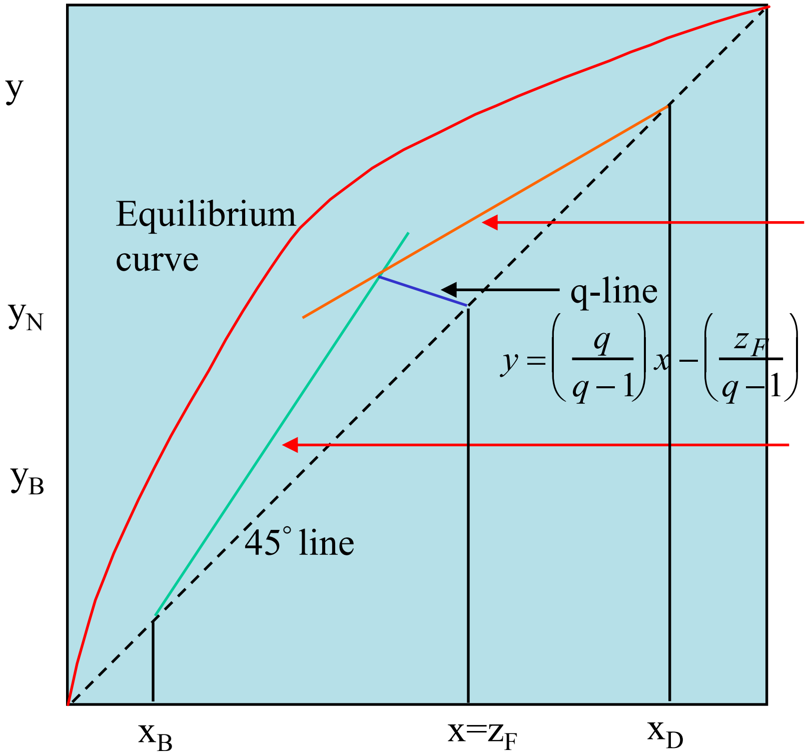
McCabe Thiele Part Two

Today we will discuss:

1) McCabe-Thiele graphical construction
2) Determination of N and X_B
3) Minimum number of stages N
4) Minimum reflux
5) Example
6) Subcooled Reflux
7) Multiple Feeds
8) Side stream products
9) Open steam
10) Non-ideal distillation: Murphree efficiency

Construction Lines for McCabe-Thiele Method

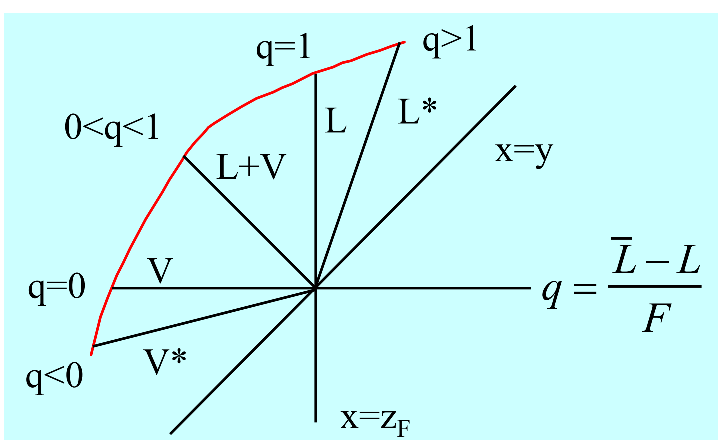


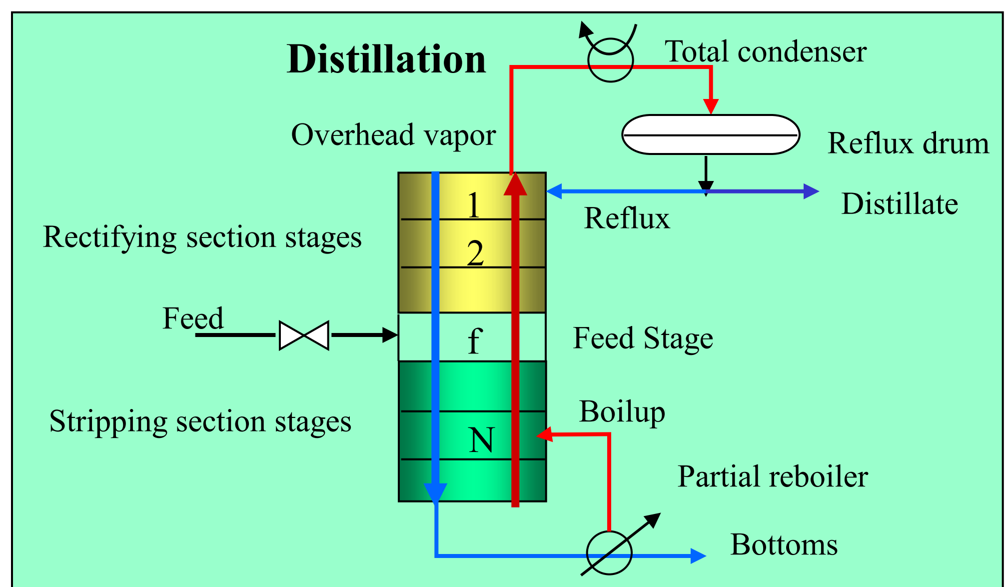
Rectifying Section:
Operating line
Slope=L/V=R/(R+1)<1

$$y = \frac{L}{V}x + \frac{D}{V}x_D$$

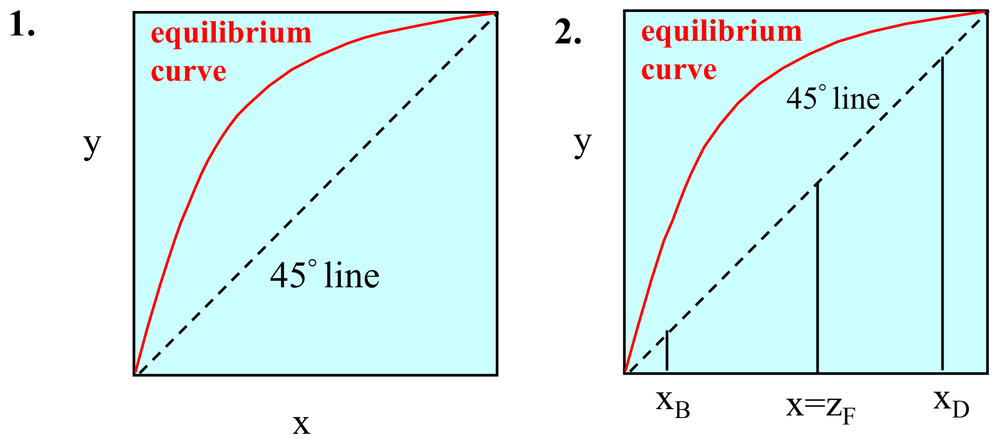
Stripping Section:
Operating line
Slope=$\bar{L}/\bar{V}$=$(V_B+1)/V_B$

$$y = \frac{\bar{L}}{\bar{V}}x - \frac{B}{\bar{V}}x_B$$

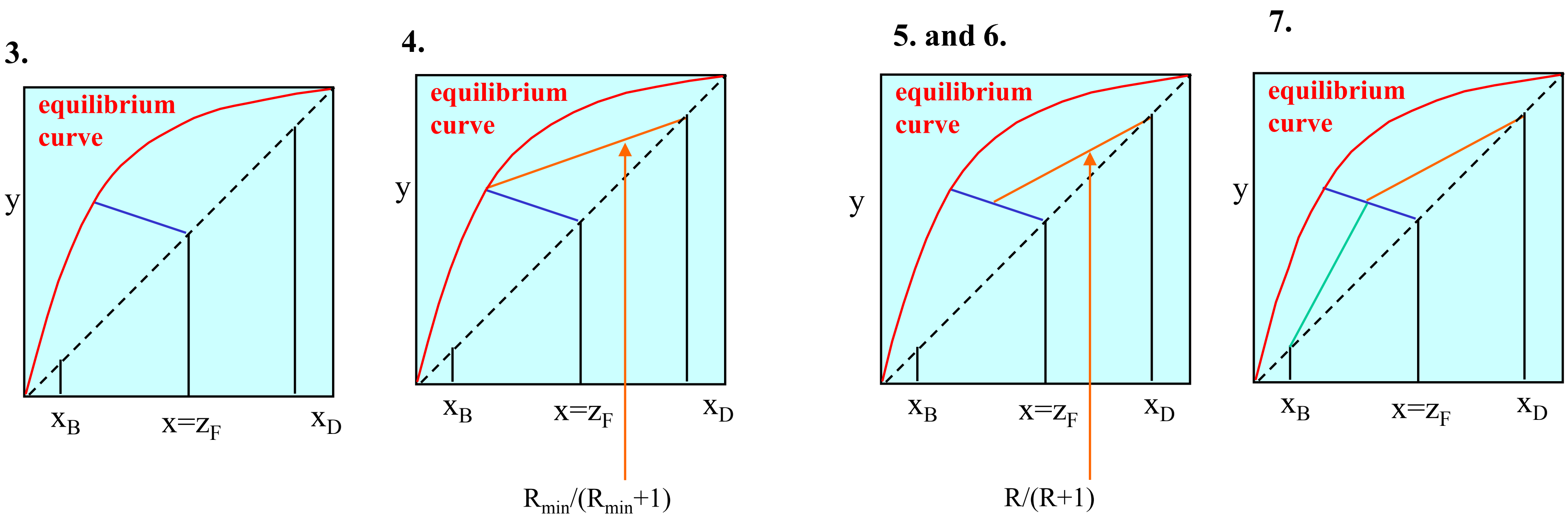




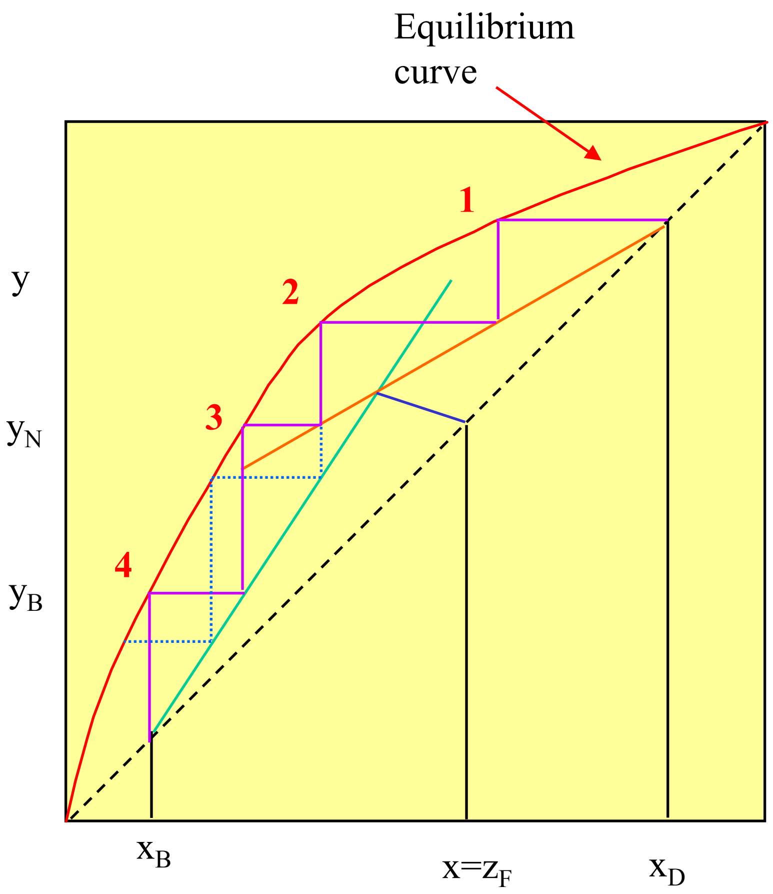
Construction for the McCabe-Thiele Method



