LIVING SHELL-LIKE STRUCTURES

adc

Joint work with

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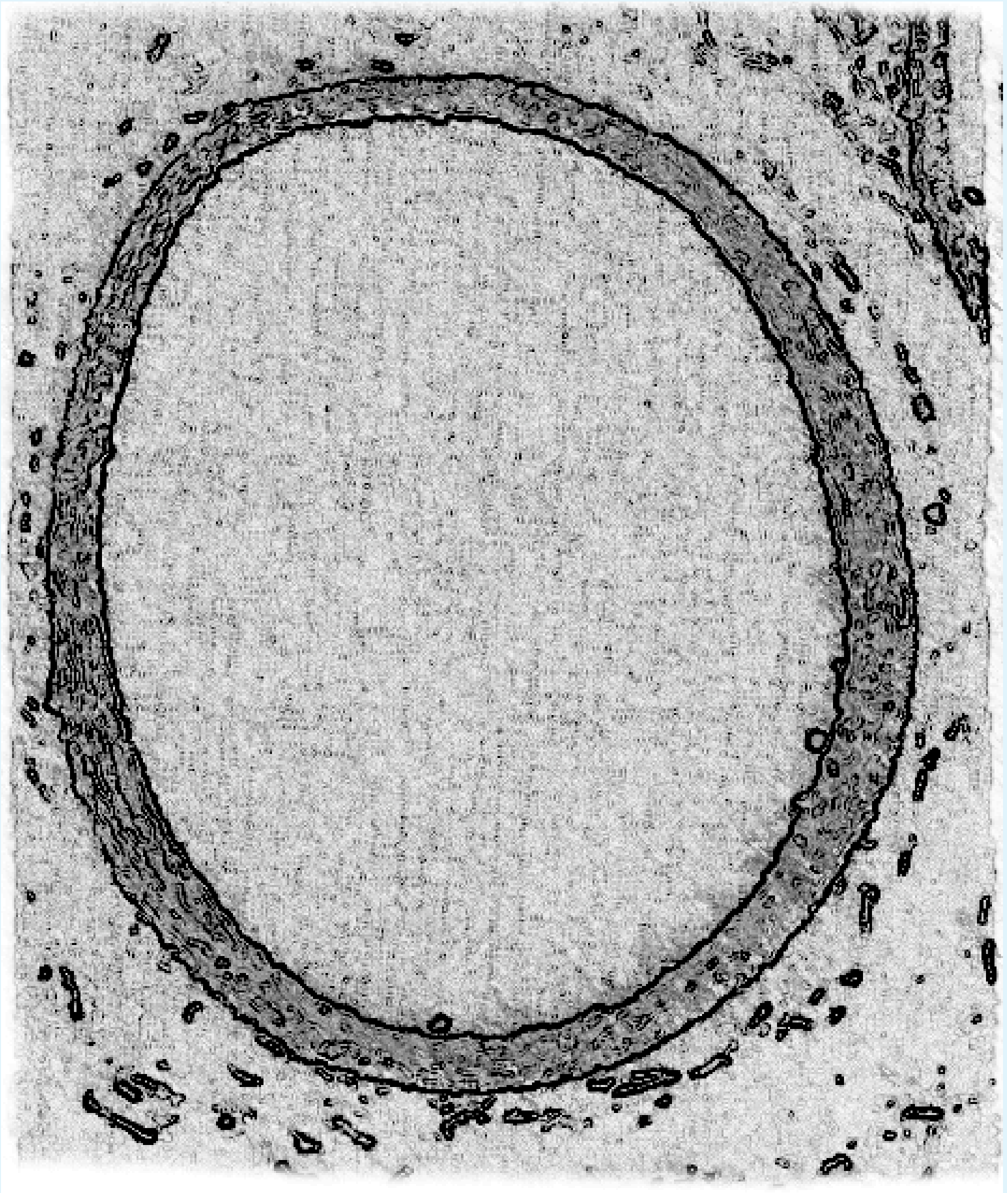
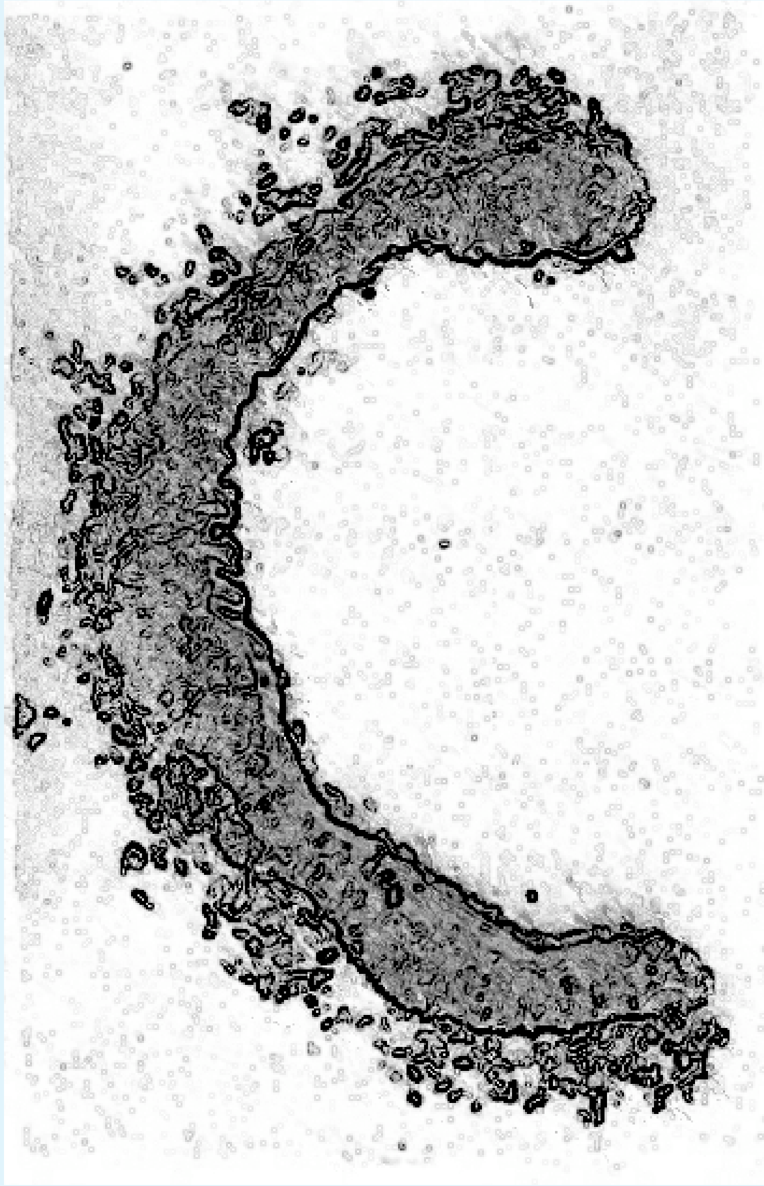
Soft shell-like structures are ubiquitous:

