Soft and rigid impact

Amabile Tatone

Dipartimento di Ingegneria delle Strutture, delle Acque e del Terreno
Università dell'Aquila - Italy

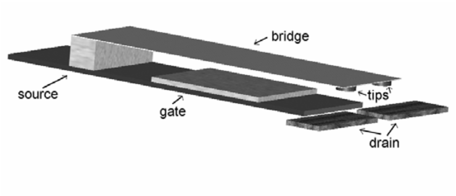
1st International Conference on Computational Contact Mechanics
Lecce, Italy, Sept. 16-18, 2009

Based on a joint work with:
Alessandro Contento and Angelo Di Egidio

Dipartimento di Ingegneria delle Strutture, delle Acque e del Terreno
Università dell'Aquila - Italy

References

- Alessandro Granaldi, Paolo Decuzzi, The dynamic response of resistive microswitches: switching time and bouncing, *J. Micromech. Microeng.*, 16, 2006.
- Z. J. Guo, N. E. McGruer, G. G. Adams, Modeling, simulation and measurement of the dynamic performance of an ohmic contact, electrostatically actuated RF MEMS switch, *J. Micromech. Microeng.*, 17, 2007



A toy model for contact simulations

Contact between a body and a rigid flat support

- rigid body
- affine body (homogeneous deformations)
- contractile affine body

Contact force constitutive laws

Repulsive force:

$$\mathbf{q}_r(\mathbf{x}, t) = \alpha_r \, d(\mathbf{x}, t)^{-\nu_r} \, \mathbf{n}$$

Damping force:

$$\mathbf{q}_d(\mathbf{x}, t) = -\beta_d \, d(\mathbf{x}, t)^{-\nu_d} \, (\mathbf{n} \otimes \mathbf{n}) \, \dot{\mathbf{p}}(\mathbf{x}, t)$$

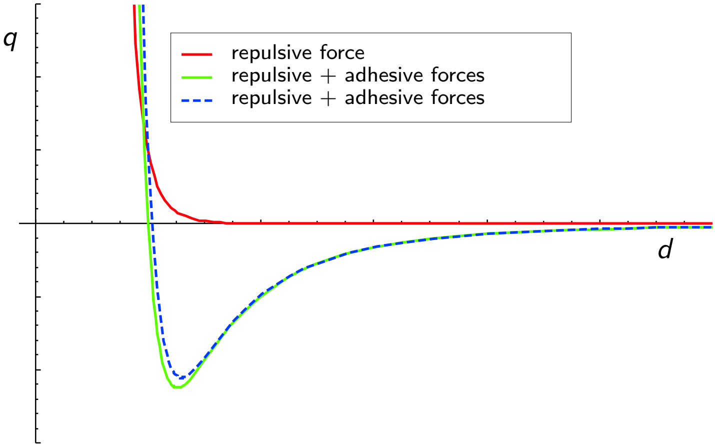
Frictional force:

$$\mathbf{q}_f(\mathbf{x}, t) = -\beta_f \, d(\mathbf{x}, t)^{-\nu_f} \, (\mathbf{I} - \mathbf{n} \otimes \mathbf{n}) \, \dot{\mathbf{p}}(\mathbf{x}, t)$$

Adhesive force:

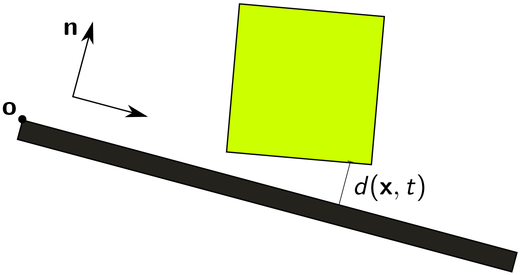
$$\mathbf{q}_a(\mathbf{x}, t) = -\beta_a \, (d(\mathbf{x}, t)^{-\nu_{aa}} - d(\mathbf{x}, t)^{-\nu_{ar}}) \, \mathbf{n}$$

Contact force constitutive laws



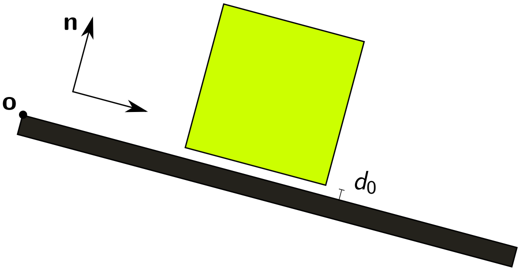
$$\nu_r = 8, \quad \nu_{aa} = 3, \quad \nu_{ar} = 6$$

