Recovering Model Consistence for Force and Velocity Measures in Robot Hybrid Control

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Abstract

Hybrid Control for robots in contact with the environment is intended to separately impose end-effector motion and contact force along different directions, determined on the base of an available environment model. Since measured forces and velocities in practice do not agree exactly with the model, some filtering procedure is required for recovering consistence, before the control phase. In this paper some issues are discussed on the filtering process, addressed as the resolution of an algebraic overdetermined linear system, solvable approximately by means of weighted pseudoinversion. A basic issue is shown to be the choice of the weight matrix that, if properly defined, allows to overcome the theoretical difficulties recently pointed out by some authors. Examples clarify the developed theory.

1. Introduction

Classical Hybrid Control strictly applies only in the case of contact with environments so stiff to impose kinematic constraints on the robot hand [1, 2, 3]. At each robot configuration directions exist in which motion is allowed, and others in which forces can be applied. Independent control is obtained along these axes, that are orthogonal if friction effects are negligible. Recently in [4, 5], by means of a suitable model structure, the hybrid strategy has been extended also to cases where the end-effector is in contact with environments with own dynamics, so that orthogonality between end-effector motions and contact forces does not hold anymore.

Whatever the robot-environment interaction model available, desired positions, velocities and forces should be planned so to be compatible with it. On the other hand, measured quantities are never exactly consistent in practice, due to errors and approximations in modeling the real set-up and to noisy measurements. Raibert and Craig [2] first proposed to recover consistence filtering out components not matching the model, to get some robustness of the control law to modeling errors. The filtering procedure used consisted simply in discarding from cartesian velocity and force measures the components along axes orthogonal to the modeled ones, using 0/1 selection matrices. This simple technique has been recognized in [6] not to be rigorous when considering 6-dimensional velocity and force vectors, composed of linear and angular terms.

In this paper, a rigorous formulation of the filtering problem is attempted. Using the framework developed in [4, 5], briefly reported in sect. 2, the filtering process is formulated in sect. 3 as the problem to minimize the error in the solution of two overdetermined systems of linear equations, one for velocities and one for forces. This algebraic problem is approximately solved by means of a weighted pseudoinversion. The weight used can be regarded as a metric tensor that induces a norm and a distance on the vector spaces of admissible velocities and forces. Thus, the solutions of the problem are the velocity and the force vectors that are the closest as possible to the measured vectors, while satisfying the model. Some theoretical problems, first pointed out in [6], are discussed in sect. 4, concerning the lack of a canonical norm or metric definition for 6-dimensional velocities and forces, and the consequent possibility to obtain different results of the filtering procedure, due to artificial norm choices. Furthermore, it is shown that non-rigorous reasonings bring to state some non-invariant behavior of filters based on positive definite weights. In order to prevent non-invariance problems, the use of certain weighting matrices that preserve invariance (Klein and Killing forms), but not positive definite, was proposed in [7]. This approach is criticized here, and the drawbacks of the resulting filters are discussed. In sect. 5 a more careful geometrical analysis of the filtering problem reveals those misunderstandings that brought to observe non-invariance in filters based on positive definite weights. A weight structure is suggested in sect. 6, that yields filters with interesting features. Finally, the behavior of all filters examined is tested on a simple but significative example.

2. Robot-Environment Interaction

In this section, a robot-environment interaction model is briefly derived, based on the approach developed in [4]. Consider a n-link \((n \geq 6)\) rigid robot in contact with an environment that is a mechanical system with
Let $r$ be the position vector of $P$, $\mathbf{0R}_n$ the rotation matrix that gives the orientation of $\mathbf{nS}$ w.r.t. to $\mathbf{0S}$, and $\mathbf{v} = (\dot{\mathbf{r}}, \dot{\omega})$ the generalized velocity vector, composed of linear and angular terms. The end-effector kinematic description from the robot side can be given using the joint variables vector $\mathbf{q} \in \mathbb{R}^n$ as

$$\mathbf{r} = \mathbf{k}(\mathbf{q}), \quad \mathbf{0R}_n = \mathbf{G}(\mathbf{q}), \quad \mathbf{v} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}.$$  