- **organelles & cell membranes**
- **lymph and blood vessels**
- **alimentary canal & respiratory ducts**
- **urinary tract**
- **uterus**

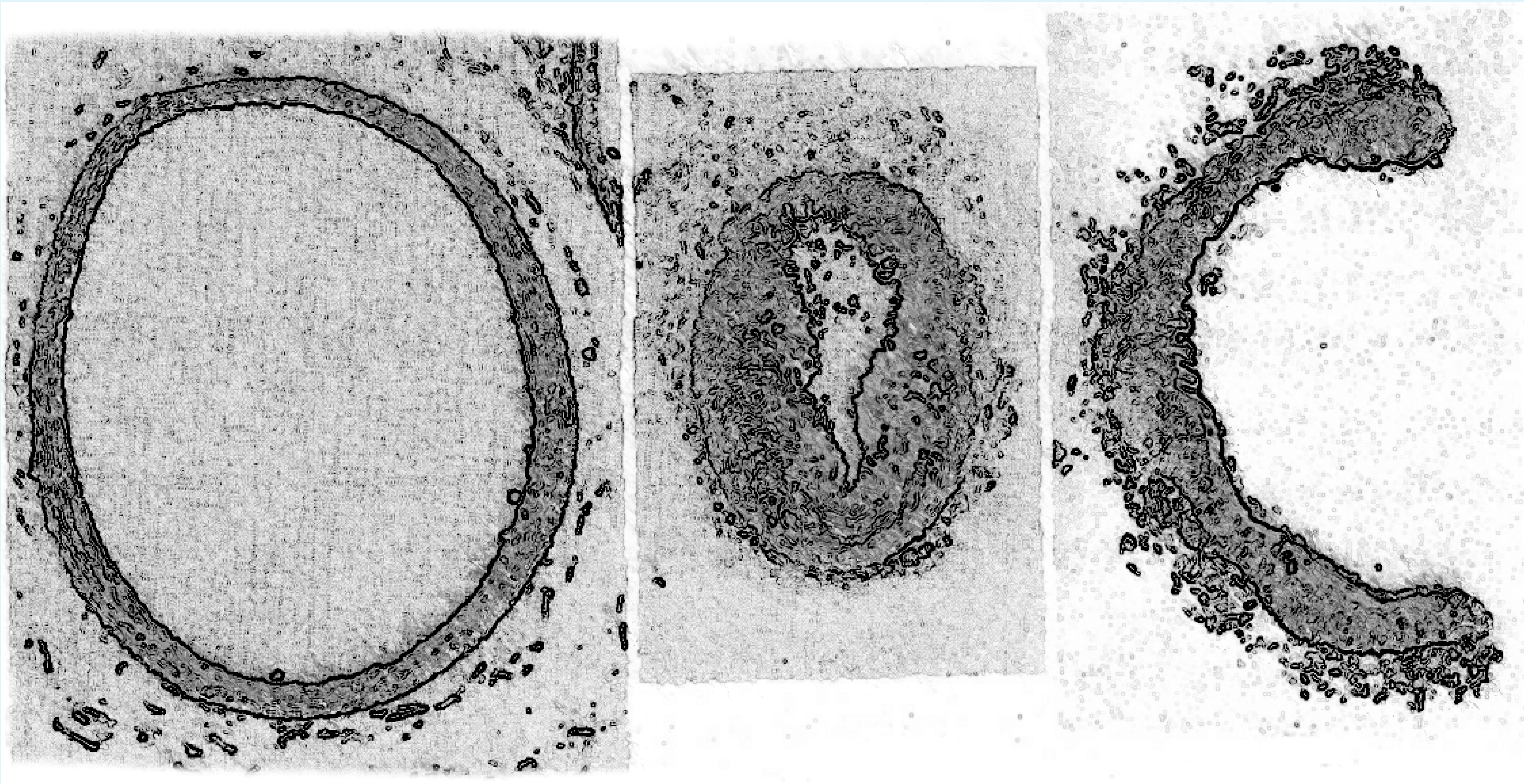
The mechanical response of all of these structures—a key feature of their physiological and pathological functioning—is subtle and elusive.



4-a



4-b



4-c

Another critical issue is their ability to grow and remodel, in a way which is both biochemically controlled and strongly coupled with the prevailing mechanical conditions.

While the characterization of the mechanical response of soft tissue is progressing at a reasonably fast pace nowadays, we find that growth mechanics is definitely the weakest link in the modelling chain.

- S. Socrate, A.P. Paskaleva, K.M. Myers, M. House. Connection between uterine contractions and cervical dilation: a biomechanical theory of cervical deformation. 1st International Conference on Mechanics of Biomaterials & Tissues, Waikoloa, HI, December 11-15, 2005.