Contact force constitutive laws



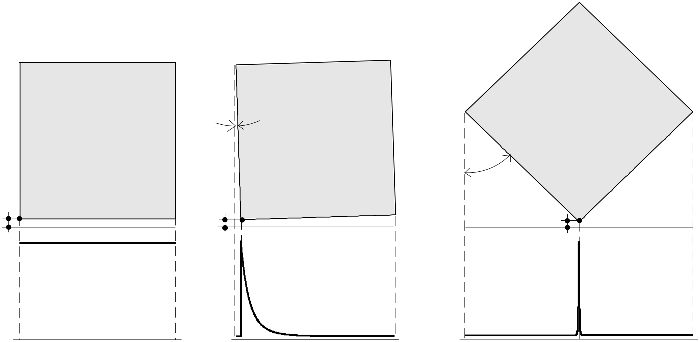
$$d(\mathbf{x}, t) := (\mathbf{p}(\mathbf{x}, t) - \mathbf{o}) \cdot \mathbf{n}$$

Contact force constitutive laws

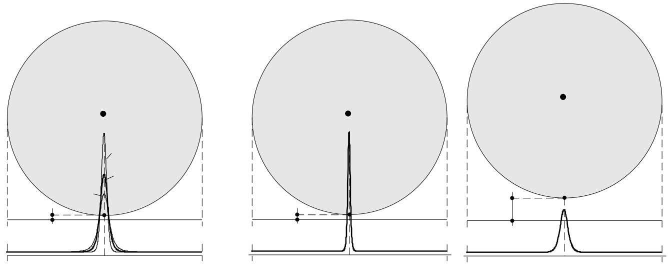


$$d(\mathbf{x}, t) := (\mathbf{p}(\mathbf{x}, t) - \mathbf{o}) \cdot \mathbf{n}$$

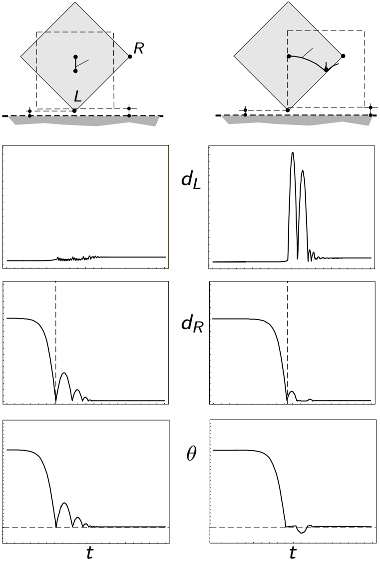
Rigid block



Rigid disk



Numerical simulations (rigid body)



001	011
002	012

Numerical simulations (rigid body)

rocking on a sloping plane

021 022 023

bouncing

031

rolling

032 033 034 035

adhesion and detachment

501 502 503 505

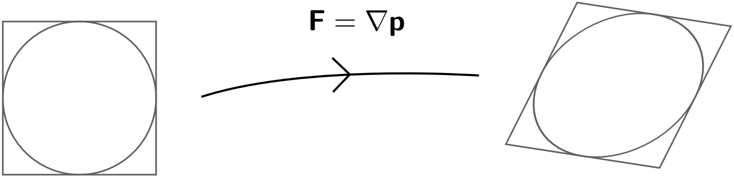
spinning top

3D-101 3D-111
3D-102 3D-112

dice throwing

3D-201 3D-211
3D-202 3D-212

Affine body



Affine body

The motion of a body $\mathcal{B}$ is described at each time t by a *transplacement* $\mathbf{p}(\cdot, t)$ defined on the *reference shape* $\mathcal{D}$:

$$\mathbf{p} : \mathcal{D} \times \mathcal{I} \rightarrow \mathcal{E}$$

characterized by the following representation:

$$\mathbf{p}(\mathbf{x}, t) = \mathbf{p}_0(t) + \nabla\mathbf{p}(t)(\mathbf{x} - \mathbf{x}_0)$$

where $\nabla\mathbf{p}(t) : \mathcal{V} \rightarrow \mathcal{V}$ is a tensor such that $\det \nabla\mathbf{p}(t) > 0$.
An affine velocity field $\mathbf{v}$ at time t has the representation:

$$\mathbf{v}(\mathbf{x}) = \mathbf{v}_0 + \nabla\mathbf{v}(\mathbf{x} - \mathbf{x}_0)$$

Along a motion at time t

$$\mathbf{v}_0 = \dot{\mathbf{p}}_0(t), \ \nabla\mathbf{v} = \nabla\dot{\mathbf{p}}(t)$$

