 degree line.
Step 2: Plot given compositions (F, B, and D)
Step 3: Draw *q-line* from $L_F$ and $V_F$
Step 4: Determine $R_{\text{min}}$ from intersection of the Rectifying section OL and the equilibrium curve.
Step 5: Determine $R$ from $R/R_{\text{min}}$
Step 6: Draw OL for *Rectifying section*
Step 7: Draw OL for *Stripping section*

**Solution:**
Step 1: From $x_D$ locate $x_1$ and $y_1$ drawing a *horizontal line* to the *equilibrium condition* for stage 1.
Step 2: Find $y_2$ drawing a *vertical line* to the rectifying OL locate the mass balance condition between $x_1$ and $y_2$.
Step 3: From $y_2$ draw a *horizontal line* to the *equilibrium condition* for stage 2 to locate $x_2$.
Step 4: Return to step 2 and cycle through steps 2 and 3 until $x_i < z_F$. Draw subsequent *vertical lines* to the stripping section OL.
Step 5: End after predetermined number of stages, or when $x_i$ is less than $x_B$. 
By returning all the exiting vapor as reflux and all the exiting liquid as boilup, the operating lines have slope of one. Although this is the minimum number of stages, no product is produced (note the feed must then go to zero).
By returning no exiting vapor as reflux and no exiting liquid as boilup the operating line intersection is as far to the left as equilibrium allows.

Although this is the minimum amount of reflux, it takes infinite stages (note the pinch point between the operating lines and equilibrium).
Minimum Reflux for Non-ideal McCabe-Thiele

Although this is the minimum amount of reflux, it takes infinite stages (note the pinch point between the operating lines and equilibrium).
Example: Determination of N and $x_B$ for McCabe-Thiele

**Given:**
100 Kmol/hr of a feed of 60% benzene and 40% heptane is to be separated by distillation. The distillate is to be 90% benzene and the bottoms 10% benzene. The feed enters the column as 30mol% vapor. Use $R$ 1.5 times the minimum. Assume a constant relative Volatility of $\infty$ of 4 and that the pressure is constant throughout the column at 1atm.

**Construction:**
Step 1: Plot equilibrium curve and 45 degree line.
The equilibrium curve is found using:

$$y = \frac{\alpha x}{1 + x(\alpha - 1)}$$

Step 2: Plot given compositions (F, B, and D)
Step 3: Draw q-line from $L_F$ and $V_F$. Use

$$q = \frac{L - L}{F} = \frac{L + L_F - L}{F} = \frac{L_F}{F} = 0.7$$

to find q. Then plot the q-line using:

$$y = \left(\frac{q}{q-1}\right)x - \left(\frac{z_F}{q-1}\right) = -2.333x + 2$$

Step 4: Determine $R_{\text{min}}$ from intersection of the rectifying section OL and the equilibrium curve. This happens at a slope of about .25

$$0.25 = \frac{R_{\text{min}}}{R_{\text{min}} + 1} \Rightarrow R_{\text{min}} = 0.333$$
Example: Determination of N and $x_B$ for McCabe-Thiele

Given:
100 Kmol/hr of a feed of 60% benzene and 40% heptane is to be separated by distillation. The distillate is to be 90% benzene and the bottoms 10% benzene. The feed enters the column as 30mol% vapor. Use $R$ 3 times the minimum. Assume a constant relative Volatility of $\infty$ of 4 and that the pressure is constant throughout the column at 1atm.

Construction:
Step 5: From $R_{\text{min}}=0.333$ and $R=3R_{\text{min}}$ we have $R=1$
And the slope of rectifying section OL is 0.5
Step 6: Draw the line with slope 0.5 which is the rectifying section OL.
Step 7. Draw the stripping section operating line from the Bottoms composition to the intersection of the rectifying section OL and the $q$-line.

Solution:
Step 1: From $x_D$ locate $x_1$ and $y_1$ drawing a horizontal line to the equilibrium condition for stage 1.
Step 2: Find $y_2$ drawing a vertical line to the rectifying OL locate the mass balance condition between $x_1$ and $y_2$.
Step 3: From $y_2$ draw a horizontal line to the equilibrium condition for stage 2 to locate $x_2$.
Step 4: Return to step 2 and cycle through steps 2 and 3 until $x_i < z_F$.

Results:
Feed at stage between 2 and 3.
5 stages (minimum stages = 3.2)
$x_B=0.05\%$ benzene
Example: Determination of $N$ and $x_B$ for McCabe-Thiele

**Given:**
100 Kmol/hr of a feed of 60% benzene and 40% heptane is to be separated by distillation. The distillate is to be 90% benzene and The bottoms 10% benzene. The feed enters the column as 30mol% vapor. Use $R$ 3 times the minimum. Assume a constant relative Volatility of $\infty$ of 4 and that the pressure is constant throughout the column at 1atm.