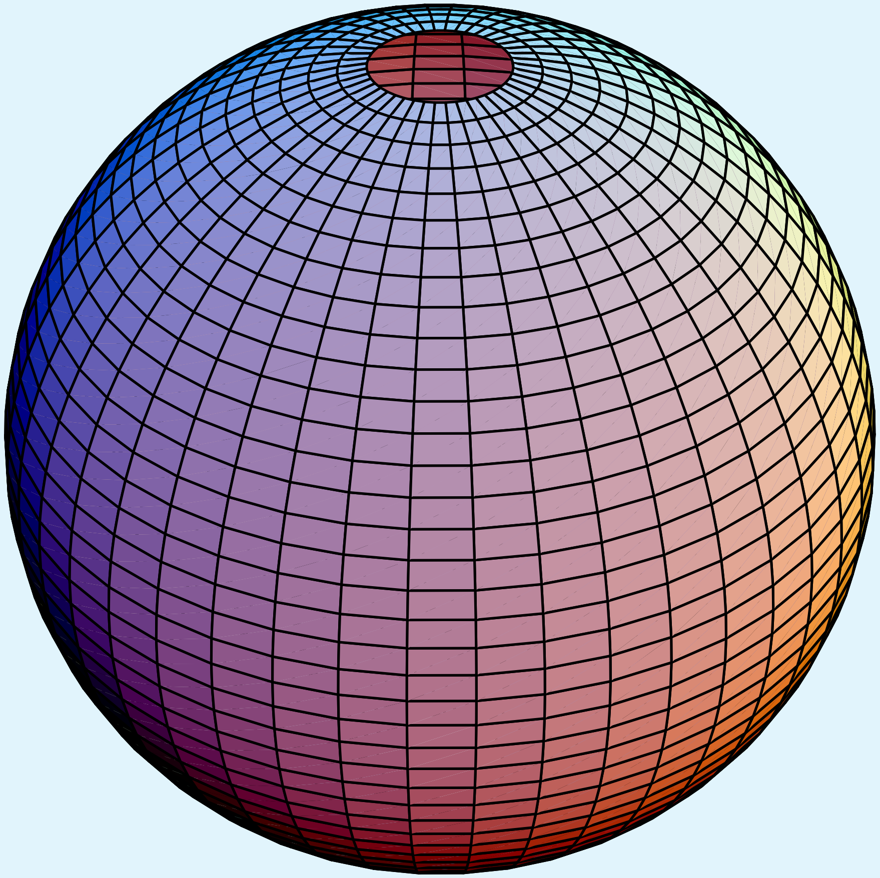
Our focus is on the two-way coupling between growth and stress, which we model within a theory in which bulk growth is governed by a novel balance law, *i.e.*, the balance of remodelling couples.

Background references

- adc, S. Quiligotti, Growth and balance. Mechanics Research Communications, **29**, pp 449–456, 2002.
- adc, Surface and bulk growth unified. Mechanics of Material Forces (P. Steinmann & G.A. Maugin, eds.), pp 53–64, Springer, New York, NY, 2005. Preprint available at http://www.ima.umn.edu/preprints/may2005/2045.pdf.

We aim at developing and implementing a layered shell theory.

As a preliminary exercise, let us indulge in spherical symmetry.



Basic kinematics

$$\mathsf{p}(x) = o + \rho(\xi)\, \mathsf{a}_r(x) \qquad\qquad (\xi := |x - o|)\,,$$

$$\nabla \mathsf{p}|_x = \frac{\rho(\xi)}{\xi}\, P(x) + \rho'(\xi)\, N(x)$$

$$(N(x) := \mathsf{a}_r(x) \otimes \mathsf{a}_r(x)\,, \quad P(x) := I - N(x))\,,$$

$$\mathbb{P}(x) = \alpha_h(\xi)\, P(x) + \alpha_r(\xi)\, N(x)\,,$$

$$F(x) := \nabla \mathsf{p}|_x\, \mathbb{P}(x)^{-1} = \lambda_h(\xi)\, P(x) + \lambda_r(\xi)\, N(x)$$

$$= \frac{\rho(\xi)}{\xi \alpha_h(\xi)}\, P(x) + \frac{\rho'(\xi)}{\alpha_r(\xi)}\, N(x)\,,$$