Affine body

Balance principle:

$$\int_{\mathcal{D}} \mathbf{b}(\mathbf{x},t)\cdot\mathbf{v}\,dV + \int_{\partial\mathcal{D}} \mathbf{q}(\mathbf{x},t)\cdot\mathbf{v}\,dA - \mathbf{S}(t)\cdot\nabla\mathbf{v}\,\mathrm{vol}(\mathcal{D}) = 0\,, \quad \forall\mathbf{v}$$

Balance equations:

$$-m\,\ddot{\mathbf{p}}_0(t) - m\,\mathbf{g} + \mathbf{f}(t) = 0$$

$$-\nabla\ddot{\mathbf{p}}(t)\,\mathbf{J} + \mathbf{M}(t) - \mathbf{S}(t)\,\mathrm{vol}(\mathcal{D}) = 0$$

Mass and Euler tensor:

$$m := \int_{\mathcal{D}} \rho \, dV$$

$$\mathbf{J} := \int_{\mathcal{D}} \rho(\mathbf{x} - \mathbf{x}_0) \otimes (\mathbf{x} - \mathbf{x}_0) \, dV$$

Total force and moment tensor:

$$\mathbf{f}(t) := \int_{\partial\mathcal{D}} \mathbf{q}(\mathbf{x}, t) \, dA$$

$$\mathbf{M}(t) := \int_{\partial\mathcal{D}} (\mathbf{x} - \mathbf{x}_0) \otimes \mathbf{q}(\mathbf{x}, t) dA$$

Material constitutive characterization

Frame indifference:

$$\mathbf{S}\cdot\mathbf{W}\,\mathbf{F}=0\quad\forall\mathbf{W}\mid\operatorname{sym}\mathbf{W}=0\quad\Rightarrow\quad\operatorname{skw}\mathbf{S}\mathbf{F}^{\mathsf{T}}=0$$

Dissipation inequality:

$$\mathbf{S}\cdot\dot{\mathbf{F}}-\frac{d}{dt}\varphi(\mathbf{F})\geq 0$$

Reduced dissipation inequality:

$$\mathbf{S}^{+}\mathbf{F}^{\mathsf{T}}\cdot\dot{\mathbf{F}}\mathbf{F}^{-1}\geq 0$$

$$\mathbf{S}^{+}:=\mathbf{S}-\widehat{\mathbf{S}}(\mathbf{F})$$

Material constitutive characterization

Hyperelastic stress:

$$\widehat{\mathbf{S}}(\mathbf{F}) \cdot \dot{\mathbf{F}} = \frac{d\varphi(\mathbf{F})}{dt}$$

Mooney-Rivlin strain energy (incompressible material):

$$\varphi(\mathbf{F}) := c_1(\imath_1(\mathbf{C}) - 3) + c_2(\imath_2(\mathbf{C}) - 3)\,.$$

$$\imath_1(\mathbf{C}) := \operatorname{tr}(\mathbf{C}), \quad \imath_2(\mathbf{C}) := \frac{1}{2}\big(\operatorname{tr}(\mathbf{C})^2 - \operatorname{tr}(\mathbf{C}^2)\big)\,.$$

Reduced dissipation inequality:

$$\mathbf{S}^{+}\mathbf{F}^{\mathsf{T}} \cdot \dot{\mathbf{F}}\mathbf{F}^{-1} \geq 0$$

$$\mathbf{S}^{+} := \mathbf{S} - \widehat{\mathbf{S}}(\mathbf{F})$$

The simplest way to satisfy *a-priori* the dissipation inequality:

$$\mathbf{S}^{+}\mathbf{F}^{\mathsf{T}} = \mu \,\mathrm{sym}\,(\dot{\mathbf{F}}\mathbf{F}^{-1})\,, \quad \mu \geq 0$$

Stress response (dissipative + energetic + reactive):

$$\mathbf{S} = \mu \,\mathrm{sym}\,(\dot{\mathbf{F}}\mathbf{F}^{-1})(\mathbf{F}^{\mathsf{T}})^{-1} + \widehat{\mathbf{S}}_0(\mathbf{F}) - \pi\,(\mathbf{F}^{\mathsf{T}})^{-1}$$

Contact forces

Surface forces per unit deformed area:

$$\mathbf{q}(\mathbf{x}, t) = \sum_j \mathbf{q}_j(\mathbf{x}, t)\ k(\mathbf{x}, t)$$