Due to the type of contact, in addition to the environment dynamic variables $s_D \in \mathbb{R}^d$ other $s_K \in \mathbb{R}^k$ kinematic variables are needed to give the end-effector pose, if relative dof’s exist at the contact. Merging the environment variables in a vector $\mathbf{s} = (s_K, s_D) \in \mathbb{R}^e$, with $e = d + k \leq 6$, the end-effector kinematics can be expressed from the environment side as

$$\mathbf{r} = \mathbf{k}_E(\mathbf{s}), \quad \mathbf{0R}_n = \mathbf{G}_E(\mathbf{s}), \quad \mathbf{v} = \mathbf{T}(\mathbf{s})\dot{\mathbf{s}}.$$  

The environment is supposed to be such that matrix $\mathbf{T}$ is always full rank, at least in the region of interest for the task execution. Kinematic and dynamic velocity contributions can be separated partitioning $\mathbf{T}(\mathbf{s})$ in $[\mathbf{T}_K(\mathbf{s}) \mathbf{T}_D(\mathbf{s})]$. Admissible exchanged forces between end-effector and workpiece form a 6-dimensional vector $\mathbf{F} = (f, m)$ of forces and torques, and can be parametrized as $\mathbf{F} = \mathbf{Y}(\mathbf{s})\lambda$, similarly to velocities. Since friction at the contact is neglected, work cannot be performed on kinematic degrees of freedom, and orthogonality of all forces to kinematic motions is stated by requiring $\mathbf{Y}^T\mathbf{T}_K = 0$. Being $\mathbf{Y}$ full rank just as $\mathbf{T}_K$, so $\lambda \in \mathbb{R}^{(6-k)}$. At each $\mathbf{s}$, the space of admissible contact forces, $\text{span}[\mathbf{Y}(\mathbf{s})]$, can be decomposed into two parts. The subspace of static reaction forces, $\text{span}[\mathbf{Y}_R(\mathbf{s})]$, is defined by $\mathbf{T}^T\mathbf{Y}_R = 0$ (orthogonal to all admissible motions) and has dimension $6 - e$. The complement, $\text{span}[\mathbf{Y}_A(\mathbf{s})]$, is a $d$-dimensional space of active forces, responsible for energy exchange between robot and environment, and is such that $\mathbf{T}_D^T\mathbf{Y}_A$ is always nonsingular. In synthesis

$$\mathbf{v} = \mathbf{T}(\mathbf{s})\dot{\mathbf{s}} = \mathbf{T}_D(\mathbf{s})\dot{\mathbf{s}}_D + \mathbf{T}_K(\mathbf{s})\dot{\mathbf{s}}_K,$$  

$$\mathbf{F} = \mathbf{Y}(\mathbf{s})\lambda = \mathbf{Y}_A(\mathbf{s})\lambda_A + \mathbf{Y}_R(\mathbf{s})\lambda_R,$$  

$\mathbf{T}^T\mathbf{Y}_R = 0$, $\mathbf{Y}^T\mathbf{T}_K = 0$, $\mathbf{T}_D^T\mathbf{Y}_A$: nonsingular.  

The components of $\mathbf{s}$ and $\lambda$ have a physical meaning in terms of lengths or angles and, respectively, in terms of forces or torques. In particular, in the case of non-dynamic environments, $\dot{\mathbf{s}}_K$ and $\lambda_R$ may directly represent tangential velocities and normal forces with respect to the constraint. Therefore, the space of parameters $\mathbf{s}$ and $\lambda$ can be viewed as a generalization of the task-space concept, often introduced in hybrid control schemes, and the choice of these variables as the controlled output of the system is quite natural.

**Remark 1.** Usually, in technical literature, when environment dynamics are considered, relative dof’s at the contact are not modeled, so that the end-effector velocity is expressed by $\mathbf{v} = \mathbf{T}_D\dot{\mathbf{s}}_D$. In this case a common claim is that all reaction forces (often called squeeze forces) are in $\ker[\mathbf{T}_D^T]$, and active forces (or move forces) are in $\text{span}[\mathbf{T}_D]$ (see e.g. [9]). All forces are admissible in this case, since $\ker[\mathbf{T}_D^T] \oplus \text{span}[\mathbf{T}_D] = \mathbb{R}^k$, and the force decomposition in the two subspaces is unique. *This claim is misleading.* In fact, while it is true that the subspace of reaction forces is univocally determined by $\ker[\mathbf{T}_D^T]$ (and so $\text{span}[\mathbf{Y}_R] \equiv \ker[\mathbf{T}_D^T]$), the statement on active forces would lead to assume $\mathbf{Y}_A = \mathbf{T}_D$, whereas this is only one possible choice. Moreover, while being a feasible numerical choice (as required in (4) $\mathbf{T}_D^T\mathbf{Y}_A = \mathbf{T}_D^T\mathbf{T}_D$ is nonsingular) it is in general meaningless from a physical viewpoint. In fact, the columns of $\mathbf{T}_D$ give generalized directions for velocities, and, in principle, cannot be used as such to give directions for forces, at least for inconsistence in measure units.

