Feedback Bending Control Experienced by Piezoelectric Actuators

Vincenzo Gattulli, Ernesto Silverii, Amabile Tatone

Dipartimento di Ingegneria delle Strutture, Acque e Terreno, Università di L'Aquila, L'Aquila, Italy

ABSTRACT

A stepped beam model is presented to predict analytically the natural frequencies and mode shapes of a uniform cantilever beam equipped with a piezoelectric patch. From the continuum model two discrete reduced-order representations are derived; their implications in the control design are discussed. An active non-collocated control scheme has been adopted to increase the cantilever damping by using an accelerometer positioned at the beam tip and a piezoelectric patch glued to the beam surface close to the support. Different output-feedbacks have been tested, and for all of them, the control effectiveness has been evaluated with both analytical and experimental models. Control spillover phenomena in some experienced cases have limited the control performance.

1. INTRODUCTION

A digital implementation of a non-collocated active control scheme for a cantilever equipped with a piezoelectric patch and an accelerometer is the objective of the present study. A model derived for a laminated continuum beam is presented, accounting for the inertia of both the piezo patch and the accelerometer. Under the hyphotesis of uniform electric field, the control action due to the piezo is modelled in two different ways. Indeed, the usual model of the bending control through the use of the Heaviside function as in Fuller, Elliot and Nelson (1996); Preumont (1999), is compared with the description of piece-wise regular displacement and stress fields. The two procedures lead to different state-space representations producing similar results as the system dimension increases. The qualitative information obtained through those representations turns out to be useful in the design of an enhanced controller. Consequently, two output feedbacks have been realized in a digital implementation, namely integral and derivative. Aiming to increase the damping, integral and derivative feedbacks are the most natural ones within the proposed and the usual model, respectively. Both feedbacks provide augmentation of the first modal damping, even though increasing the voltage up to a certain value, instability of higher modes occurs. On this respect, integral feedback shows a better performance.

2. EQUATIONS OF MOTION

From the balance equations derived for a laminated continuum presented in Tatone et al. (1999), assuming as constitutive relation $M = YJv''$, a planar Eulero-Bernoulli stepped beam model (Fig. 1) have been developed to predict analytically the natural frequencies and mode shapes of a uniform cantilever equipped with a piezoelectric patch. The model describes the stepped portion including the inertia and the stiffness of the piezoelectric patch.

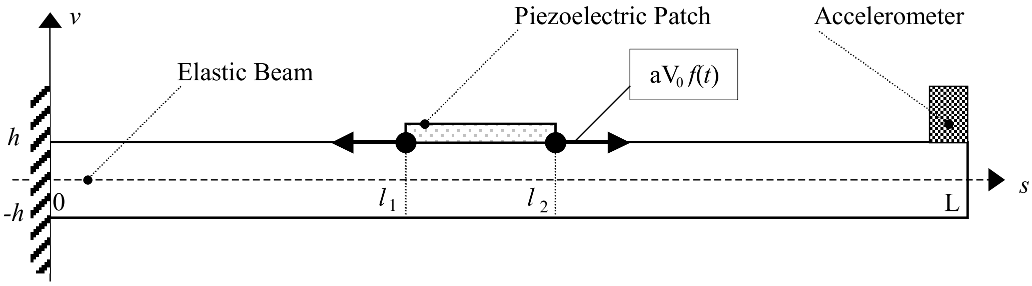

Figure 1: Schetch of the cantilever equipped by piezo-patch and accelerometer

Denoting by ρ and ρ_p the mass density for unit lenght of the alluminium and the piezoelectric lamina respectively and neglecting the inertia momentum, except for the tip accelerometer, the equations of motion result to be

$$YJ\,v^{IV} + \rho\,\ddot{v} = 0 \qquad \text{in }]0, l_1[\qquad (1a)$$

$$YJ\,v^{IV} - hN_p'' + (\rho + \rho_p)\ddot{v} = 0 \qquad \text{in }]\,l_1, l_2[\qquad (1b)$$

$$YJ\,v^{IV} + \rho\,\ddot{v} = 0 \qquad \text{in }]\,l_2, \mathrm{L}[\qquad (1c)$$