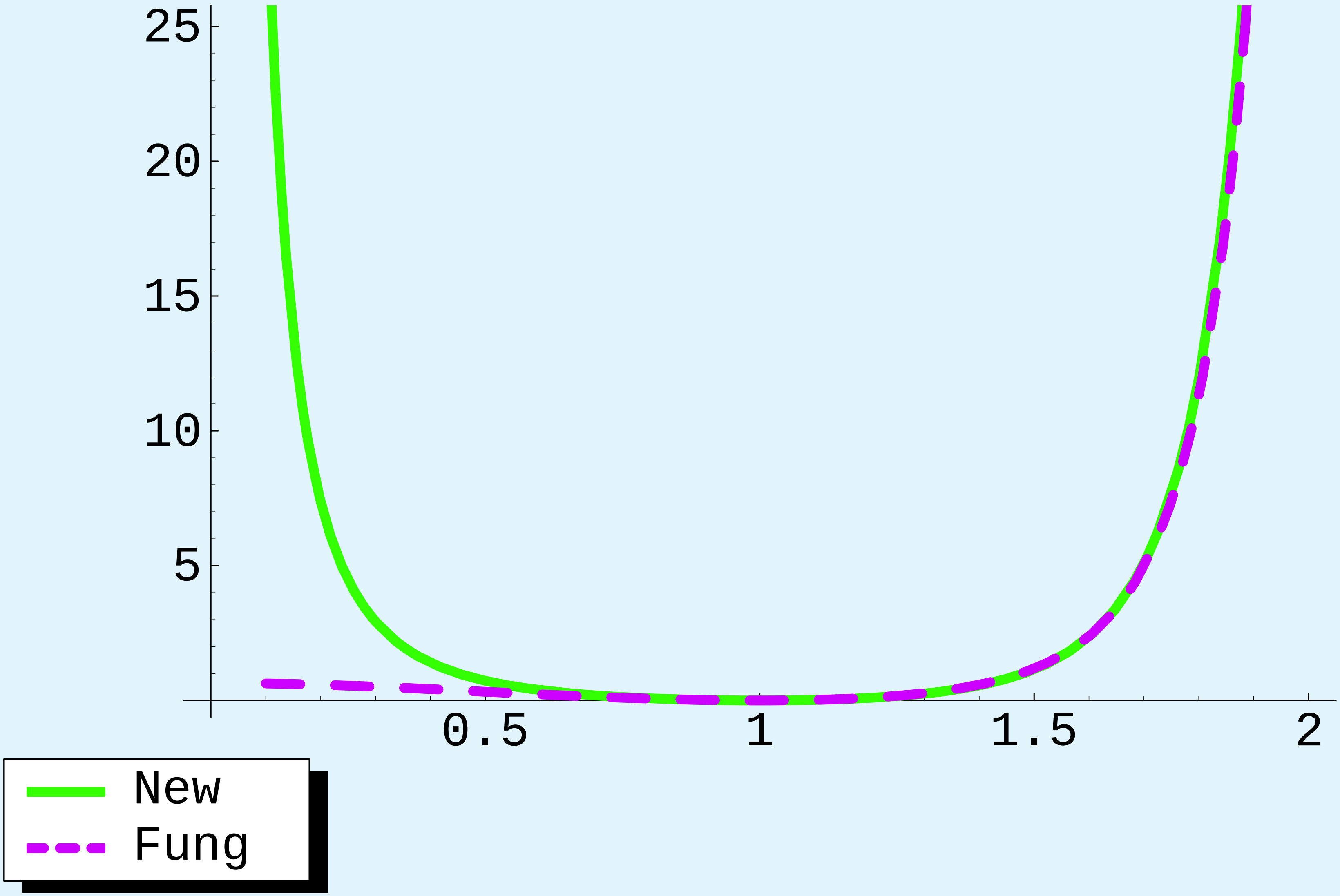
$$\mathbb{V}(x) := \dot{\mathbb{P}}(x)\, \mathbb{P}(x)^{-1} = \frac{\dot{\alpha}_h(\xi)}{\alpha_h(\xi)}\, P(x) + \frac{\dot{\alpha}_r(\xi)}{\alpha_r(\xi)}\, N(x)\,.$$