Area change factor:

$$k(\mathbf{x}, t) := \|\nabla \mathbf{p}(t)^{-T} \mathbf{n}_{\partial\mathcal{D}}(\mathbf{x})\| \det \nabla \mathbf{p}(t)$$

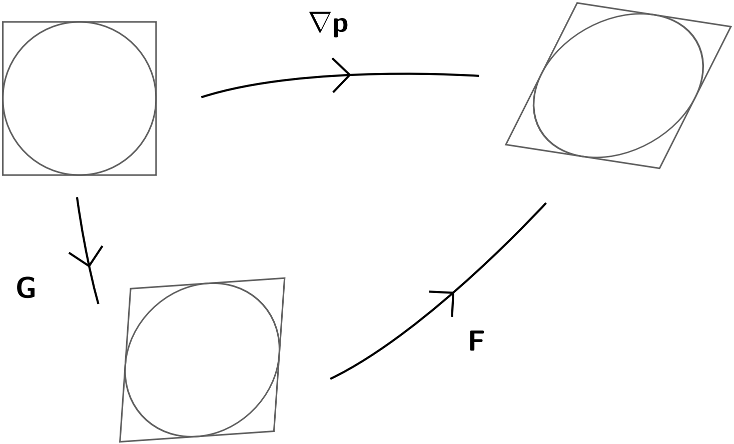
$\mathbf{n}_{\partial\mathcal{D}}(\mathbf{x})$ outward unit normal vector

Numerical simulations (elastic body)

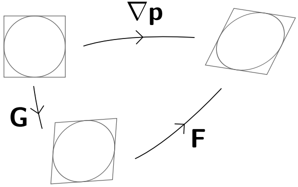
elastic bouncing, rolling and oscillations

041	112			
200	214	215	216	217
318	319			

Affine contractile body



Affine contractile body



Kröner-Lee decomposition:

$$\mathbf{F}(t) := \nabla\mathbf{p}(t)\,\mathbf{G}(t)^{-1}$$

Contraction velocity:

$$\mathbf{V} = \dot{\mathbf{G}}\mathbf{G}^{-1}$$

Affine contractile body

Balance principle:

$$\int_{\mathcal{D}} \mathbf{b}(\mathbf{x}, t) \cdot \mathbf{v}\, dV + \int_{\partial\mathcal{D}} \mathbf{q}(\mathbf{x}, t) \cdot \mathbf{v}\, dA - \mathbf{S}(t) \cdot \nabla\mathbf{v}\, \mathrm{vol}(\mathcal{D})$$
$$+\big(\mathbf{Q}(t) \cdot \mathbf{V} - \mathbf{A}(t) \cdot \mathbf{V}\big)\, \mathrm{vol}(\mathcal{D}) = 0\,, \quad \forall(\mathbf{v}, \mathbf{V})$$

Balance equations:

$$-m\, \ddot{\mathbf{p}}_0(t) - m\, \mathbf{g} + \mathbf{f}(t) = 0$$

$$-\nabla\ddot{\mathbf{p}}(t)\, \mathbf{J} + \mathbf{M}(t) - \mathbf{S}(t)\, \mathrm{vol}(\mathcal{D}) = 0$$

$$\mathbf{Q}(t) - \mathbf{A}(t) = 0$$

Material constitutive characterization

Frame indifference:

$$\mathbf{S} \cdot \mathbf{W}\,\nabla\mathbf{p} = 0 \quad \forall\mathbf{W} \mid \operatorname{sym}\mathbf{W} = 0 \quad \Rightarrow \quad \operatorname{skw}\mathbf{S}\nabla\mathbf{p}^{\mathsf{T}} = 0$$

Dissipation inequality:

$$\mathbf{A} \cdot \dot{\mathbf{G}}\mathbf{G}^{-1} + \mathbf{S} \cdot \nabla\dot{\mathbf{p}} - \frac{d}{dt}\big(\varphi(\mathbf{F})\det\mathbf{G}\big) \geq 0$$

Material constitutive characterization

Reduced dissipation inequality:

$$\mathbf{S}^{+}\nabla\mathbf{p}^{\mathsf{T}} \cdot \dot{\mathbf{F}}\mathbf{F}^{-1} + \mathbf{A}^{+} \cdot \dot{\mathbf{G}}\mathbf{G}^{-1} \geq 0$$

$$\mathbf{S}^{+} := \mathbf{S} - \widehat{\mathbf{S}}(\mathbf{F})\,, \quad \mathbf{A}^{+} := \mathbf{A} + \mathbf{F}^{\mathsf{T}}\mathbf{S}\mathbf{G}^{\mathsf{T}} - (\det\mathbf{G})\varphi(\mathbf{F})\mathbf{I}$$