**Example.** Consider the crank in fig. 1, whose rotation axis is parallel to $\mathbf{x}$ axis, and suppose the end-effector rigidly grasping the knob. If the knob is idle on the crank, the end-effector has two dof’s of motion, and its linear and angular velocities are independent. Instead, when the knob is fixed with the crank, the end-effector is constrained to move with only one dof, and linear and angular quantities are tightly coupled. Since this fact causes many interesting problems, the fixed knob example will be used in the following as a testbed. Given the definition of $\mathbf{s}$ as the angle shown in fig. 1 and being $\mathbf{d}$ the crank length, the end-effector velocity description can be written as

$$\mathbf{v} = \begin{bmatrix} 0 & -\mathbf{d} \sin \mathbf{s} & \mathbf{d} \cos \mathbf{s} & 1 & 0 & 0 \end{bmatrix}^T \dot{\mathbf{s}} = \mathbf{T} \dot{\mathbf{s}}.$$  

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If the crank dynamics is negligible, the variable $s$ can be considered kinematic ($s = s_K$) and active forces do not exist. The reaction forces can be expressed as

$$F_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -\sin s & \cos s & 0 & 0 \\ 0 & \cos s & \sin s & 0 & 0 \\ 0 & -d & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \lambda_R = Y_R(s)\lambda_R. \quad (6)$$

When the crank mass and inertia cannot be neglected, the variable $s$ is dynamic ($s = s_D$), and active forces $F_A = Y_A\lambda_A$ must be added, where the column vector $Y_A$ is such that $T_D Y_A \neq 0$. Different choices are possible. As stressed in Remark 1, most authors would propose $T_D = Y_A$, so that

$$F_A = \begin{bmatrix} 0 & -d\sin s & d\cos s & 1 & 0 & 0 \end{bmatrix}^T \lambda_A. \quad (7)$$

Our claim is that this choice is theoretically meaningless. In fact, no measure unit exists for $\lambda_A$ that renders $F_A$ really a force! Moreover, the product $T_D Y_A = 1 + d^2$, useful for expressing robot-environment energy exchange, is a sum of nonhomogeneous quantities. Two well-founded alternatives are

$$F_{A,1} = \begin{bmatrix} 0 & -\sin s & \cos s & 0 & 0 & 0 \end{bmatrix}^T \lambda_{A,1} \quad (8)$$
$$F_{A,2} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T \lambda_{A,2},$$

where $\lambda_{A,1}$ is a force and $\lambda_{A,2}$ is a torque.

3. Measure Filtering

In hybrid tasks, the trajectory specification consists of desired time histories for both the end-effector position and contact force, and must be planned so to satisfy the available robot-environment interaction model. On the other hand, measured quantities are never exactly in agreement with the model, due to errors and approximations in it. As a result, a basic issue in hybrid control is the recovery of measure-model consistency before feed back measures in the control system. Since usually in technical literature the knowledge of tangential and normal directions to the constraining environment at each robot configuration is assumed, more attention is devoted to the filtering of velocity and force measures.

Viewing a task as a succession of workpiece modifications, in our framework it is quite natural to take the environment variables $s$ and $\lambda$ as controlled output, rather than the robot variables. Thus, the trajectory is specified in terms of $s_{des}(t)$ and $\lambda_{des}(t)$, and measured values of $s$ and $\lambda$ are needed to implement an error driven controller (see [5]). In this setting, the classical problem of filtering velocity and force outputs is equivalent to find actual parameter values ($\hat{s}_m, \hat{\lambda}_m$) for given measures $v_m$ and $F_m$ (the problem of finding $s_m$, given the measured end-effector pose, is not addressed here). The subscript $m$ stands for variables computed from measures. We assume that at each configuration the description of the robot-environment interaction is given by means of matrices $T$ and $Y$, both full rank. The solution ($s_m, \lambda_m$) to the filtering problem is obtained by solving the two algebraic overdetermined linear systems

$$v_m = T \hat{s} \quad \text{and} \quad F_m = Y \hat{\lambda}. \quad (9)$$

In general, mismatches between model and real world and the presence of measure noise, forbid exact solution. The problem can be resolved approximately by minimizing the vector errors

$$e_v = v_m - T \hat{s} \quad \text{and} \quad e_F = F_m - Y \hat{\lambda}. \quad (10)$$