The relevant boundary and continuity conditions associated with the problem (1) are

$$YJ\,v'''(l_1^-) - YJ\,v'''(l_1^+) + hN_p'(l_1^+) = 0 \qquad (2a)$$

$$YJ\,v'''(l_2^-) - YJ\,v'''(l_2^+) - hN_p'(l_2^-) = 0 \qquad (2b)$$

$$-YJ\,v''(l_1^-) + YJ\,v''(l_1^+) - hN_p(l_1^+) = 0 \qquad (2c)$$

$$-YJ\,v''(l_2^-) + YJ\,v''(l_2^+) + hN_p(l_2^-) = 0 \qquad (2d)$$

$$YJ\,v'''(L) + YJm_a/\rho\, v^{IV}(L) = 0 \qquad (2e)$$

$$YJ\,v''(L) - YJ\,m_a/\rho\, h_a^2 v^{V}(L) = 0 \qquad (2f)$$

where m_a is the accelerometer mass and h_a its eccentricity. The boundary condition (2) are completed by the continuity on v and v'. In the following, the piezoelectric lamina is used as actuator, consequently, under the hypothesis of uniform electric field, the axial force N_p results to be proportional to the applied voltage V and the elongation of the piezoelectric lamina ε_p, for the adhesion condition are respectively

$$N_p = b\varepsilon_p - aV\,; \qquad \varepsilon_p = -hv'' \qquad (3)$$

Substitution of (3) into (2) under the assumption that both the curvature v'' and the voltage V are constant in] l_1, l_2[, simplifies the boundary value problem (1) and (2). Moreover, neglecting the additional mass of the piezo produces the following *type-1* problem,

$$\rho\,\ddot{v} + EI\,v'''' = 0 \tag{4}$$

with boundary conditions

$$v = v' = 0 \qquad \text{in } s = 0 \tag{5a}$$
$$[v] = [v'] = [v'''] = 0 \qquad \text{in } s = l_1 \text{ and } s = l_2 \tag{5b}$$
$$[v''] + \beta(v'(l_1^+) - v'(l_2^-))/2(l_2 - l_1) = \alpha/2V_0 f(t) \qquad \text{in } s = l_1 \tag{5c}$$
$$[v''] + \beta(v'(l_1^+) - v'(l_2^-))/2(l_2 - l_1) = -\alpha/2V_0 f(t) \qquad \text{in } s = l_2 \tag{5d}$$
$$v'' = v''' + \gamma v^{IV} = 0 \qquad \text{in } s = L \tag{5f}$$

where for the function $v(s)$ and its derivative, the square bracket indicates $[v(s_0)]=v(s_0^-)-v(s_0^+)$ and the coefficients are $\alpha = ah/YJ$, $\beta = bh^2/YJ$ and $\gamma = m_a/YJ$.
Alternatively the problem (4) (5) can be expressed in the entire domain with the simple boundary conditions (5a) and (5f) as (*type-2* problem)

$$\rho\,\ddot{v} + EI\,v'''' = -\mu'(s) \tag{6}$$

where $\mu(s)$ is the control momentum distribution on the piezo sub-domain given by

$$\mu'(s) = [\delta'(s - l_1) - \delta'(s - l_2)]\,N_p \frac{h}{2} \tag{7}$$

where the symbol δ indicates the δ-Dirac function

2.1 Discrete Models

A finite discrete representation is derived for the two problems previously presented. In the *type-1* problem the eingenfunction basis has been enriched with a set of so-called quasi-static functions. The solution shows fast convergence respect to the discontinuity due to the the concentrated control momentum. For the *type-2* problem, the more common procedure of expressing the solution only in the eigenfunction basis is used. In this case the convergence in the discontinuity is slower.
Searching the eigenfuctions for the *type-1* problem requires to solve the associated homogeneous problem, so that (5) can be written as $\mathcal{B}(v_0) + \mathcal{B}(v_t)\,f(t) = Q\,f(t)$ where the transverse displacement has been described by the sum of a quasi-static function and the the dynamic part of the response as $v(s,t) = v_0(s,t) + v_t(s)\,f(t)$. To evaluate the quasi-static functions, the problem $\mathcal{B}(v_t) = Q$ can be solved producing in the three sub-domains the quasi-static functions:
$v_{t1}(s) = 0$; $v_{t2}(s) = -\alpha/(\beta+4)V_0(s-l_1)^2$; $v_{t3}(s) = -\alpha/(\beta+4)V_0(l_2-l_1)(2s-l_2-l_1)$.

The boundary conditions results to be homogenous ($\mathcal{B}(v_0) = 0$). Neglecting the damping terms in (4) and solving the associated eigenvalue problem, the eigenfunctions φ_{ij} in the j sub-domains with $j = \{1,2,3\}$ can be determined, describing the transverse displacements as

$$v_j(s,t) = v_{0j}(s,t) + v_{tj}(s)\, f(t) = \sum_{i=1}^{\infty} \varphi_{ij}(s) \cdot q_i(t) + v_{tj}(s)\, f(t) \tag{8}$$

and an ordinary-differential problem describing the modal amplitudes can be obtained

$$\ddot{q}_i(t) + 2\omega_i \zeta_i \dot{q}_i(t) + \omega_i^2 q_i(t) = p_i \ddot{f}(t) \tag{9}$$

where the partecipating coefficients p_i of the control action to the modal dynamics are

$$p_i = -\int_0^L \varphi_i(s) v_t(s) ds \Bigg/ \int_0^L \varphi_i(s)\, ds \tag{10}$$