Minimum number of stages is determined by stepping off between the equilibrium curve and the 45 degree line. The result is 3.2 stages.
If the liquid reflux is colder than the bubble-point temperature, then it will condense some vapor in the top stage. This changes the reflux ratio to the internal reflux ratio.
McCabe-Thiele Method: Subcooled Reflux

The amount of extra reflux that is produced depends on the heat capacity of the liquid, and the heat of vaporization of the vapor.

\[ R' \Delta H_{vap} = RC_P^L \Delta T_{sub} \]

The total amount of reflux, called the internal reflux is the sum of the reflux ratio and the vapor condensed by the subcooled reflux:

\[ R_{int} = R + R' \]

\[ R_{int} = R(1 + \frac{C_P^L \Delta T_{sub}}{\Delta H_{vap}}) \]

\[ dH = C_PdT + V(1 - T\alpha)dP \]
If the liquid reflux is obtained from a partial condenser, then the reflux is produced as the liquid in equilibrium with the vapor distillate in the condenser.
McCabe-Thiele Method: Partial Condenser

The vapor distillate composition then determines the $y_D$ and stages are stepped off from the intersection of $y_D$ and the equilibrium curve.

Equation:

$$y = \left(\frac{q}{q-1}\right)x - \left(\frac{Z_F}{q-1}\right)$$
The McCabe-Thiele method for cascades can be applied to systems with more than two sections. Here, we show a cascade with 2 feeds: A 3 section cascade.

How do you make the McCabe-Thiele graphical construction for such a cascade?
McCabe-Thiele Method: Multiple Feeds

First, note that each feed stream changes the slope of the operating line from section to section.

The feed stream changes the flow rates in the stages above and below it. Consequently, it changes the mass balances and the slopes of the operating lines.
The flow rates above Feed 1 are constant due to constant molar overflow (CMO). The feed changes the slope depending on the feed condition. Flow rates in the intermediate section are constant, but change when Feed 2 is introduced.
Example: Feed 1 a saturated vapor of composition \( z_{F1} \), and Feed 2 a saturated liquid of composition \( z_{F2} \).
Occasionally a cascade is configured such that an intermediate side stream of intermediate composition is removed from the column.

How do we analyze this configuration?

Use the multiple mass balance envelopes and assume a constant molar overflow condition.

If we perform a material balance in the light key around the stages above the side stream including the condenser:

\[ V_{n+1}y_{n+1} = L_n x_n + Dx_D \]

Which we can rearrange to find:

\[ y_{n+1} = \frac{L_n}{V_{n+1}} x_n + \frac{D}{V_{n+1}} x_D \]

For \( L \) and \( V \) constant from stage to stage, then:

\[ y = \frac{L}{V} x + \frac{D}{V} x_D \]

Operating line above side stream
If we perform a material balance in the light key around the stages above the side stream including the side stream and condenser:

\[ V_{n+1} y_{n+1} = L_n x_n + L_s x_s + D x_D \]

Which we can rearrange to find:

\[ y_{n+1} = \frac{L_n}{V_{n+1}} x_n + \frac{L_s x_s + D x_D}{V_{n+1}} \]

For \( L \) and \( V \) constant from stage to stage, then:

\[ y = \frac{L'}{V} x + \frac{L_s x + D x}{V} \]

The two operating lines intersect at:

\[ x = x_s \]
McCabe-Thiele Method: Side Stream

**Equilibrium curve**

**Rectifying Section:**
Operating line
Slope = $\frac{L}{V} = \frac{R}{R+1} < 1$

**Intermediate section:**
Operating line
Slope = $\frac{L'}{V}$

**Stripping Section:**
Operating line
Slope = $\frac{L}{V'} = \frac{V_B+1}{V_B}$

**Side Stream:**
Liquid withdrawn

**Distillation**

- **Overhead vapor**
- **Rectifying section stages**
- **Feed**
- **Stripping section stages**
- **Total condenser**
- **Reflux drum**
- **Distillate**
- **Partial reboiler**
- **Bottoms**

$x = x_N$

$x = x_s$

$x = x_D$

$x = x_F$
Consider the cascade shown on the left:

In this example, the reboiler is replaced by a source of hot steam or an inert gas. In this case, the vapor entering the bottom stage of the column has no light key and so \( y_B \) is zero, although \( x_B \) is non-zero.