In order to perform minimization, some scalar error size must be defined. In earliest implementations of filters, simple pseudoinversion was used to solve problem (9), thus minimizing the Euclidean error norm

$$\|e\| \triangleq \sqrt{e^T e} = (\|e_v\|^2 + \|e_F\|^2)^{1/2}. \quad (11)$$

This solution is theoretically erroneous, since nonhomogeneous linear and angular components are added together, with the practical consequence that the result of pseudoinversion depends on the measure units chosen, and this is not desirable! This problem of noninvariance with respect to changes in measure units was pointed out by Lipkin and Duffy in [7].

The algebraic formulation of the minimization problem can be put in the form

$$\min_{\hat{s}} e_v^T W e_v \quad \min_{\hat{\lambda}} e_F^T \Psi e_F, \quad (12)$$

in which the weighting matrices $W$ and $\Psi$, assumed symmetric, must be such to avoid inconsistencies due to measure units. Moreover, they must be positive definite in order to provide unique solution to the problem in all situations. The filtering problem is solved by weighted pseudoinversion

$$\hat{s}_m = T_W^T v_m = (T_W^T W)^{-1} T_W^T W v_m, \quad (13a)$$
$$\hat{\lambda}_m = Y_\Psi^T F_m = (Y_\Psi^T \Psi Y)^{-1} Y_\Psi^T \Psi F_m. \quad (13b)$$

Operators $T_W$ and $Y_\Psi$ are the filters, and their features can be shaped by properly choosing the weights. Geometrical interpretations can be given to the result of pseudoinversion. The weights $W$ and $\Psi$ behave as metric tensors in the space of velocities and forces, and define the scalar product and the norm as

$$\langle v_1, v_2 \rangle \triangleq v_1^T W v_2, \quad \|v\| \triangleq (v^T W v)^{1/2},$$
$$\langle F_1, F_2 \rangle \triangleq F_1^T \Psi F_2, \quad \|F\| \triangleq (F^T \Psi F)^{1/2}. \quad (14)$$

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Observe that for consistence must be $\Psi = W^{-1}$.¹

The orthogonality definition — null scalar product—in the velocity space and in the force space is quite artificial. A natural orthogonality is intrinsically defined only between velocity and force vectors (the force space is dual to the velocity space), since it expresses the concept of null power flux supplied by a force along a given direction. In the pseudoinversion process only those components of $v_m$ and $F_m$ that are, respectively, in \(\text{span}[T]\) and in \(\text{span}[Y]\), are taken into account for the computation of parameters $s$ and $\lambda$. Measured components along orthogonal directions (according to the metric $W$) are filtered out.

**Remark 2.** The weight matrix $W$ plays no role in pseudoinversion if $v_m \in \text{span}[T]$ (and, dually, if $F_m \in \text{span}[Y]$). In this case there are no components to be cut, and the solution of the problem is exact. ■

**Remark 3.** The filters (13) are slightly different from those usually discussed in literature, where the input and output spaces are the same. In our setting, the outputs of (13a) and (13b) are in the environment parameter space (the space of variables $s$ and $\lambda$), as required by the control laws presented in [5]. ■

## 4. Non-Invariance w.r.t. Translations

Cartesian velocities and forces are computed, starting from measures on the robot arm, with respect to a given point of interest $P$ on the end-effector. Whenever a different terminal point is selected, matrices $T$ and $Y$ undergo a transformation. Lipkin and Duffy [7] first questioned the non-invariance of unweighted pseudoinversion w.r.t. translations of the terminal point. However, it is shown here that the same criticism can be moved to any filter based on a constant symmetric positive definite weight $W$.