Differently, considering the *type-2* problem expressed by (6) with boundary conditions (5a) and (5f) expanding on the basis of the cantilever beam eigenfunctions φ_i(s) the following modal equations can be obtained:

$$\ddot{\breve{q}}_i(t) + 2\omega_i \zeta_i \dot{\breve{q}}_i(t) + (\omega_i^2 + y_i) q_i(t) = \breve{p}_i f(t) \tag{11}$$

where

$$y_i = \frac{\dfrac{bh^2\, (\varphi_i'(l_2) - \varphi_i'(l_1))^2}{4\,(l_2 - l_1)}}{\rho \displaystyle\int_0^L \varphi_i^2(s)\, ds} \; ; \qquad \breve{p}_i = \frac{-a\dfrac{h}{2} V_0 (\varphi_i'(l_2) - \varphi_i'(l_1))}{\rho \displaystyle\int_0^L \varphi_i^2(s)\, ds} \tag{12}$$

2.2 Output-Feedback Control

Defining the state-space representation for both the problems with $\mathbf{x} = \{q_i, \dot{q}_i\}$ and $\mathbf{z} = \{\breve{q}_i, \dot{\breve{q}}_i\}$ yields to

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}_1\, \mathbf{x} + \mathbf{b}_1\, \ddot{u} \\ y &= \mathbf{c}_1\, \mathbf{x} + d\, u \end{aligned} \quad (\textit{type-1}) \qquad \begin{aligned} \dot{\mathbf{z}} &= \mathbf{A}_2\, \mathbf{z} + \mathbf{b}_2\, u \\ y &= \mathbf{c}_1\, \mathbf{x} \end{aligned} \quad (\textit{type-2}) \tag{13}$$

where the output y of the system are the measurement furnished by the tip accelerometer and the matrices $\mathbf{A}_1$ and $\mathbf{A}_2$, the column vectors $\mathbf{b}_1$ and $\mathbf{b}_2$ and the row vectors $\mathbf{c}_1$ and $\mathbf{c}_2$ can be easily evaluated through the eqs. (9-12) respectively and $u = f(t)$ represents the voltage time-dependence applied at the piezo.

A class of output-feedbacks have been considered. In particular through a conditioner, displacement, velocity and acceleration of the tip beam are available for the feedback with a sufficient degree of accuracy. In order to enhance the system damping the two problem gives different information for the control design, indeed in the *type-1* model the double derivative of control $\ddot{u}$ enters as input of the system with partecipating coefficients given by (11) while in the *type-2* model the control u partecipates through the coefficients of (13). Therefore both integral and velocity feedbacks are expected the most effective as

$$u = g \int y\,\mathrm{d}t \quad (\textit{type-1}\text{: integral feedback}) \qquad u = g\,\dot{y} \quad (\textit{type-2}\text{: velocity feedback}) \qquad (14)$$

In (14) for the *type-1* control the output y is the tip displacement while in the *type-2* the output is the tip velocity. The control effectivness have been tested increasing the control gain g. However other two different *type-2* output feedbacks have been evaluated and tested where the output has been the tip displacement and acceleration. The results regarding these last ones have been less effective and they are omitted for sake of brevity.

3. ANALYTICAL AND EXPERIMENTAL RESULTS

The two analytical models (13) have been built identifyng the coefficients through both direct measure and an identification process of the first 13 natural frequencies and modal damping based on the fitting of experimented transfer function between the piezo and the tip acceleration. In particular the system constant values of the model (1) (2) and (3) results to be: YJ =4.58627 Nm^2, ρ= 0.2236 Kg m^{-1}, ρ_p= 0.081 Kg m^{-1}, m_a= 0.02 Kg, h_a = 0.01m, L=0.505 m, l_1= 0.0675 m, l_2= 0.1183 m, h=0.0015875 m, a=0.3143 N/V, b=299.4 kN. The tickness of the alluminia lamina is $2h$. The models have permitted to evaluate the expected spillover effects due to the non-collocated nature of the active control configuration realized with the output-feedbacks (14).