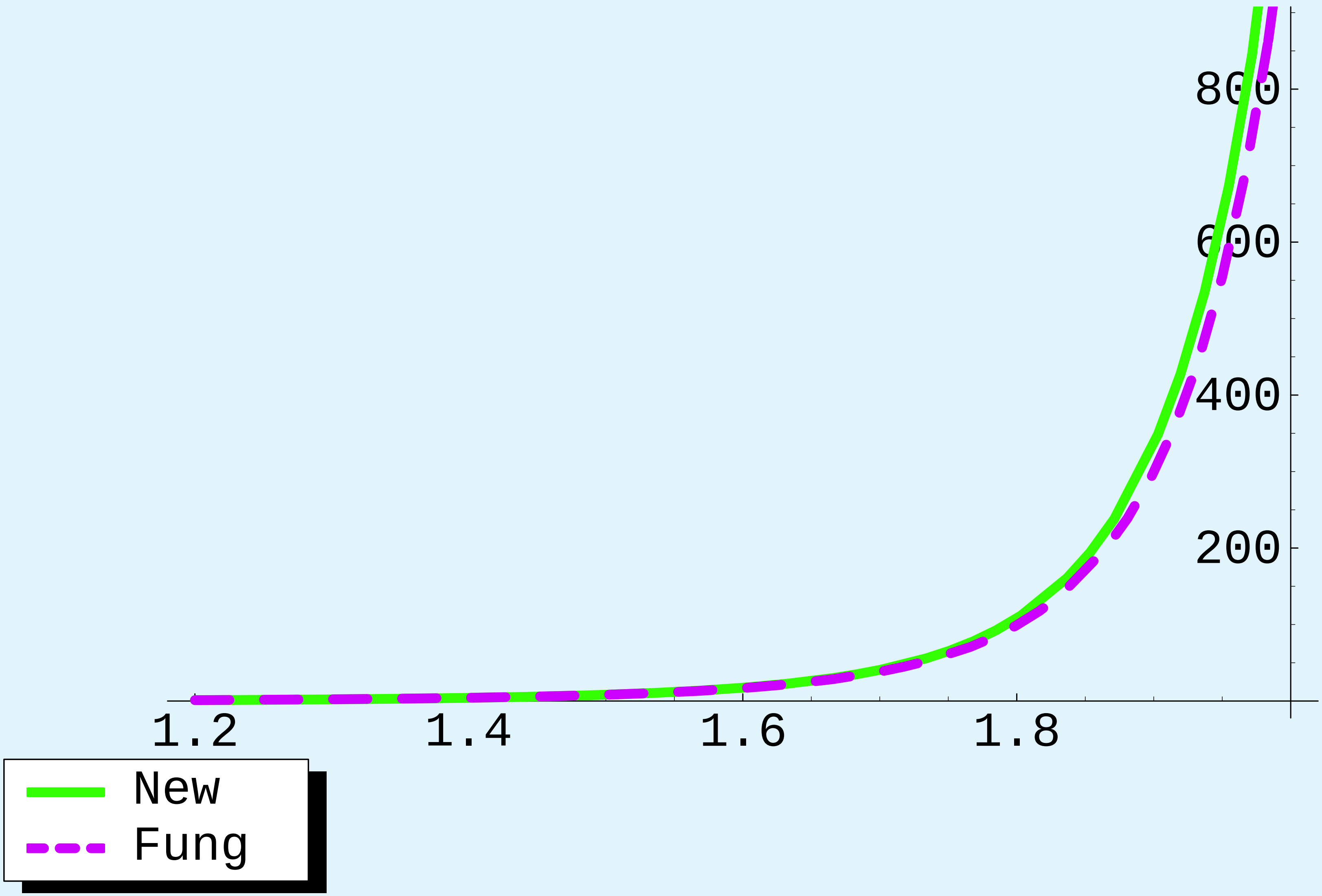
Basic energetics

$$\psi(\lambda_h, \lambda_r; \alpha_h, \alpha_r) = \varphi(\lambda_h, \lambda_r)\, \alpha_h^2 \alpha_r\,,$$