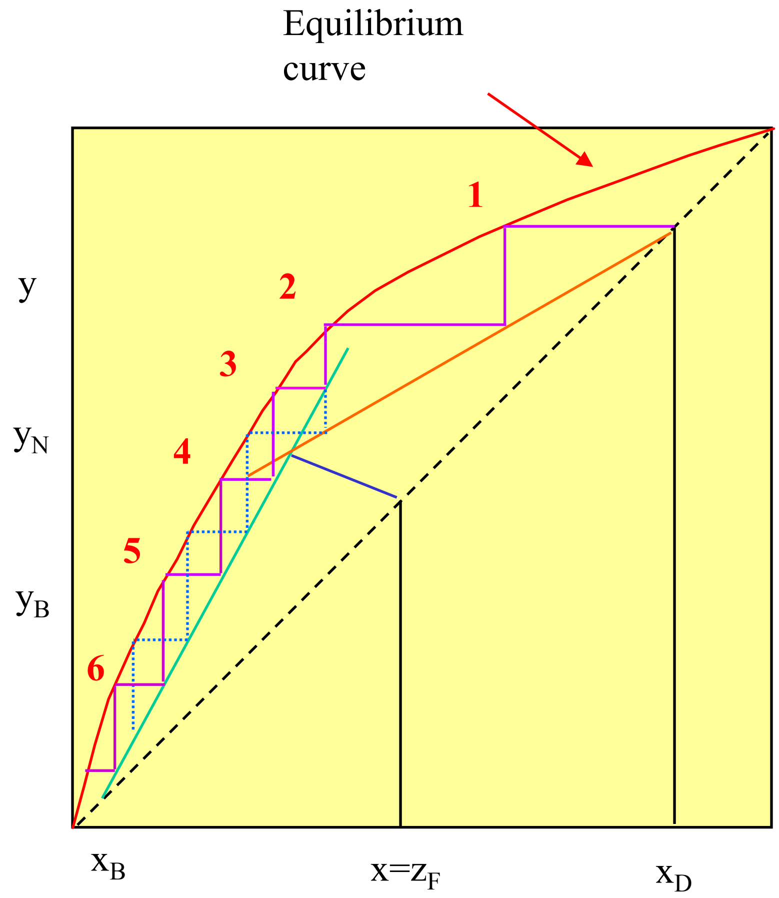
Step 1: Plot **equilibrium curve** and 45 degree line.
Step 2: Plot given compositions (F, B, and D)
Step 3: Draw **q-line** from L_F and V_F
Step 4: Determine R_{min} from intersection of the rectifying section OL and the equilibrium curve.
Step 5: Determine R from R/R_{min}
Step 6: Draw OL for **Rectifying section**
Step 7: Draw OL for **Stripping section**



Feed Location for the McCabe-Thiele Method

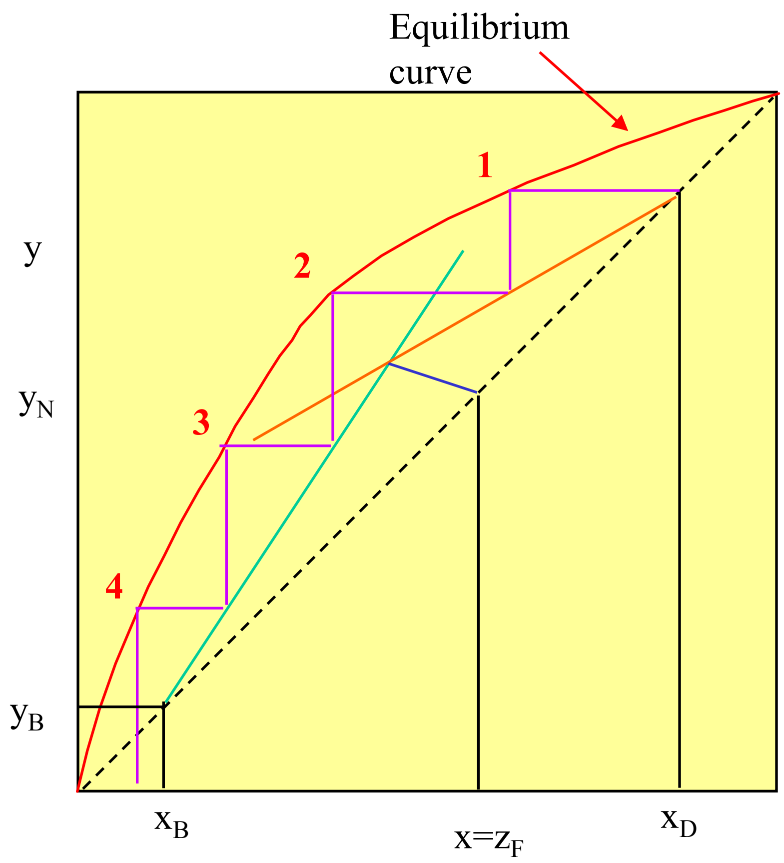


Feed stage located one tray too low.



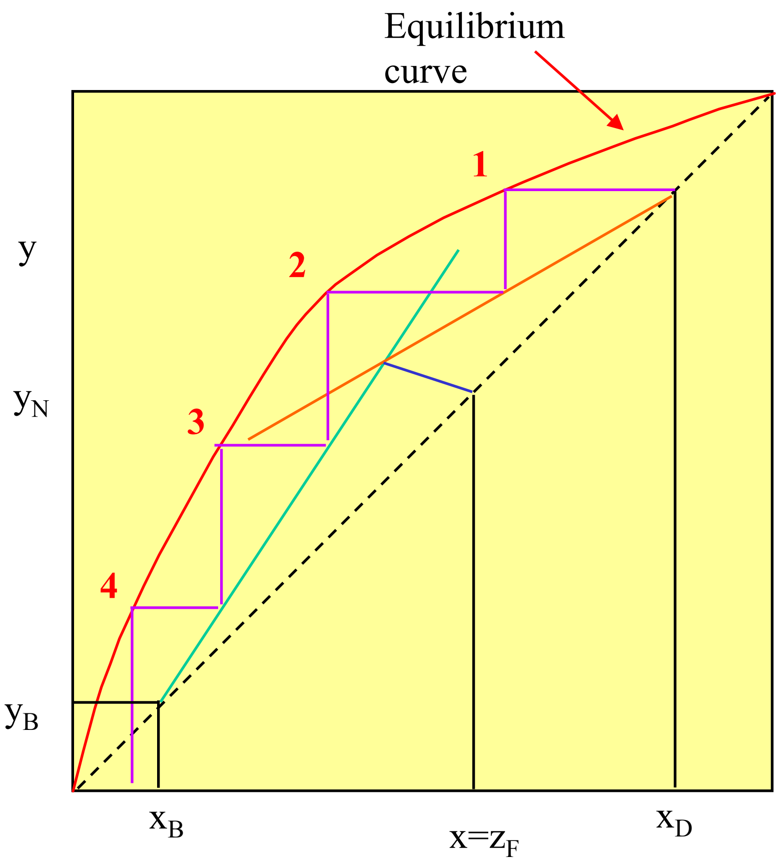
Feed stage located one tray too high.

Optimum Feed Location for McCabe-Thiele



Optimum feed stage location.

Determination of N and x_B for McCabe-Thiele



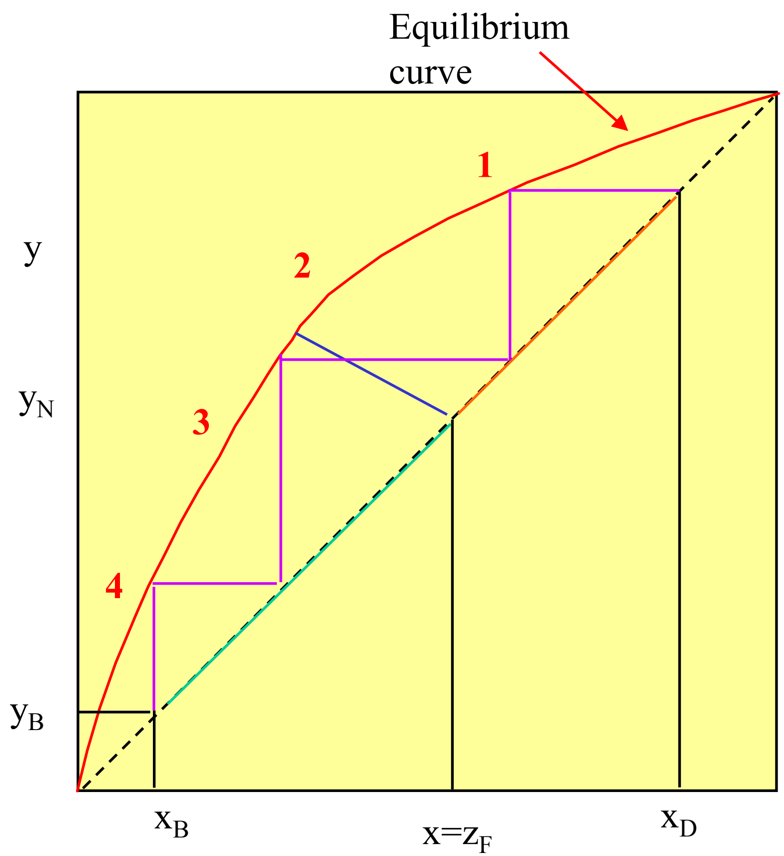
Construction:

Step 1: Plot **equilibrium curve** and 45 degree line.
Step 2: Plot given compositions (F, B, and D)
Step 3: Draw **q-line** from L_F and V_F
Step 4: Determine R_{min} from intersection of the Rectifying section OL and the equilibrium curve.
Step 5: Determine R from R/R_{min}
Step 6: Draw OL for **Rectifying section**
Step 7: Draw OL for **Stripping section**

Solution:

Step 1: From x_D locate x_1 and y_1 drawing a horizontal line to the equilibrium condition for stage 1.
Step 2: Find y_2 drawing a vertical line to the rectifying OL locate the mass balance condition between x_1 and y_2.
Step 3: From y_2 draw a horizontal line to the equilibrium condition for stage 2 to locate x_2.
Step 4: Return to step 2 and cycle through steps 2 and 3 until $x_i < z_F$. Draw subsequent vertical lines to the stripping section OL.
Step 5: End after predetermined number of stages, or when x_i is less than x_B.

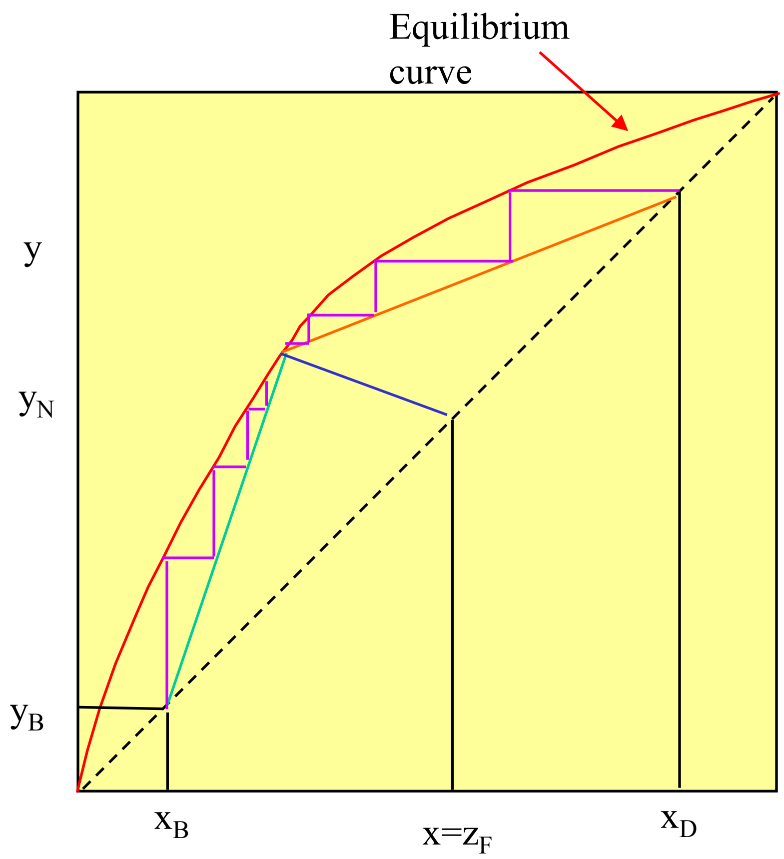
Minimum Number of Stages for McCabe-Thiele



By returning all the exiting vapor as reflux and all the exiting liquid as boilup the operating lines have slope of one.

Although this is the minimum number of stages, no product is produced (note the feed must then go to zero).