Suppose to change the selection of the terminal point on the end-effector from $P$ to $P'$. Note that the angular velocity and the linear force are independent on this change. Let $b$ be the segment joining $P$ to $P'$. The vector product operator $b \times$ can be represented by a skew symmetric matrix $S$ that depends on the end-effector pose, and hence on $s$. The end-effector velocity and force at $P'$ are related to those at $P$ by

$$v_{P'} = Qv_P \quad \text{and} \quad F_{P'} = Q^{-T}F_P,$$

where

$$Q = \begin{bmatrix} I_3 & -S(s) \\ O_3 & I_3 \end{bmatrix} = Q(s).$$

The matrices $T$ and $Y$, defined w.r.t. $P$, modify as

$$\hat{T} = QT \quad \text{and} \quad \hat{Y} = Q^{-T}Y.$$  (17)

In the following only the non-invariance problem for velocity filters is presented. The case of forces is dual.

A filter like (13a) is considered invariant if, for any given $v_P$ and $v_{P'} = Qv_P$, one has

$$(T^TWT)^{-1}T^T Wv_P = (T^T W \hat{T})^{-1}T^T Wv_{P'}.$$  (18)

Observe first that when $v_P \in \text{span}[T]$ (and so $v_{P'} \in \text{span}[T]$) then (18) holds, since in this case the weight plays no role, as stressed in Remark 2. Instead, for vectors $v_P$ not satisfying the model, it is easy to verify that eq. (18) holds if and only if $W$ is such that

$$Q^T(s)WQ(s) = W.$$  (19)

In order to find conditions on $W$ that make (19) satisfied, rewrite this in a block partitioned form

$$\begin{bmatrix} I_3 & O_3 \\ S & I_3 \end{bmatrix} \begin{bmatrix} W_{lin} & W_{mix} \\ W_{mix}^T & W_{ang} \end{bmatrix} \begin{bmatrix} I_3 \\ -S \end{bmatrix} = W.$$  (20)

Then, the following conditions are found

$$W_{lin}S = O_3, \quad -W_{mix}^T S + S W_{mix} = O_3,$$  (21)

that must hold for any $S$ skew symmetric. The first of (21) requires $W_{lin} = 0$ and the second $W_{mix} = wI_3$ with $w$ scalar, while no condition is given on $W_{ang}$. It follows that no positive definite matrix $W$ can be found that satisfies (19).

Lipkin and Duffy in [7] proposed the use of two classic bilinear forms as weights in the filters: the Klein form, represented by

$$W_{Kl} = \begin{bmatrix} O_3 & I_3 \\ I_3 & O_3 \end{bmatrix},$$  (22)

and the Killing form, represented by

$$W_{K,v} = \begin{bmatrix} O_3 & O_3 \\ O_3 & I_3 \end{bmatrix} \quad \text{and} \quad W_{K,F} = \begin{bmatrix} I_3 & O_3 \\ O_3 & O_3 \end{bmatrix},$$  (23)

respectively for velocities and forces. Both the Klein and Killing forms, and every linear combination of them, satisfy conditions (21) (and dual conditions for forces). However, since the Klein form is non-definite and the Killing form is semi-definite, they do not define metric tensors, and their use in (13) is not allowed in many situations. In fact, inversion is not feasible when these forms, applied to admissible end-effector velocities or forces, give a zero result. In particular, note that filters based on the Killing form do not use measures

¹ Given a weight $W$ for velocity vectors, the matrix $W^{1/2}$ can be considered as a change of coordinates that allows the use of the Euclidean norm. The corresponding change of coordinates for forces, that preserves the force-velocity product, is $W^{-1/2}$ and the metric tensor for force vectors is $\Psi = W^{-1}$. 1279
of linear velocity and of force momentum. Then, besides wasting informations, the resulting filter can not be used in cases in which angular velocity or linear force are constrained to be zero.

The invariance problem can be overcome by using the algebraic expedient of employing a weight matrix that is function of the point choice on the end-effector. In fact, defining the weight at \( P \) as

\[
W_P = Q^{-T} W_P Q^{-1},
\]  

in which \( W_P \) is the weight at \( P \), equality (18) is always satisfied (condition (19), rewritten with \( W_P \) in the left and \( W_P \) in the right hand side, is also satisfied). This proposal was made in [8], though no theoretical justification was provided. In the next section, eq. (24) assumes a precise geometrical meaning.