Does the slope of the rectifying section operating line increase or decrease?
McCabe-Thiele Method: Open Steam

**Equilibrium curve**

- **Rectifying Section:**
  - Operating line
  - Slope: $\frac{L}{V} = \frac{R}{R+1} < 1$

- **Stripping Section:**
  - Operating line
  - Slope: $\frac{L}{V}$

**Distillation**

- **Total condenser**
- **Reflex drum**
- **Overhead vapor**
- **Distillate**
- **Reflex**
- **Feed Stage**
- **Feed**
- **Steam or inert hot gas (y=0)**
- **Bottoms**

**Legend:**

- $y$: Mole fraction of a component in the vapor phase
- $y_N$: Mole fraction of a component in the feed
- $y_B$: Mole fraction of a component in the bottoms
- $x$: Mole fraction of a component in the liquid phase
- $x_{ZF}$: Mole fraction of a component at the feed stage
- $x_D$: Mole fraction of a component at the distillate stage
- $x=z$: Mole fraction of a component at the feed stage

**Diagram:**

- Saturated liquid feed

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Lecture 13: McCabe-Theile
Non-equilibrium McCabe-Thiele: Murphree Efficiency

The Murphree Plate Efficiency gives the ratio of the actual composition difference between two sequential plates, and that predicted by equilibrium.

For the vapor efficiency:

$$E_{MV} = \frac{y_n - y_{n+1}}{y_n^* - y_{n+1}} = \frac{AB}{EB}$$

For the liquid efficiency:

$$E_{ML} = \frac{x_n - x_{n+1}}{x_n^* - x_{n+1}} = \frac{AB'}{E'B'}$$
We have already developed the McCabe-Thiele Graphical Method for cascades. The same equations we used for the operating lines, q-line, and equilibrium curve can be used to solve for the compositions in each stage algebraically.

\[
y = \frac{\alpha x}{1 + x(\alpha - 1)}
\]

Rectifying Section:

Operating line
Slope = \(\frac{L}{V} = \frac{R}{R+1} < 1\)

\[
y = \frac{R}{R+1} x + \frac{1}{R+1} x_D
\]

Stripping Section:

Operating line
Slope = \(\frac{L}{V} = \frac{V_B+1}{V_B}\)

\[
y = \frac{L}{V} x - \frac{B}{V} x_B
\]

45° line

Equilibrium curve

q-line

\[
y = \left(\frac{q}{q-1}\right)x - \left(\frac{z_F}{q-1}\right)
\]
To carry out the algebraic method we need to determine the slopes of the operating lines algebraically. This can be done finding the intersections between the q-line and equilibrium curve, and the q-line and the rectifying section operating line.

\[ y = \frac{\alpha x}{1 + x(\alpha - 1)} \]

\[ y = \left( \frac{q}{q-1} \right)x - \left( \frac{z_F}{q-1} \right) \]

\[ y = \left( \frac{q}{q-1} \right)x - \left( \frac{z_F}{q-1} \right) = \frac{\alpha x}{1 + x(\alpha - 1)} \]
The slope of the *operating line* for the rectifying section with minimum reflux can be determined from the rise over run. We can then also find the minimum reflux from this slope.

\[ y = \left( \frac{q}{q-1} \right)x - \left( \frac{z_F}{q-1} \right) = \frac{\alpha x}{1 + x(\alpha - 1)} \]

From the minimum reflux, and \( R/R_{\text{min}} \) we can determine the reflux \( R \).

We determine the slope of the rectifying section *operating line* from:

\[ \text{slope} = \frac{R}{R + 1} \]
We can find the intersection of the operating line and the q-line to determine the stripping section operating line:

\[
y = \left(\frac{q}{q-1}\right)x - \left(\frac{z_F}{q-1}\right) = \frac{R}{R+I}x + \frac{I}{R+I}x_D
\]

From the minimum reflux, and \(R/R_{\text{min}}\) we can determine the reflux R.

We determine the slope of the stripping section operating line from:

\[
\frac{y_{QR} - y_B}{x_{QR} - x_B} = \text{slope}
\]
McCabe-Thiele: Algebraic Method

1. In total condenser \( y_1 = x_0 \)

2. \( x_1 \) is determined from the equilibrium curve:
   \[
y_1 = \frac{\alpha x_1}{1 + x_1 (\alpha - 1)}
   \]

3. \( y_2 \) is determined from operating line for the rectifying section:
   \[
y_2 = \frac{R}{R + 1} x_1 + \frac{1}{R + 1} x_D
   \]

4. Repeat steps 2 and 3 until \( x_n \) is less than \( x_{QR} \) (you are on a point of the equilibrium curve to the left of the intersection of the OL and the q-line).

5. \( y_3 \) is determined from operating line for the stripping section:
   \[
y_3 = \frac{L}{V} x_2 - \frac{B}{V} x_B
   \]

6. \( x_3 \) is determined from the equilibrium curve:
   \[
y_3 = \frac{\alpha x_3}{1 + x_3 (\alpha - 1)}
   \]

7. Repeat steps 5 and 6 until \( x_n \) is less than \( x_B \)
McCabe-Thiele Algebraic Method: Examples

x_D = 0.9, x_B = 0.1, z_F = 0.5, q = 0.8

1. Alpha = 4, R = R_{min}
2. Alpha = 4, R = 2R_{min}
3. Alpha = 4, R = 4R_{min}
4. Alpha = 4, R = 20R_{min}
5. Alpha = 1.1, R = 3R_{min}