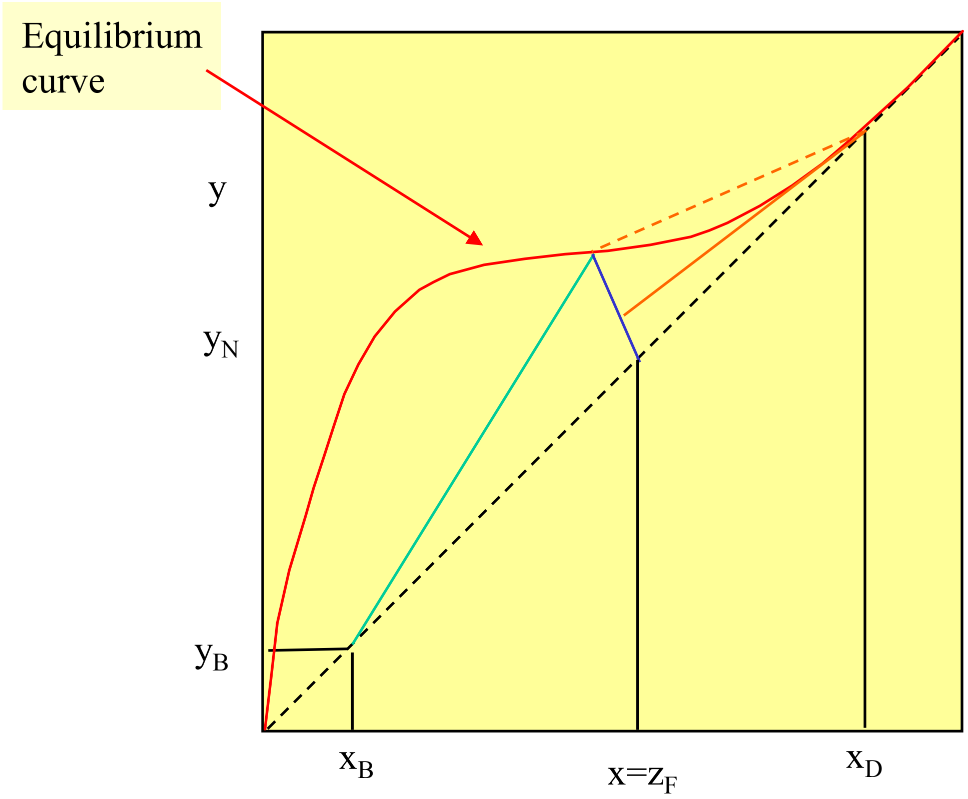
Minimum Reflux for McCabe-Thiele



By returning no exiting vapor as reflux and no exiting liquid as boilup the operating line intersection is as far to the left as equilibrium allows.

Although this is the minimum amount of reflux, it takes infinite stages (note the pinch point between the operating lines and equilibrium).

Minimum Reflux for Non-ideal McCabe-Thiele

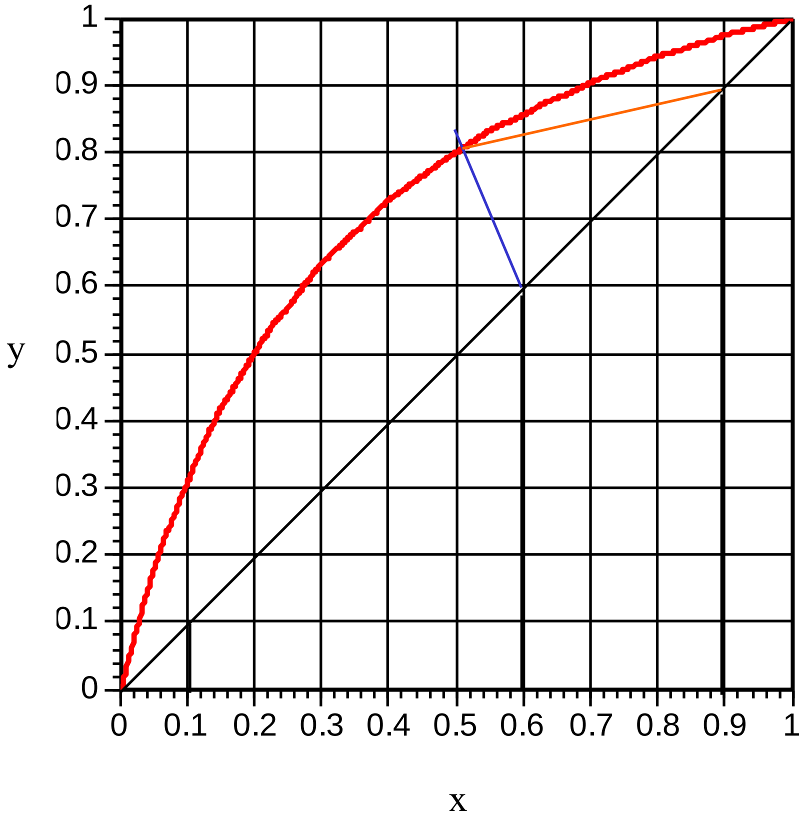


Although this is the minimum amount of reflux, it takes infinite stages (note the pinch point between the operating lines and equilibrium).

Example: Determination of N and x_B for McCabe-Thiele

Given:

100 Kmol/hr of a feed of 60% benzene and 40% heptane is to be separated by distillation. The distillate is to be 90% benzene and The bottoms 10% benzene. The feed enters the column as 30mol% vapor. Use R 1.5 times the minimum. Assume a constant relative Volatility of ∝ of 4 and that the pressure is constant throughout the column at 1atm.



Construction:

Step 1: Plot **equilibrium curve** and 45 degree line.
The equilibrium curve is found using:

$$y = \frac{\alpha x}{1 + x(\alpha - 1)}$$

Step 2: Plot given compositions (F, B, and D)
Step 3: Draw **q-line** from L_F and V_F. Use

$$q = \frac{\overline{L} - L}{F} = \frac{L + L_F - L}{F} = \frac{L_F}{F} = 0.7$$

to find q. Then plot the q-line using:

$$y = \left(\frac{q}{q-1}\right)x - \left(\frac{z_F}{q-1}\right) = -2.333x + 2$$

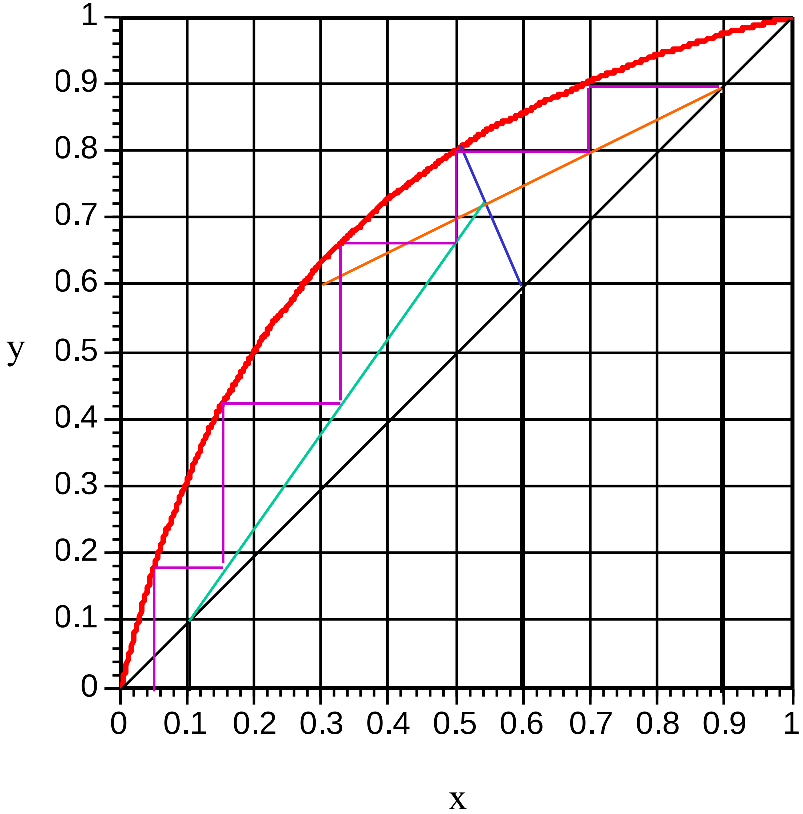
Step 4: Determine R_{min} from intersection of the rectifying section OL and the equilibrium curve. This happens at a slope of about .25

$$0.25 = \frac{R_{\min}}{R_{\min} + 1} \Rightarrow R_{\min} = 0.333$$

Example: Determination of N and x_B for McCabe-Thiele

Given:

100 Kmol/hr of a feed of 60% benzene and 40% heptane is to be separated by distillation. The distillate is to be 90% benzene and The bottoms 10% benzene. The feed enters the column as 30mol% vapor. Use R 3 times the minimum. Assume a constant relative Volatility of $\propto$ of 4 and that the pressure is constant throughout the column at 1atm.



Construction:

Step 5: From R_{min}=0.333 and R=3R_{min} we have R=1
And the slope of rectifying section OL is 0.5
Step 6: Draw the line with slope 0.5 which is the rectifying section OL.
Step 7. Draw the stripping section operating line from the Bottoms composition to the intersection of the rectifying section OL and the q-line.

Solution:

Step 1: From x_D locate x_1 and y_1 drawing a horizontal line to the equilibrium condition for stage 1.
Step 2: Find y_2 drawing a vertical line to the rectifying OL locate the mass balance condition between x_1 and y_2.
Step 3: From y_2 draw a horizontal line to the equilibrium condition for stage 2 to locate x_2.
Step 4: Return to step 2 and cycle through steps 2 and 3 until $x_i<z_F$.

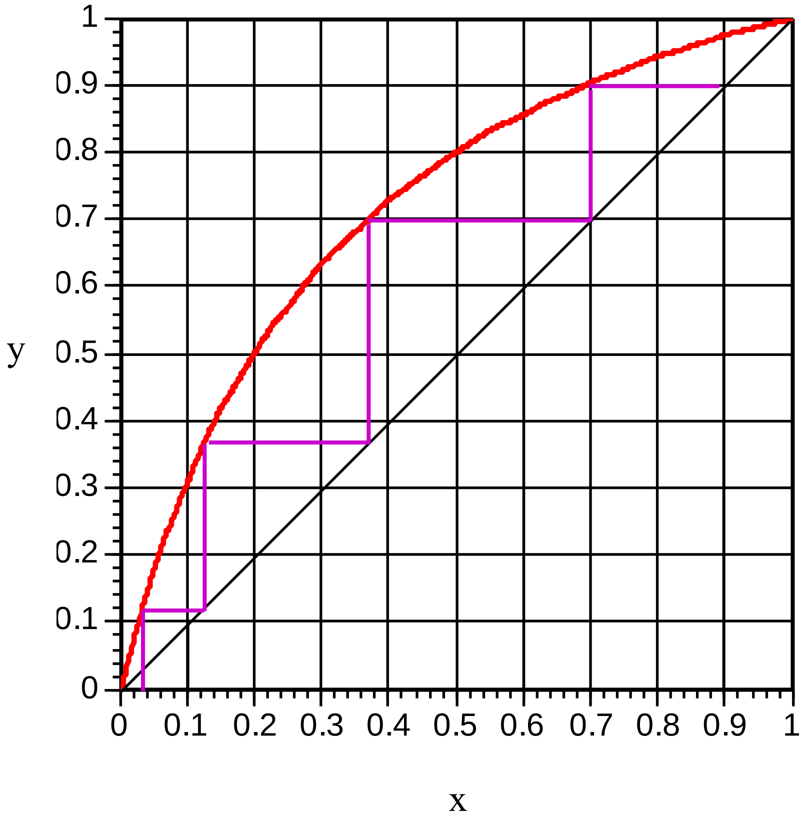
Results:

Feed at stage between 2 and 3.
5 stages (minimum stages = 3.2)
x_B=0.05% benzene

Example: Determination of N and x_B for McCabe-Thiele

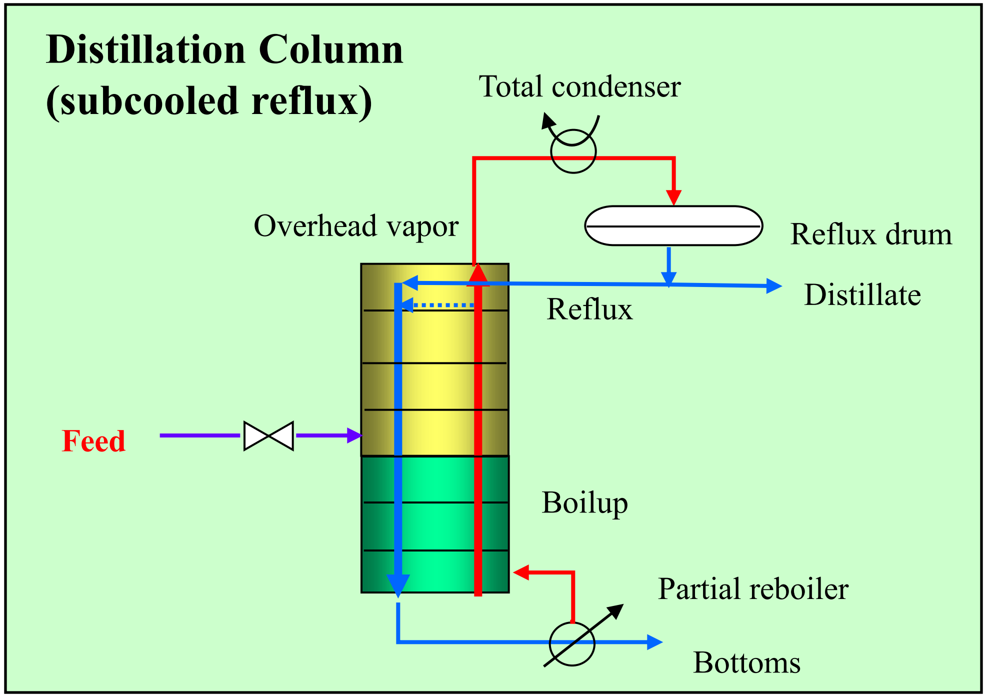
Given:

100 Kmol/hr of a feed of 60% benzene and 40% heptane is to be separated by distillation. The distillate is to be 90% benzene and The bottoms 10% benzene. The feed enters the column as 30mol% vapor. Use R 3 times the minimum. Assume a constant relative Volatility of $\propto$ of 4 and that the pressure is constant throughout the column at 1atm.