5. Geometrical Insights

In this section it is shown that the non-invariance problem discussed above comes out from a non rigorous algebraic approach. Instead, a proper geometrical analysis makes clear the arisen misunderstandings. Again, only the case of velocity is considered.

The main issue is to recognize that generalized velocities \( v_P \) and \( v_{\hat{P}} \) at two distinct points \( P \) and \( \hat{P} \) on the end-effector are two representations of the same state of motion. The choice of a point on the end-effector defines a coordinate representation for the vector space of admissible velocities at a given pose. On the other hand, concepts like norm and metric in a vector space must be independent of the coordinate system. Hence, two vectors \( v_P \) and \( v_{\hat{P}} \), representing the same end-effector velocity, must have the same norm: \( \|v_P\| = \|v_{\hat{P}}\| \). Moreover, in general a metric tensor has different matricial representations in the two coordinate systems. Then, if \( W_P \) and \( \hat{W}_{\hat{P}} \) represent the same metric tensor in \( P \) and in \( \hat{P} \)-coordinates, they must yield

\[
\|v_P\| = (v_{\hat{P}}^T \hat{W}_{\hat{P}} v_{\hat{P}})^{\frac{1}{2}} = (v_{\hat{P}}^T \hat{W}_{\hat{P}} v_{\hat{P}})^{\frac{1}{2}} = \|v_{\hat{P}}\|.
\]  

Substituting \( v_P = Q^{-1} v_{\hat{P}} \) in (25), the coordinate transformation law for the metric tensor is found, obtaining exactly eq. (24), which receives now a precise geometrical interpretation.

It is now evident the misunderstanding that has led, in the previous section, to state non-invariance of any metric and norm definition. In fact, it is not correct to use the same matrix \( W \) for representing a metric tensor in two different coordinate systems, as it has been done in (18). This has brought to an apparent non-invariance and to an erroneous invariance condition (19). Weighting matrices that satisfy (19), like the Klein and Killing forms, are particular bilinear forms, whose matricial representation is invariant w.r.t. coordinate transformation (24).

6. A Proposal for the Weight Matrix

Due to the lack of a canonical metric definition for generalized velocity and force vectors, an arbitrary definition is unavoidable to formalize the minimization problem. This can be made by choosing, at a given point \( P \), a positive definite weighting matrix \( W \), which as well preserve consistence in measure units in the scalar product. A quite direct and interesting structure is the diagonal one

\[
W_\alpha = \begin{bmatrix} I_3 & O_3 \\ O_3 & \alpha^2 I_3 \end{bmatrix},
\]  

which defines the norm

\[
\|v\| = \left( \|v_{\text{lin}}\|^2 + \alpha^2 \|v_{\text{ang}}\|^2 \right)^{1/2}.
\]  

The factor \( \alpha \) has the dimension of a length, for making homogeneous the added terms in (27). Since weighted pseudoinversion minimizes the velocity error norm, \( \alpha \) represents the designer confidence attached to the angular matching relative to the translational one, when the velocity output is computed at \( P \). Computation of the force weight \( W_\alpha^{-1} \) is trivial.

Remark 4. When the environment is such that for a given choice of \( P \) linear and angular quantities can be parametrized independently, the behavior of the filter is independent on \( \alpha \). In fact, in such situations the two terms of the error norm (27) can be minimized separately using disjoint sets of parameters.

Remark 5. For \( \alpha \to \infty \) the weight (26) produces the same filtering of the Killing form, giving all the credit to angular velocity measures. In this case, if angular velocities are small or zero, the filtering procedure fails. As for forces, \( \alpha \to \infty \) gives all credit to the linear component.

A translation of \( P \) in \( \hat{P} \) changes of representation of metric tensor (26) in \( \hat{W}_\alpha \) as

\[
\begin{bmatrix} I_3 & O_3 \\ -S I & O \end{bmatrix} \begin{bmatrix} I_3 & O_3 \\ O & \alpha^2 I \end{bmatrix} \begin{bmatrix} I_3 & S \\ -S I & \alpha^2 I \end{bmatrix} = \begin{bmatrix} I_3 & S \\ -S I & \alpha^2 I \end{bmatrix}.
\]  