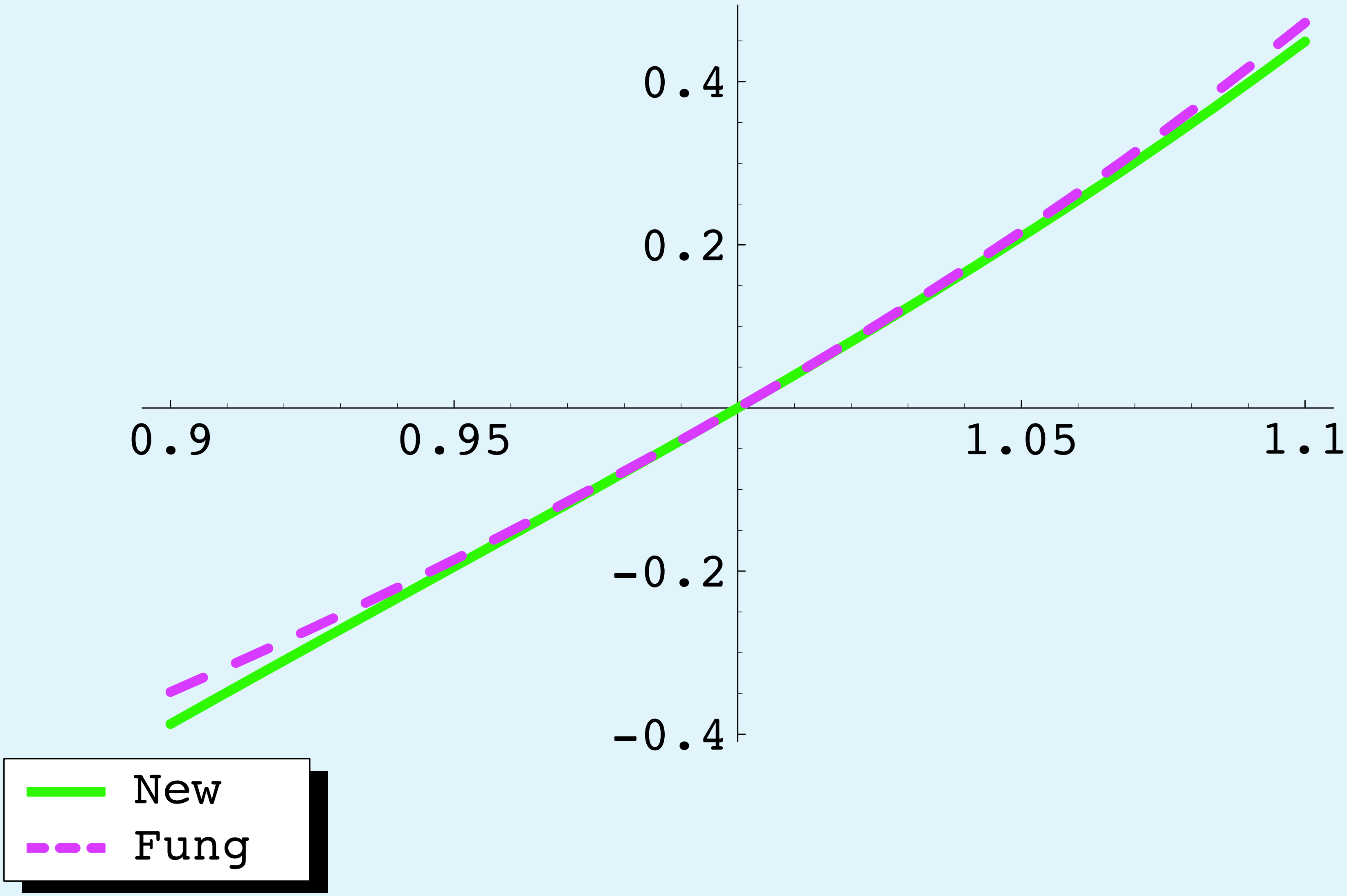
$$\det F = \lambda_h^2 \lambda_r = 1 \;\Rightarrow$$

$$\varphi(\lambda_h) = \frac{1}{2}\left(\exp\left(\frac{\mathfrak{a}}{4}\,(\lambda_h^2 - 1)^2\right) - 1\right)\left(\lambda_h + \frac{1}{\lambda_h^2}\right).$$


25
20
15
10
5
0.5
1
1.5
2
New
Fung



800
600
400
200
1.2
1.4
1.6
1.8
New
Fung


0.4
0.2
-0.2
-0.4
0.9
0.95
1.05
1.1
New
Fung

Basic dynamics

$$\mathbb{s}_h = \frac{1}{6}\frac{\partial\varphi}{\partial\lambda_h} - \frac{\pi}{\lambda_h},$$

$$\mathbb{s}_r = -\frac{\lambda_h^3}{3}\frac{\partial\varphi}{\partial\lambda_h} - \frac{\pi}{\lambda_r},$$

$$\mathbb{c}_h = \varphi + \pi - \frac{\lambda_h}{6}\frac{\partial\varphi}{\partial\lambda_h} + \mathbb{c}_h^+,$$

$$\mathbb{c}_r = \varphi + \pi + \frac{\lambda_h}{3}\frac{\partial\varphi}{\partial\lambda_h} + \mathbb{c}_r^+.$$