Minimum number of stages is determined by stepping off between the equilibrium curve and the 45 degree line.The result is 3.2 stages.

McCabe-Thiele Method: Subcooled Reflux



If the liquid reflux is colder than the bubble-point temperature, then it will condense some vapor in the top stage. This changes the reflux ratio to the internal reflux ratio.

McCabe-Thiele Method: Subcooled Reflux

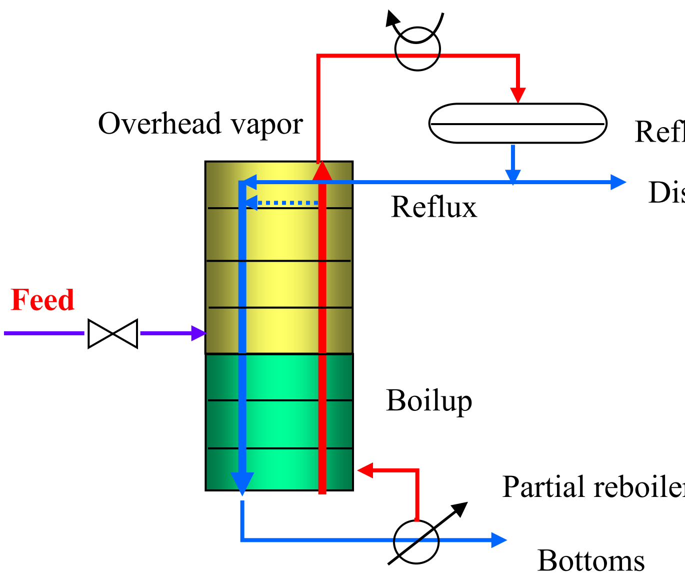
The amount of extra reflux that is produced depends on the heat capacity of the liquid, and the heat of vaporization of the vapor.

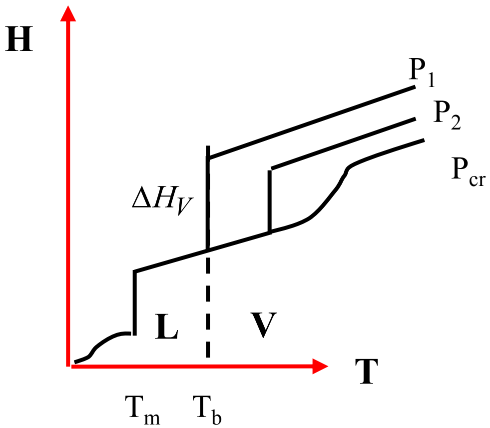
$$R' \Delta H^{vap} = R C_P^L \Delta T_{sub}$$

The total amount of reflux, called the internal reflux is the sum of the reflux ratio and the vapor condensed by the subcooled reflux:

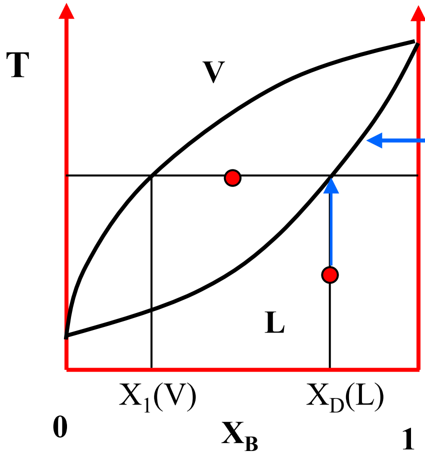
$$R_{\text{int}} = R + R'$$

$$R_{\text{int}} = R\left(1 + \frac{C_P^L \Delta T_{sub}}{\Delta H^{vap}}\right)$$

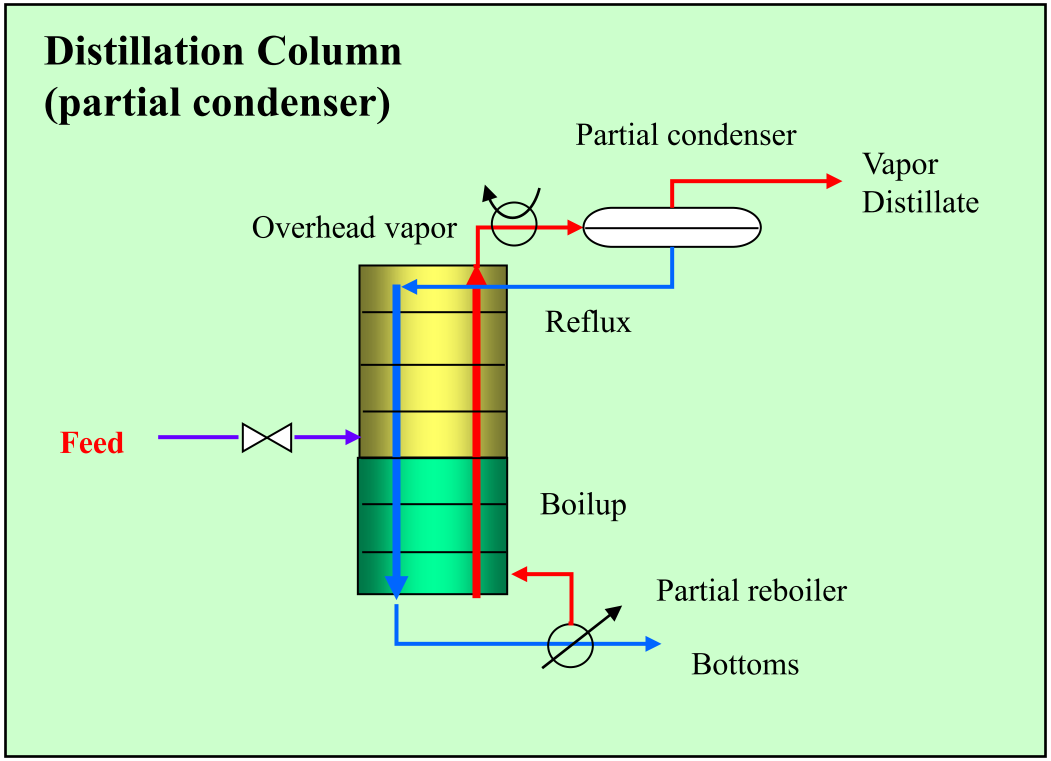




$$dH = C_P dT + V(1 - T\alpha)dP$$



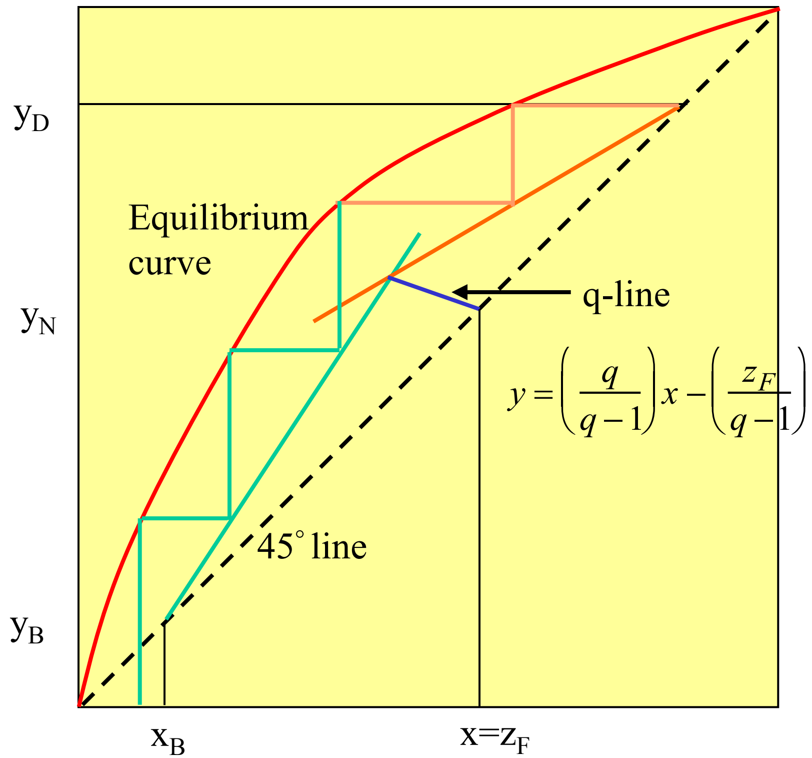
McCabe-Thiele Method: Partial Condenser

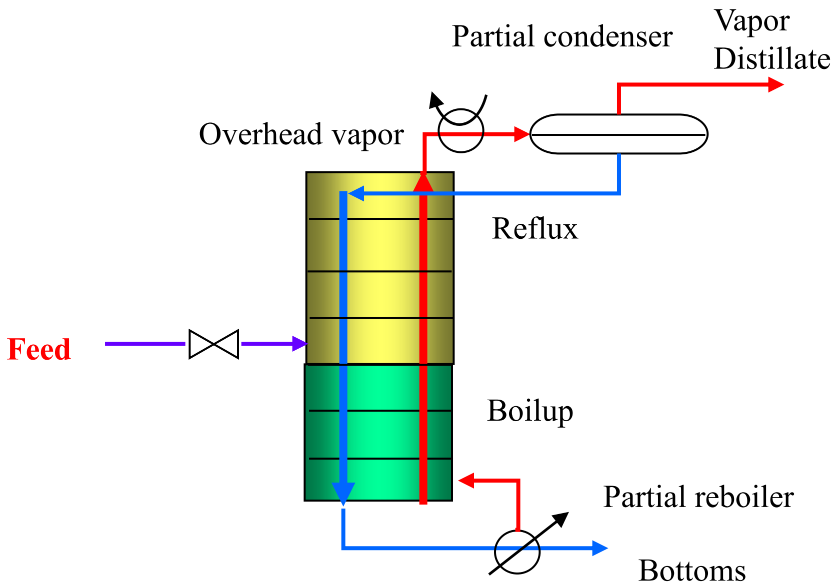


If the liquid reflux is obtained from a partial condenser, then the reflux is produced as the liquid in equilibrium with the vapor distillate in the condenser.

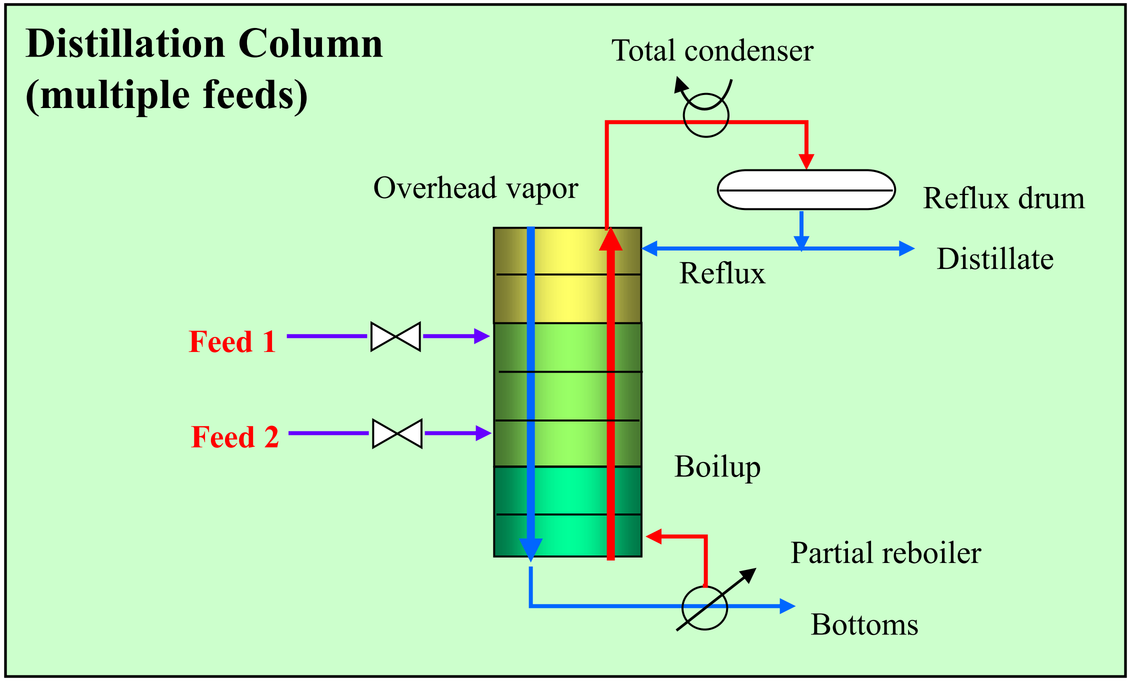
McCabe-Thiele Method: Partial Condenser

The vapor distillate composition then determines the y_D and stages are stepped off from the intersection of y_D and the equilibrium curve.