7. Applications to the Case Study

In this section the behavior of filters based on the different weighting matrices previously analyzed, is tested for the case of the crank with fixed knob.
First of all, the use of Klein form in this case gives

$$\mathbf{T}^T \mathbf{W}_{K;T} = 0 \quad \text{and} \quad \det(\mathbf{Y}_R^T \mathbf{W}_{K;Y_R}) = 0,$$

and so the filter structure (13) cannot be applied. This situation occurs when linear and angular components of end-effector velocity or force are related by vector product. In fact, since the Klein form applied to a vector $\mathbf{v} = (\mathbf{r}, \omega)$ gives the scalar product $2\mathbf{r} \cdot \omega$, if a vector product relation exists $\mathbf{r} = \omega \times \mathbf{d}$, the Klein form gives $2(\mathbf{r} \times \mathbf{d}) \cdot \omega = 0$. Simple computations show that a translation of the point $P$ on the end-effector leaves (29) unchanged. Invariance is preserved.

Using the Killing form, for velocity vectors, we have

$$\mathbf{T}^T \mathbf{W}_{K,v} = 1, \quad \mathbf{T}^T \mathbf{W}_{K,v} \mathbf{v}_m = \omega_{z,m},$$

and so the result of the filtering (13a) is $\delta_m = \omega_{z,m}$. Linear velocity measures are completely ignored, thus leaving unexploited this information that would increase robustness w.r.t. errors in measures. Applying the Killing form for filtering forces we have that $\mathbf{Y}_R^T \mathbf{W}_{K,F} \mathbf{Y}_R$ is singular, so that (13b) cannot be used. Again, translation of $P$ leaves unchanged the results.

Using now in (13a) the weight (26), we obtain

$$\delta_m = \frac{1}{(d^2 + \alpha^2)} \left[ d^2 \left( \frac{-\dot{r}_y, \sin \theta + \dot{r}_z, \cos \theta}{d} \right) + \alpha^2 \omega_{z,m} \right].$$

This is a convex linear combination of two angular terms, with coefficients $\rho = d^2/(d^2 + \alpha^2)$ and $\bar{\rho} = \alpha^2/(d^2 + \alpha^2) = 1 - \rho$. In the ideal case of exact model and noise-free measurements the two terms are equal, as easily checked, so that the result of (31) is independent on $\alpha$. The values $\alpha = 0$ and $\alpha = \infty$ assign maximal confidence, respectively, to linear and angular velocity measures, while $\alpha = d$ is suitable when the same credit is given to both them, yielding exactly the mean value. Note that, being absent in the model, linear velocity $\dot{r}_{x,m}$ and angular components $\omega_{y,m}$ and $\omega_{z,m}$ do not appear in (31). If present in the measures, these are filtered out as errors, independently of $\alpha$.

As for forces, in the case of negligible dynamics, using (13b) simple computations yield

$$\begin{align*}
\lambda_{R,1} &= f_{x,m}, \\
\lambda_{R,4} &= M_{y,m}, \\
\lambda_{R,3} &= f_{y,m} \cos s + f_{z,m} \sin s, \\
\lambda_{R,5} &= M_{z,m}, \\
\lambda_{R,2} &= (1 - \rho)(-f_{y,m} \sin s + f_{z,m} \cos s) + \rho \frac{M_{z,m}}{d}.
\end{align*}$$

Note that in $\lambda_{R,2}$, the only one dependent on $\alpha$ (through $\rho$), the two terms of the combination are equal in the case of error-free model and measures. Now for $\alpha = \infty$ ($\rho = 0$) the maximal confidence is given to linear force measures, i.e. the first term of the combination. If active forces are considered, then $\mathbf{Y} = [\mathbf{Y}_R \mathbf{Y}_A]$ becomes a nonsingular square matrix, and the weight plays no role in (13b) since pseudo inversion collapses into standard inversion. Translations of $P$, taken into account by properly modifying the weight matrix, do not affect the results of the filtering procedure.

8. Conclusions

Some issues on the problem of filtering force and velocity measures, in hybrid control, for recovering their consistence with the environment model, are discussed in this paper. By means of a suitable description of the robot-environment interaction, the filtering process is formulated as a minimization problem, solvable by means of weighted pseudo inversion. The selection of the weight matrix is shown to be a basic issue in the filter design. In fact, a suitable choice, along with a proper geometrical interpretation of its role, overcome the theoretical difficulties recently pointed out by some authors. Results are tested on a case study.

References