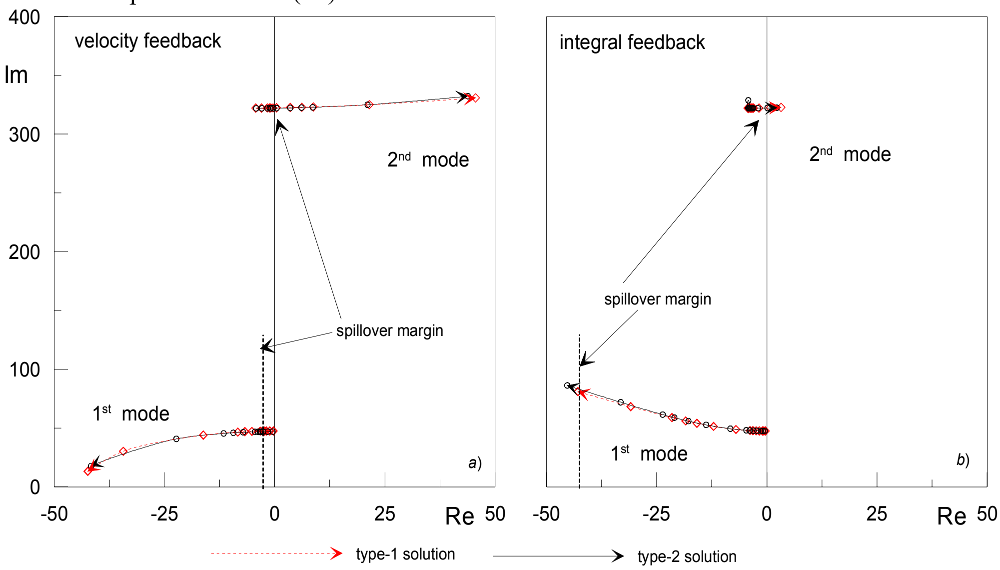

Figure 2: Root-locus of 1st and 2nd mode: a) velocity output-feedback b) integral output-feedback

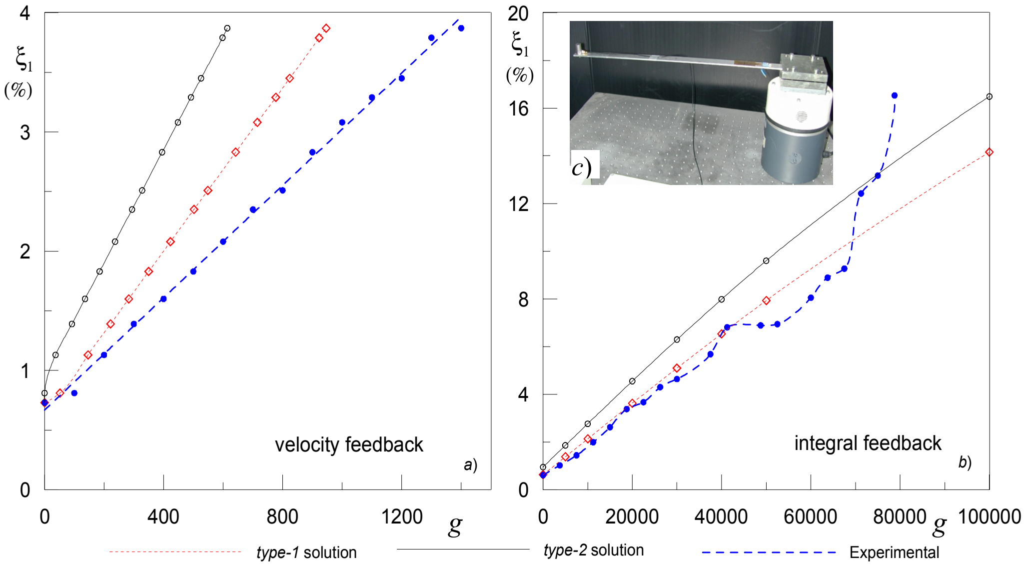

Figure 3: Experimental set-up of the cantilever equipped by piezo-patch and accelerometer

The root-locus of the first two eigenvalues of the cloosed-loop systems (13) are shown for the velocity and integral feedbacks in Figures.2a,b respectively. Both models predict a better performance of the integral feedback because the instability margin due to spillover of the second eigenpair is larger and more damping in the first mode can be induced. The result have been confirmed by experiments. Figure 3a,b depicts the experienced damping in the first mode before the relevant spillover instability. Indeed increasing the control gain g for both controllers it is reached an unstable behavior, but for velocity feedback the maximum measured damping is around 4% while larger values are obtained for integral feedback. The experimental results, obtained by the set-up shown in Fig.3c, are in better agreement with the prediction of the *type-1* model than of the *type-1* one.

4. CONCLUSIONS

The common model of the bending control induced by a piezoelectric patch has been compared with an enriched model based on description of the continuum in three sub-domains. The two models conduct to different state-space representation of the control. The integral output feedback designed on the qualitative information given by the richer model has shown better performance with respect instability due to spillover phenomena. The proposed model seems to produce also quantitative results closer to the experienced cases.

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