McCabe-Thiele Method: Multiple Feeds



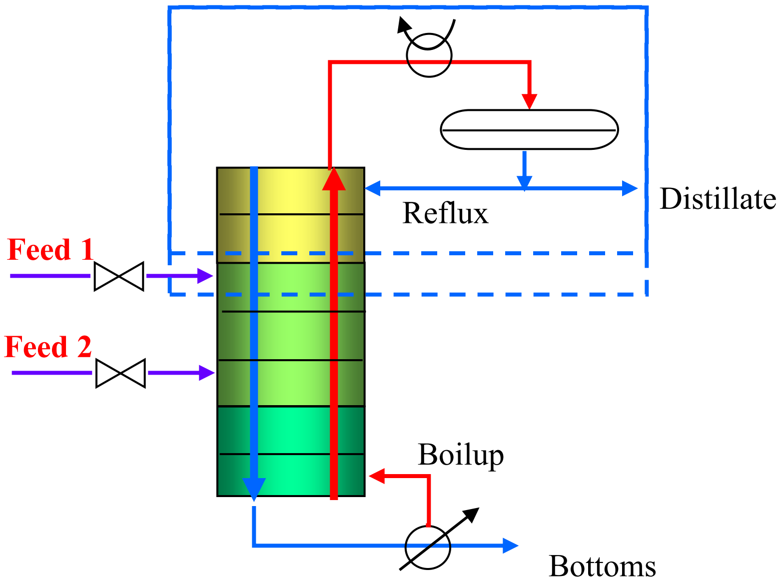
The McCabe-Thiele method for cascades can be applied to systems with more than two sections. Here, we show a cascade with 2 feeds: A 3 section cascade.

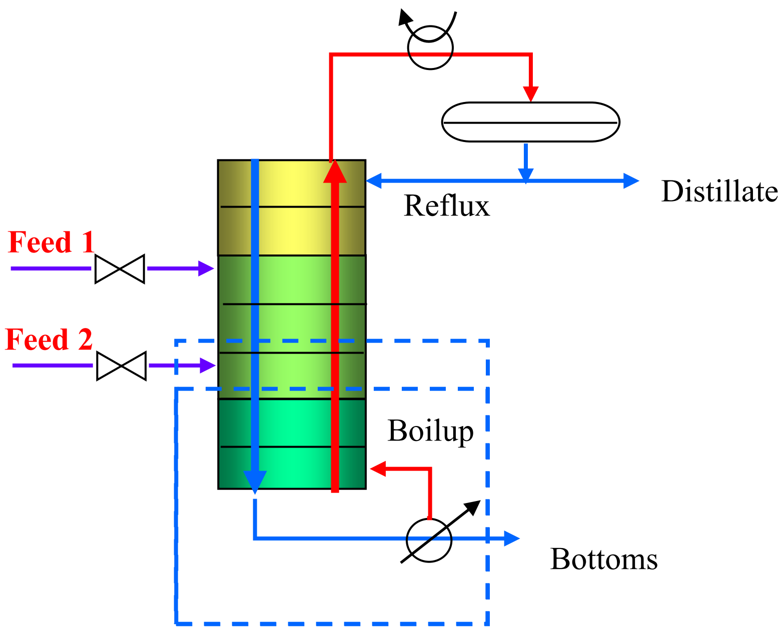
How do you make the McCabe-Thiele graphical construction for such a cascade?

McCabe-Thiele Method: Multiple Feeds

First, note that each feed stream changes the slope of the operating line from section to section.

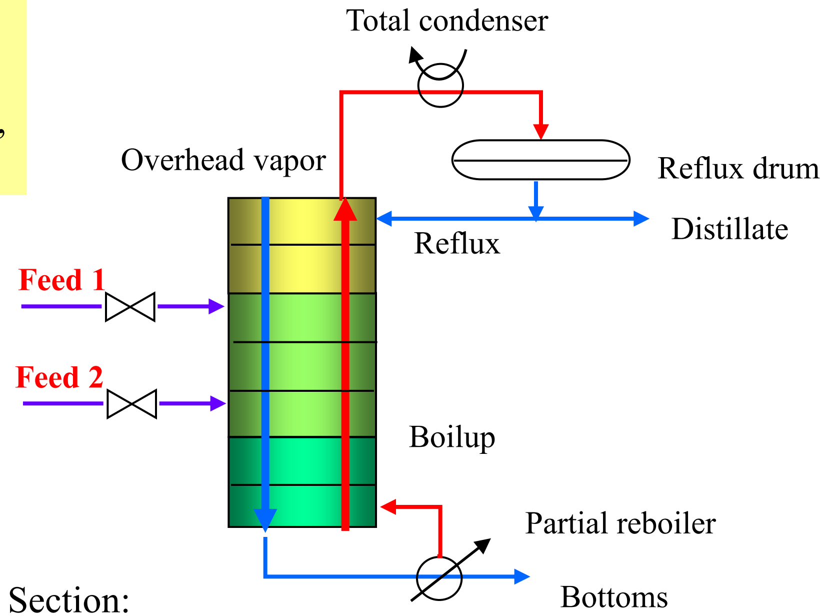
The feed stream changes the flow rates in the stages above and below it. Consequently, it changes the mass balances and the slopes of the operating lines.

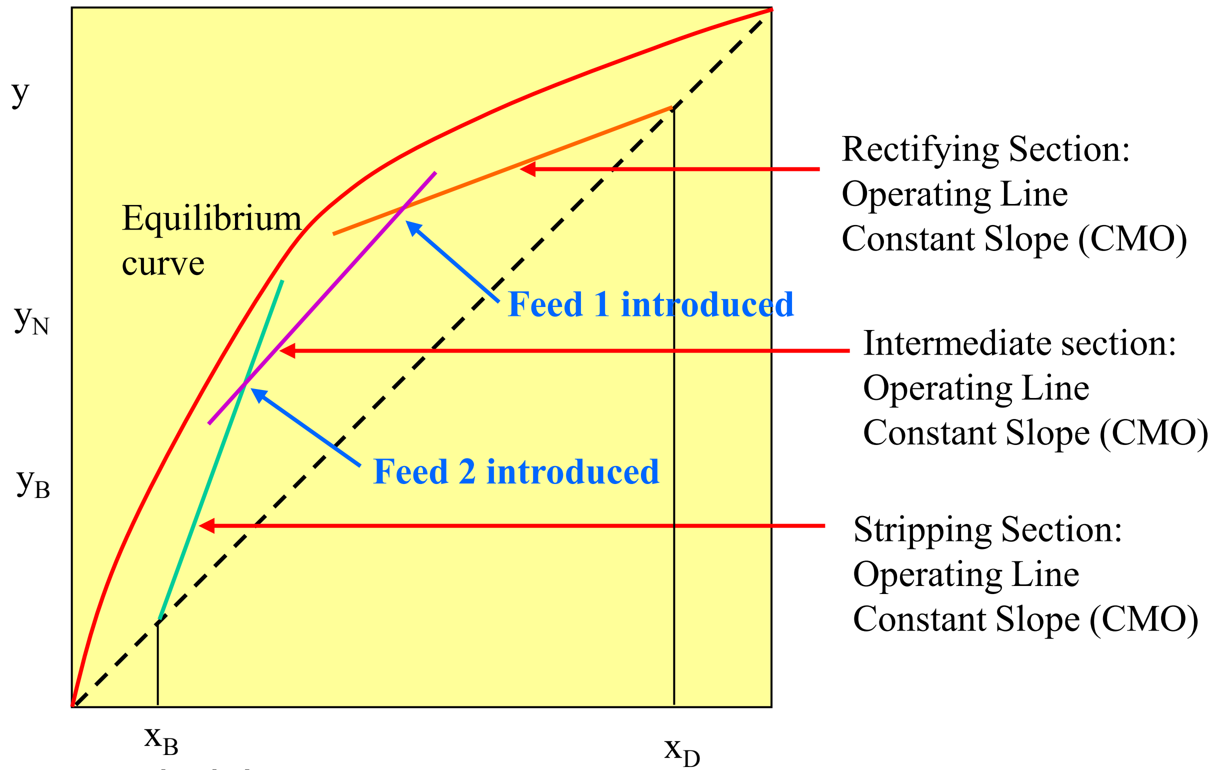




McCabe-Thiele Method: Multiple Feeds

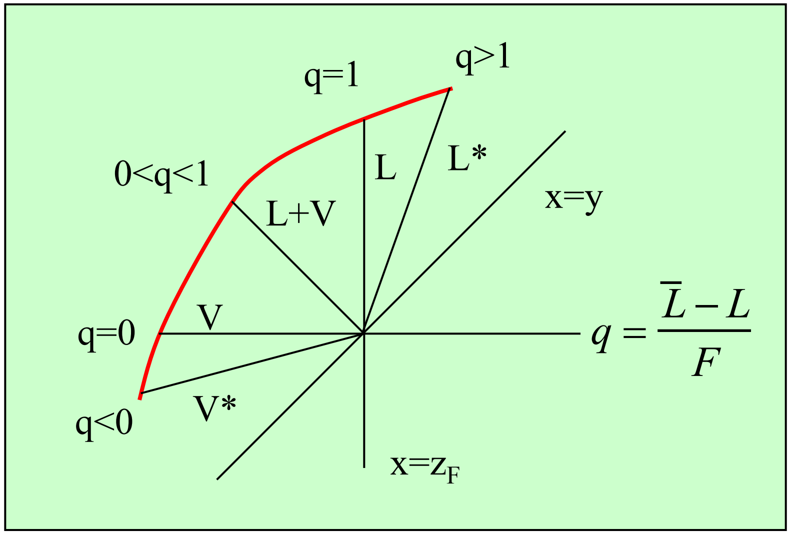
The flow rates above Feed 1 are constant due to constant molar overflow (CMO). The feed changes the slope depending on the feed condition. Flow rates in the intermediate section are constant, but change when Feed 2 is introduced.

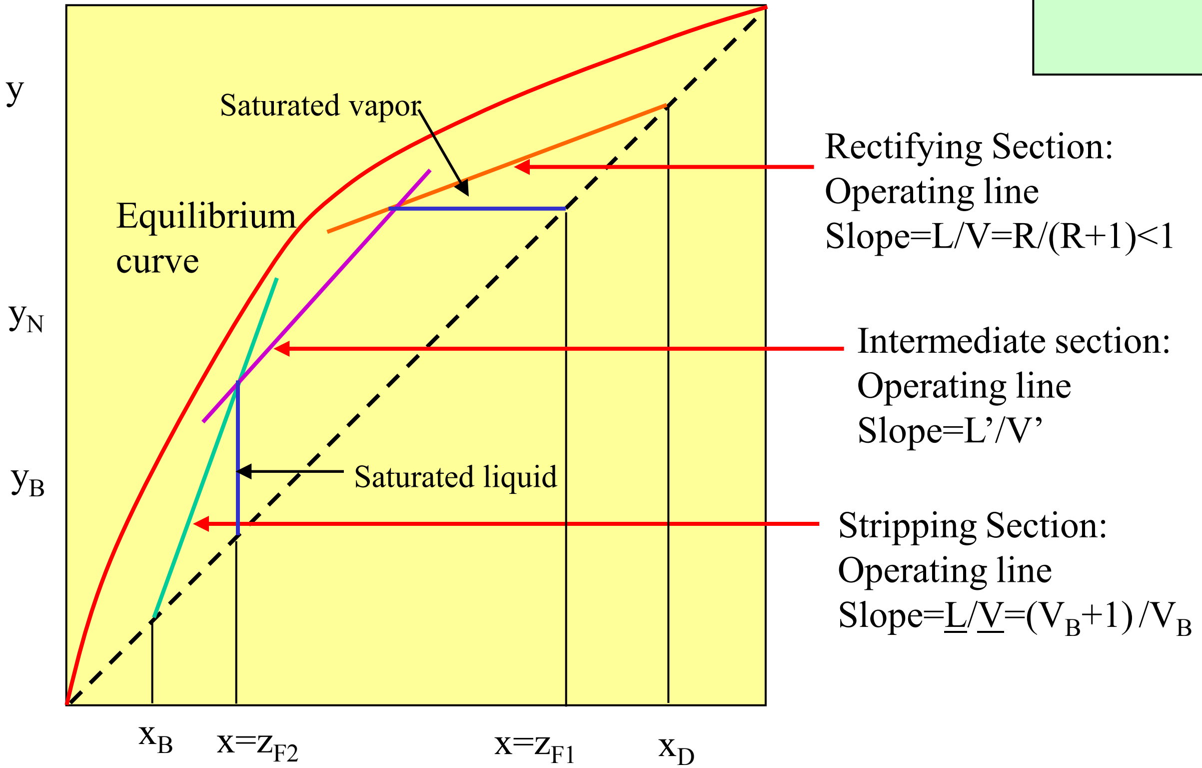




McCabe-Thiele Method: Multiple Feeds

Example: Feed 1 a saturated vapor of composition z_{F1}, and Feed 2 a saturated liquid of composition z_{F2}