Hyperelastic stress:

$$\widehat{\mathbf{S}}(\mathbf{F})\mathbf{G}^{\mathsf{T}} \cdot \dot{\mathbf{F}} = \frac{d\varphi(\mathbf{F})}{dt}$$

Material constitutive characterization

The simplest way to satisfy *a-priori* the dissipation inequality:

$$\mathbf{S}^{+}\nabla\mathbf{p}^{\mathsf{T}} = \mu \ \mathrm{sym}\,(\dot{\mathbf{F}}\mathbf{F}^{-1})\,, \quad \mu \geq 0$$
$$\mathbf{A}^{+} = \mu_\gamma\,\dot{\mathbf{G}}\mathbf{G}^{-1}\,, \quad \mu_\gamma \geq 0$$

Stress characterization:

$$\mathbf{S} = \mu \ \mathrm{sym}\,(\dot{\mathbf{F}}\mathbf{F}^{-1})(\nabla\mathbf{p}^{\mathsf{T}})^{-1} + \widehat{\mathbf{S}}_0(\mathbf{F}) - \pi\,(\nabla\mathbf{p}^{\mathsf{T}})^{-1}$$
$$\mathbf{A} = \mu_\gamma\,\dot{\mathbf{G}}\mathbf{G}^{-1} - \big(\mathbf{F}^{\mathsf{T}}\mathbf{S}\mathbf{G}^{\mathsf{T}} - (\det\mathbf{G})\varphi(\mathbf{F})\mathbf{I}\big)$$

Material constitutive characterization

Equations of motion:

$$-m\,\ddot{\mathbf{p}}_0 - m\,\mathbf{g} + \mathbf{f} = 0$$

$$-\nabla\ddot{\mathbf{p}}\,\mathbf{J} + \mathbf{M} - \mathbf{S}\,\mathrm{vol}(\mathcal{D}) = 0$$

$$\mu_\gamma\,\dot{\mathbf{G}}\mathbf{G}^{-1} = \mathbf{F}^\mathsf{T}\mathbf{S}\mathbf{G}^\mathsf{T} - (\det\mathbf{G})\,\varphi(\mathbf{F})\mathbf{I} + \mathbf{Q}$$

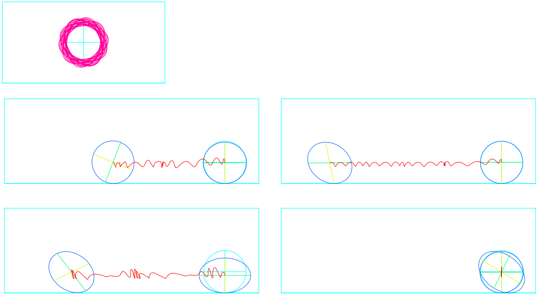
Numerical simulations (contractile body)

oscillating driving Q

12g1 12g2 12g3

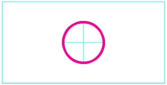
Numerical simulations (contractile body)

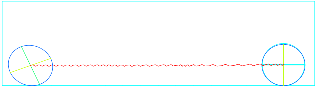
oscillating driving G

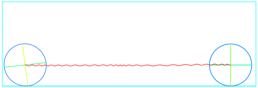


Numerical simulations (contractile body)

oscillating driving G







References

- Simo J.C., Wriggers P. and Taylor R.L., "A perturbed Lagrangian formulation for the finite element solution of contact problems," *Comp. Methods Appl. Mech. Engrg.*, **51**, 163–180 (1985).
- Wriggers-Za Wriggers P. and Zavarise G., "Chapter 6, Computational Contact Mechanics," in *Encyclopedia of Computational Mechanics*, Stein E., de Borst R., Hughes T.J.R., editors, John Wiley & Sons (2004).

References

- Di Carlo A. and Quiligotti S., “Growth and Balance,” *Mech. Res. Comm.*, **29**, 449–456 (2002).
- Di Carlo A., “Surface and bulk growth unified,” in *Mechanics of Material Forces*, Steinmann P. and Maugin G. A., editors, Springer, New York, 53–64 (2005).
- Nardinocchi P. and Teresi L., “On the active response of soft living tissues,” *J. Elasticity*, **88**, 27–39 (2007).

Supplementary references

- Gianfranco Capriz, Paolo Podio-Guidugli, Whence the boundary conditions in modern continuum physics?, *Atti Convegni Lincei n. 210*, 2004
- Antonio Di Carlo, Actual surfaces versus virtual cuts, *Atti Convegni Lincei n. 210*, 2004