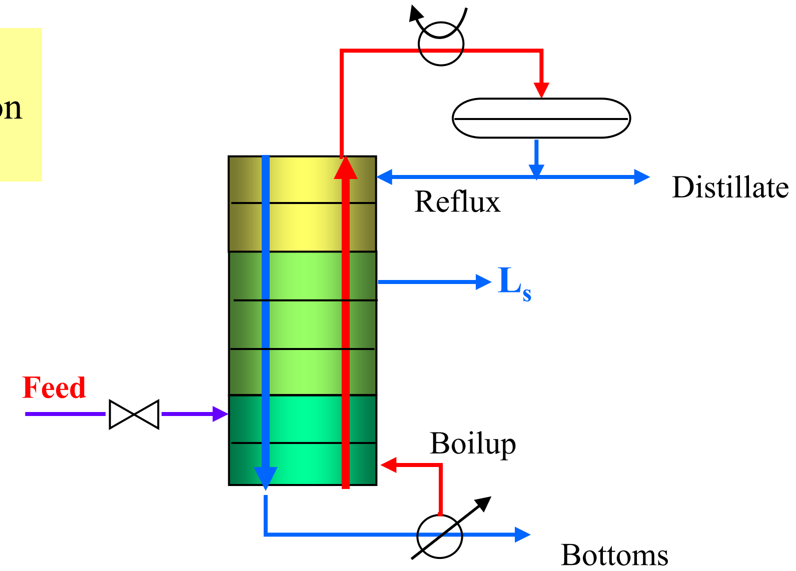


McCabe-Thiele Method: Side Stream

Occasionally a cascade is configured such that an intermediate side stream of intermediate composition is removed from the column.

How do we analyze this configuration?

Use the multiple mass balance envelopes and assume a constant molar overflow condition.

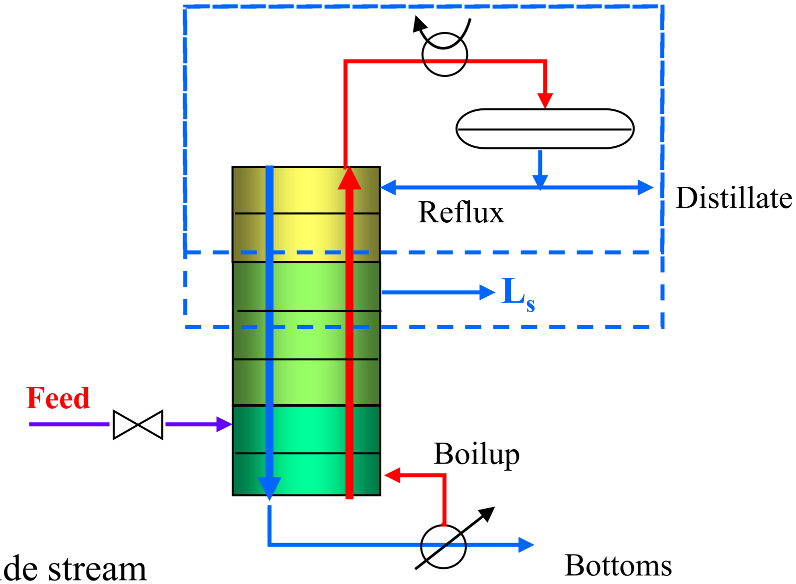


If we perform a material balance in the light key around the stages above the side stream including the condenser:

$$V_{n+1}y_{n+1} = L_n x_n + D x_D$$

Which we can rearrange to find:

$$y_{n+1} = \frac{L_n}{V_{n+1}} x_n + \frac{D}{V_{n+1}} x_D$$



For L and V constant from stage to stage, then:

$$y = \frac{L}{V} x + \frac{D}{V} x_D$$

Operating line above side stream

McCabe-Thiele Method: Side Stream

If we perform a material balance in the light key around the stages above the side stream including the side stream and condenser:

$$V_{n+1}y_{n+1} = L_n x_n + L_s x_s + Dx_D$$

Which we can rearrange to find:

$$y_{n+1} = \frac{L_n}{V_{n+1}} x_n + \frac{L_s x_s + Dx_D}{V_{n+1}}$$

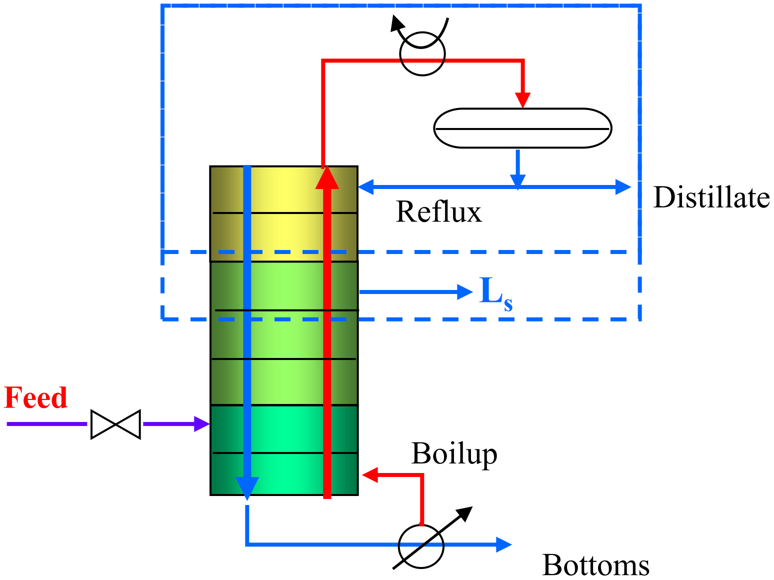
For L and V constant from stage to stage, then:

$$y = \frac{L'}{V} x + \frac{L_s x + Dx}{V}$$

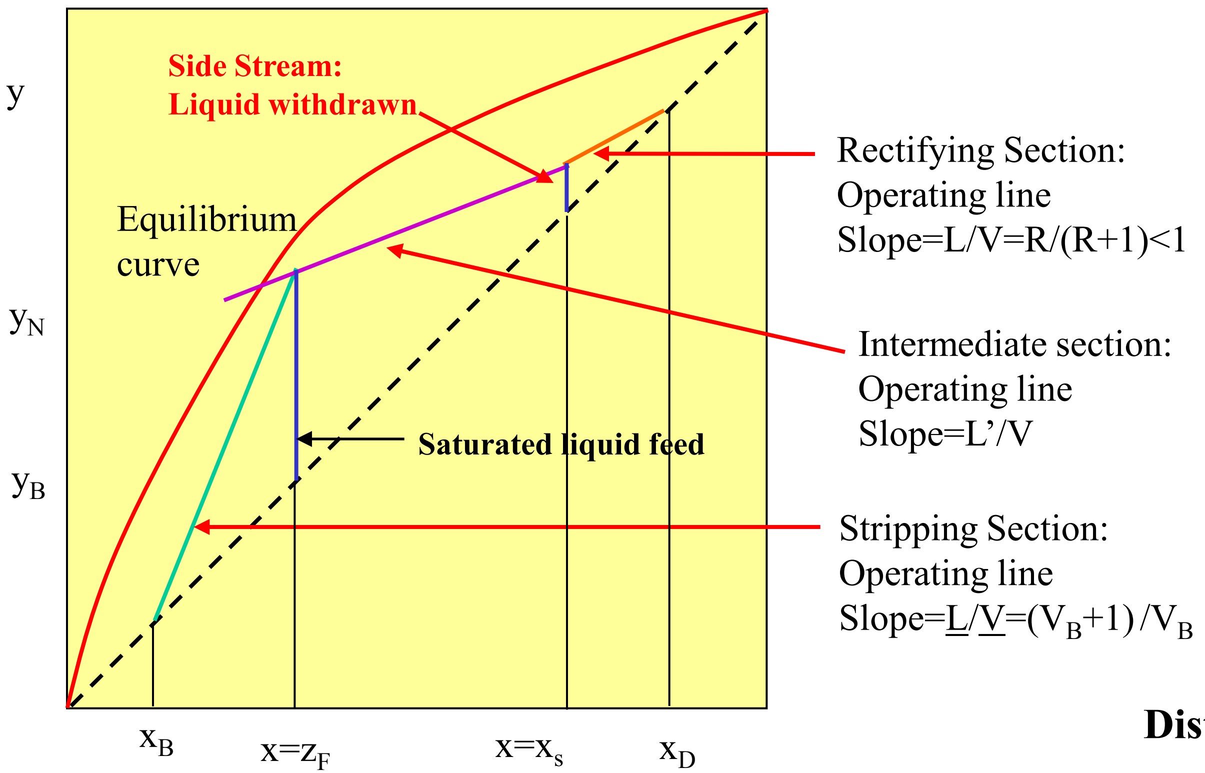
Operating line below side stream

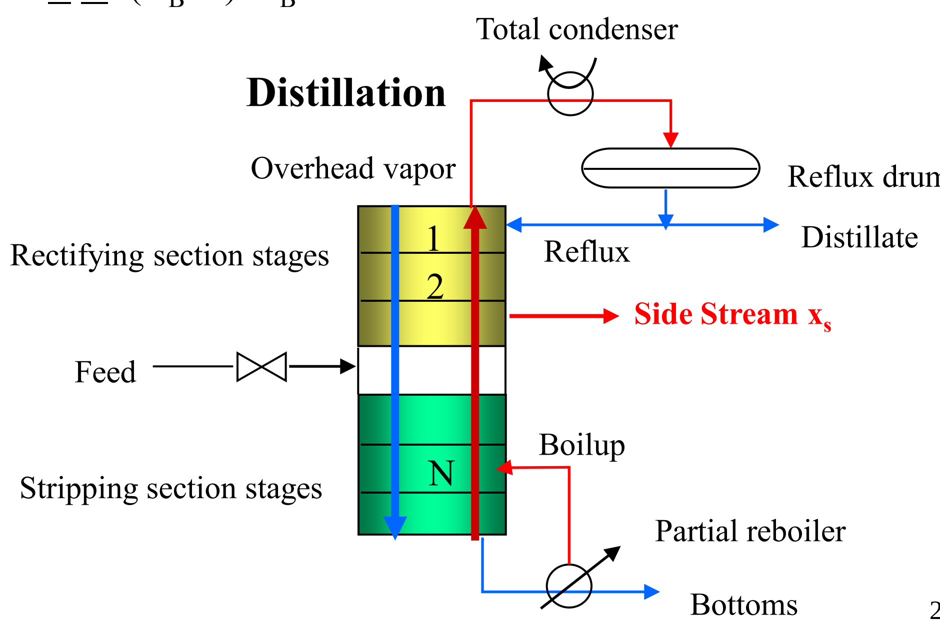
The two operating lines intersect at :

$$x = x_s$$



McCabe-Thiele Method: Side Stream

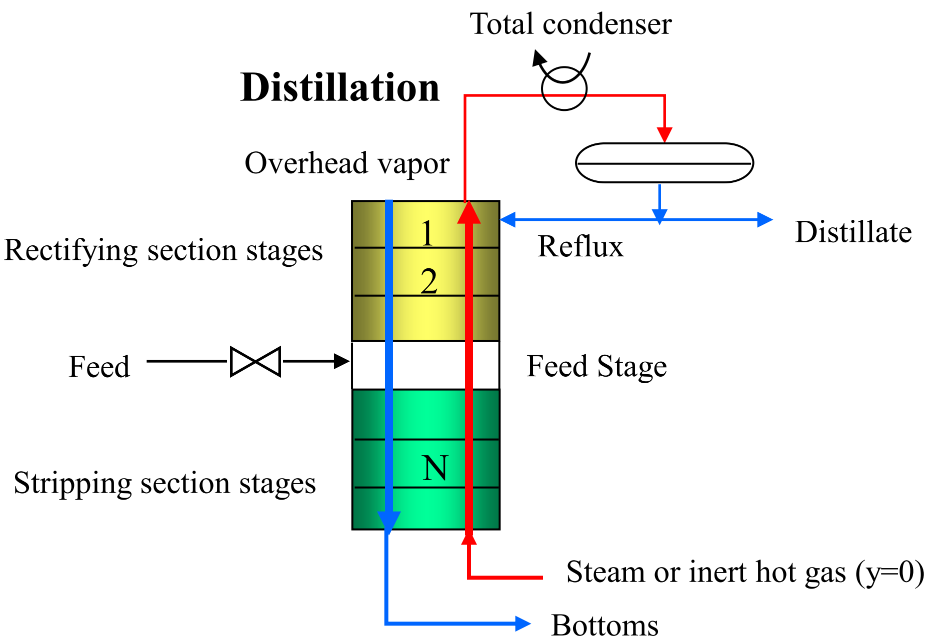




McCabe-Thiele Method: Open Steam

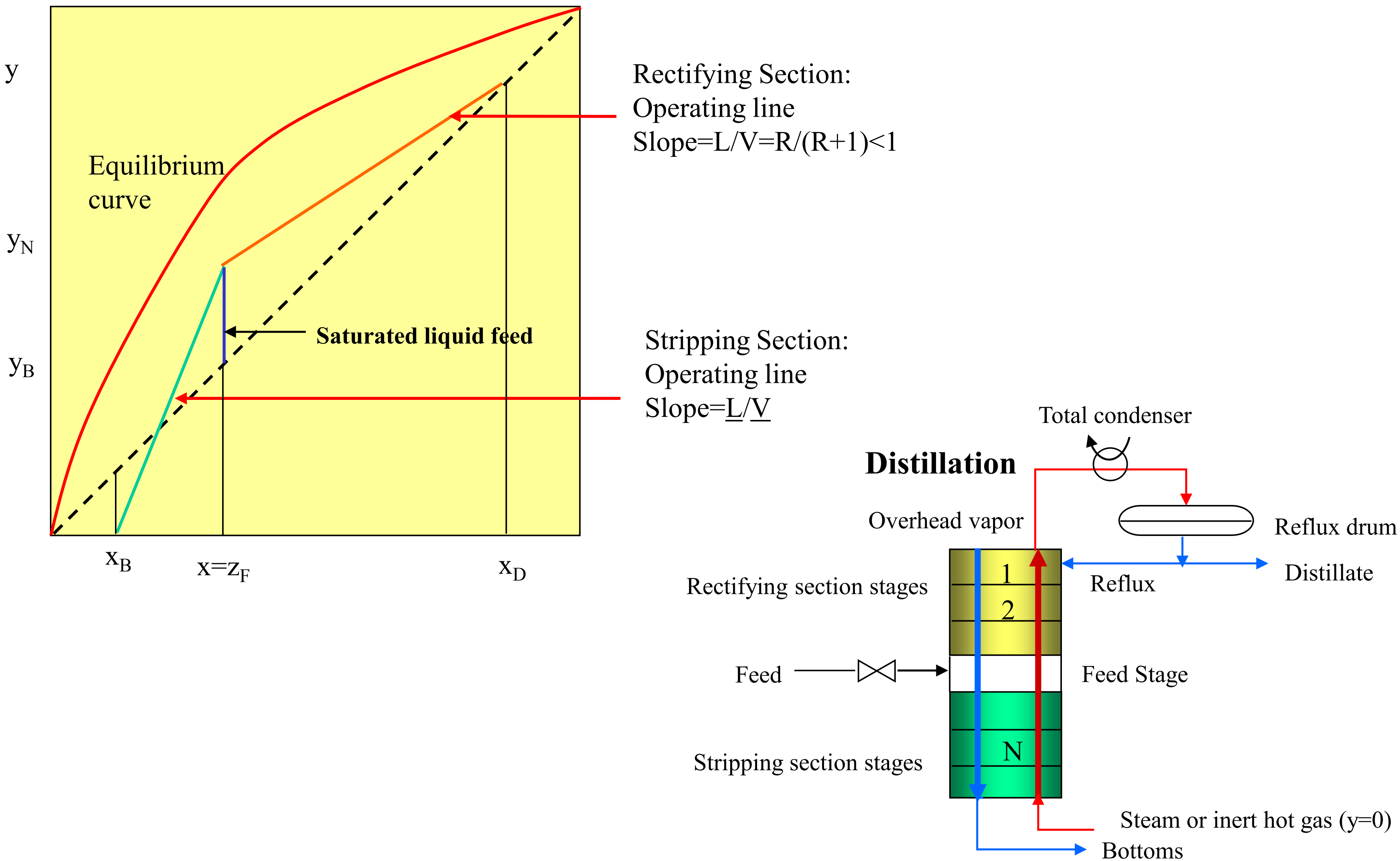
Consider the cascade shown on the left:

In this example, the reboiler is replaced by a source of hot steam or an inert gas. In this case, the vapor entering the bottom stage of the column has no light key and so y_B is zero, although x_B is non-zero.

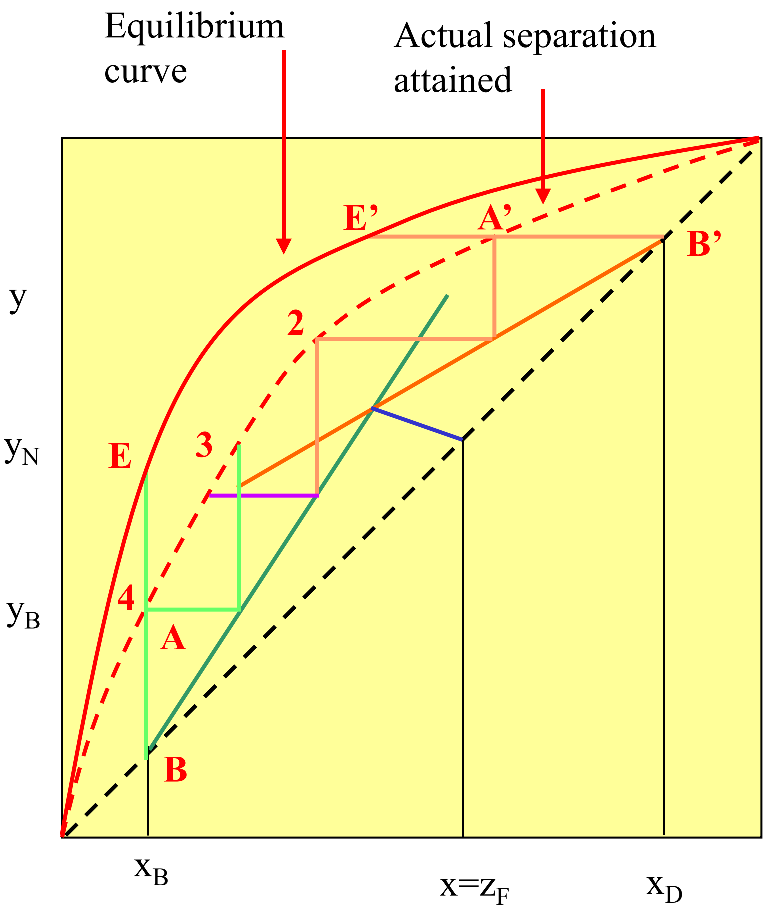


Does the slope of the rectifying section operating line increase or decrease?

McCabe-Thiele Method: Open Steam



Non-equilibrium McCabe-Thiele: Murphree Efficiency



Component distribution obtained less than theoretical limit described by equilibrium

The Murphree Plate Efficiency gives the ratio of the actual composition difference between two sequential plates, and that predicted by equilibrium.

For the vapor efficiency:

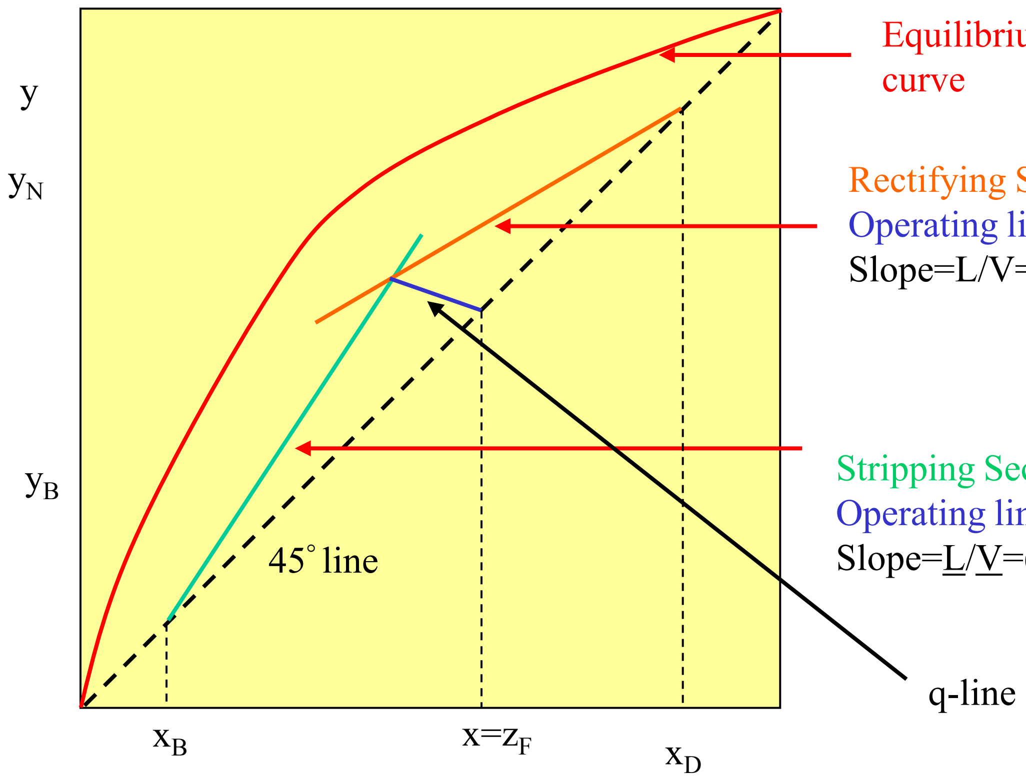
$$E_{MV} = \frac{y_n - y_{n+1}}{y_n^* - y_{n+1}} = \frac{\overline{AB}}{\overline{EB}}$$

For the liquid efficiency:

$$E_{ML} = \frac{x_n - x_{n+1}}{x_n^* - x_{n+1}} = \frac{\overline{A'B'}}{\overline{E'B'}}$$

McCabe-Thiele Algebraic Method

We have already developed the McCabe-Thiele **Graphical Method** for cascades. The same equations we used for the operating lines, q-line, and equilibrium curve can be used to solve for the compositions in each stage **algebraically**.



Equilibrium curve

$$y = \frac{\alpha x}{1 + x(\alpha - 1)}$$

Rectifying Section:
Operating line
Slope=L/V=R/(R+1)<1

$$y = \frac{R}{R+1}x + \frac{1}{R+1}x_D$$

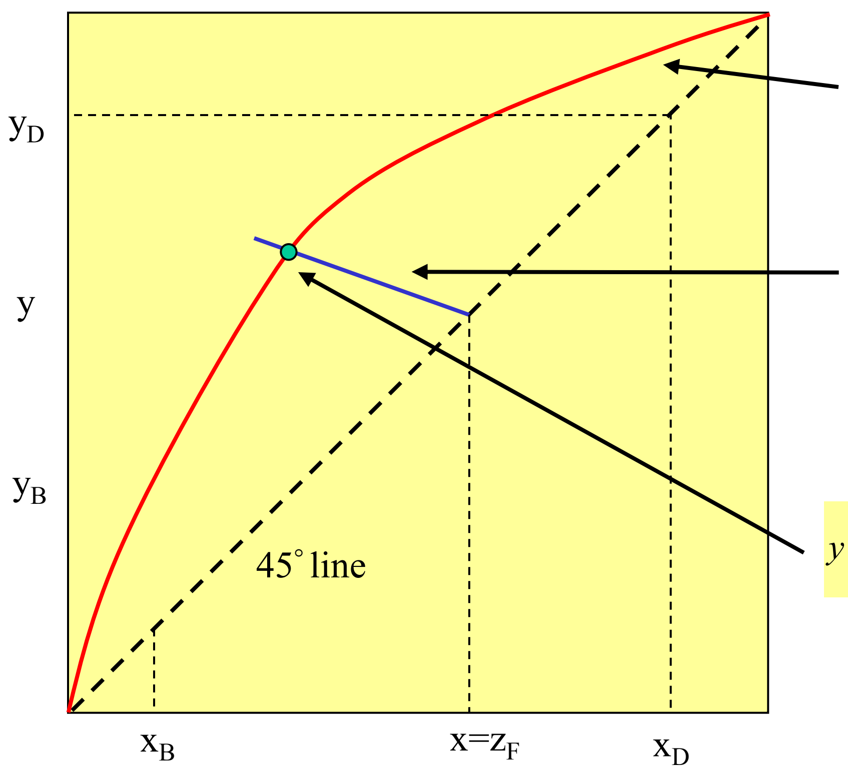
Stripping Section:
Operating line
Slope=$\bar{L}/\bar{V}$=$(V_B+1)/V_B$

$$y = \frac{\bar{L}}{\bar{V}}x - \frac{B}{\bar{V}}x_B$$

$$y = \left(\frac{q}{q-1}\right)x - \left(\frac{z_F}{q-1}\right)$$

McCabe-Thiele: Minimum Reflux

To carry out the algebraic method we need to determine the slopes of the operating lines algebraically. This can be done finding the intersections between the q-line and equilibrium curve, and the q-line and the rectifying section operating line.



$$y = \frac{\alpha x}{1 + x(\alpha - 1)}$$

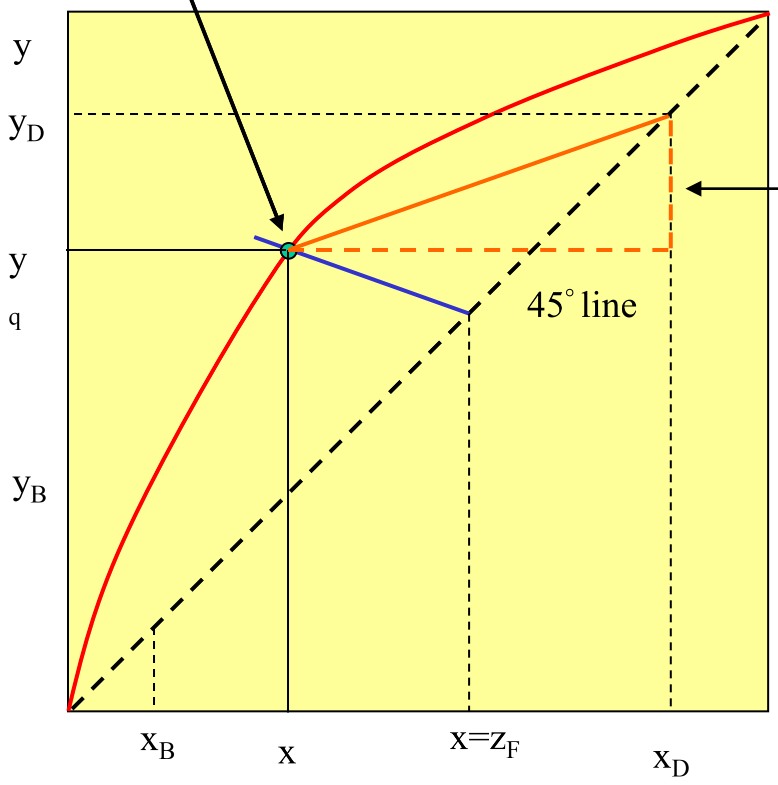
$$y = \left(\frac{q}{q-1}\right)x - \left(\frac{z_F}{q-1}\right)$$

$$y = \left(\frac{q}{q-1}\right)x - \left(\frac{z_F}{q-1}\right) = \frac{\alpha x}{1 + x(\alpha - 1)}$$

McCabe-Thiele: Rectifying Section Operating Line

The slope of the operating line for the rectifying section with minimum reflux can be determined from the rise over run. We can then also find the minimum reflux from this slope.

$$y = \left(\frac{q}{q-1}\right)x - \left(\frac{z_F}{q-1}\right) = \frac{\alpha x}{1 + x(\alpha - 1)}$$



$$\frac{y_D - y_q}{x_D - x_q} = \frac{R_{min}}{R_{min} + 1}$$

From the minimum reflux, and R/R_{min} we can determine the reflux R.

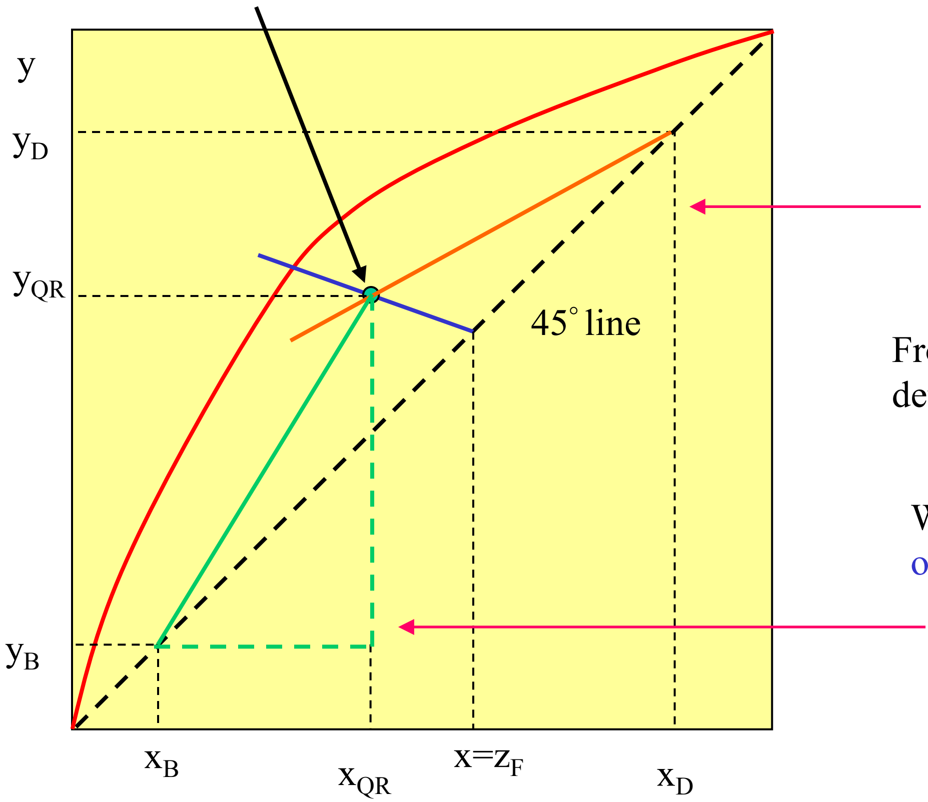
We determine the slope of the rectifying section operating line from:

$$slope = \frac{R}{R+1}$$

McCabe-Thiele: Rectifying Section Operating Line

We can find the intersection of the operating line and the q-line to determine the stripping section operating line:

$$y = \left(\frac{q}{q-1}\right)x - \left(\frac{z_F}{q-1}\right) = \frac{R}{R+1}x + \frac{1}{R+1}x_D$$



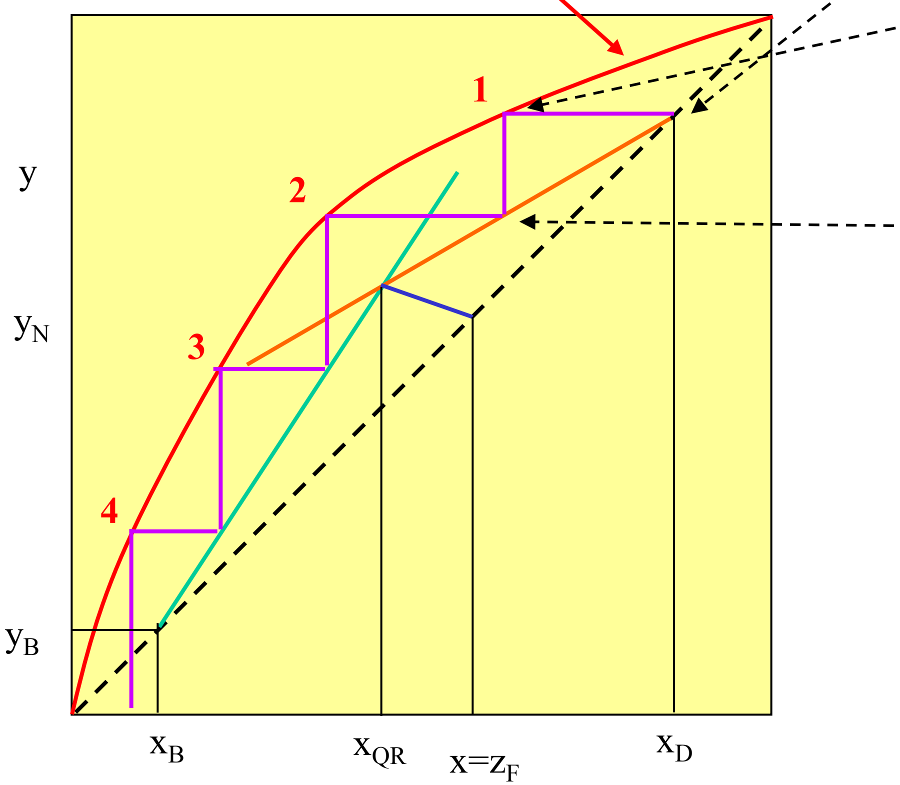
$$slope = \frac{R}{R+1}$$

From the minimum reflux, and R/R_{min} we can determine the reflux R.

We determine the slope of the stripping section operating line from:

$$\frac{y_{QR} - y_B}{x_{QR} - x_B} = slope$$

McCabe-Thiele: Algebraic Method



1. In total condenser $y_1=x_0$

2. x_1 is determined from the equilibrium curve:

$$y_1 = \frac{\alpha x_1}{1+x_1(\alpha-1)}$$

3. y_2 is determined from operating line for the rectifying section:

$$y_2 = \frac{R}{R+1}x_1 + \frac{1}{R+1}x_D$$

4. Repeat steps 2 and 3 until x_n is less than x_{QR} (you are on a point of the equilibrium curve to the left of the intersection of the OL and the q-line).

5. y_3 is determined from operating line for the stripping section:

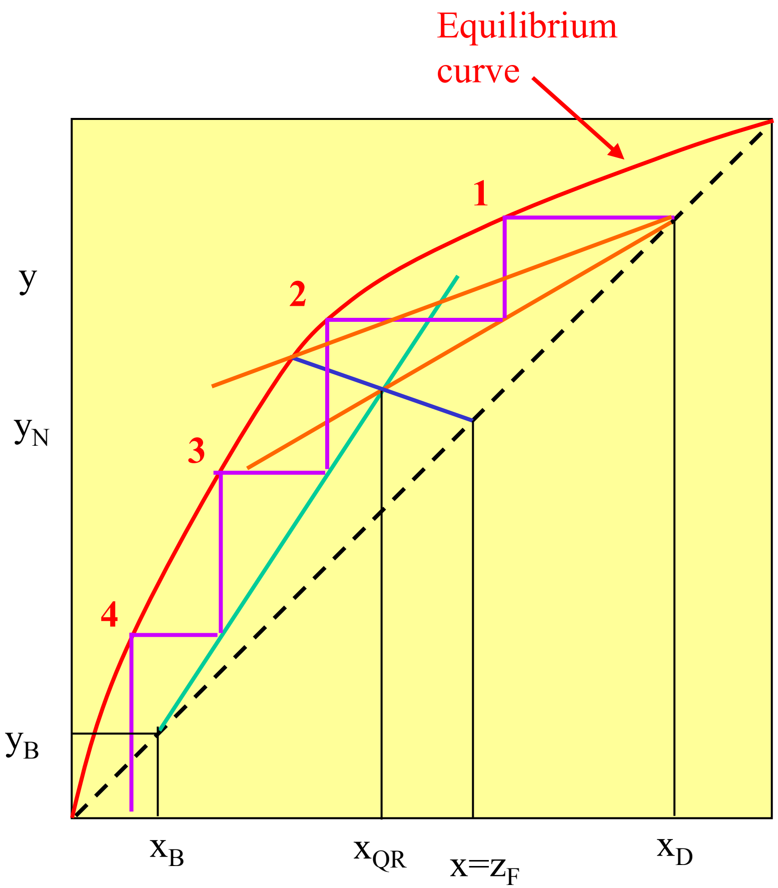
$$y_3 = \frac{\bar{L}}{\bar{V}}x_2 - \frac{B}{\bar{V}}x_B$$

6. x_3 is determined from the equilibrium curve:

$$y_3 = \frac{\alpha x_3}{1+x_3(\alpha-1)}$$

7. Repeat steps 5 and 6 until x_n is less than x_B

McCabe-Thiele Algebraic Method: Examples



x_D=0.9, x_B=0.1, z_F=0.5, q=0.8

1. Alpha = 4, $R=R_{min}$
2. Alpha=4 $R=2R_{min}$
3. Alpha=4 $R=4R_{min}$
4. Alpha=4 $R=20R_{min}$
5. Alpha=1.1 $R=3R_{min}$