AIMETA`03
XVI Congresso AIMETA di Meccanica Teorica e Applicata
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SMORZAMENTO PASSIVO DI VIBRAZIONI ATTRAVERSO LAMINE PIEZOELETTRICHE

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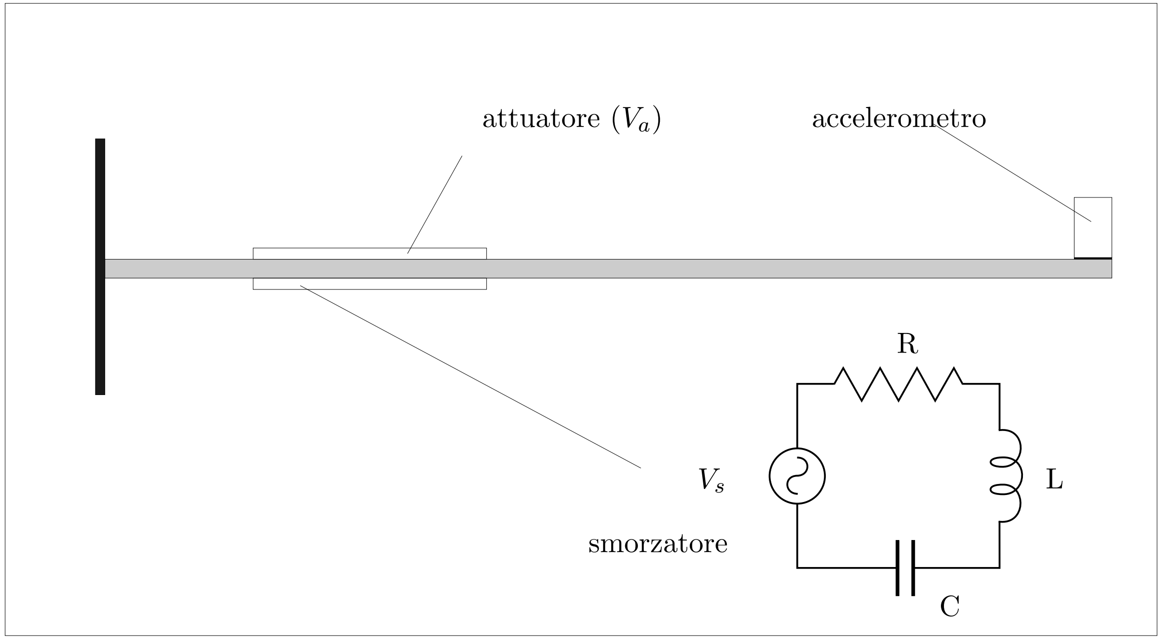
Obiettivi

- riduzione dell'ampiezza delle oscillazioni dissipando energia meccanica attraverso un circuito elettrico
- realizzazione di un circuito risonante RLC con elevati valori di induttanza (richiesti a basse frequenze)
- simulazione di un'induttanza con un *giratore*
- confronto della risposta sperimentale con la risposta calcolata



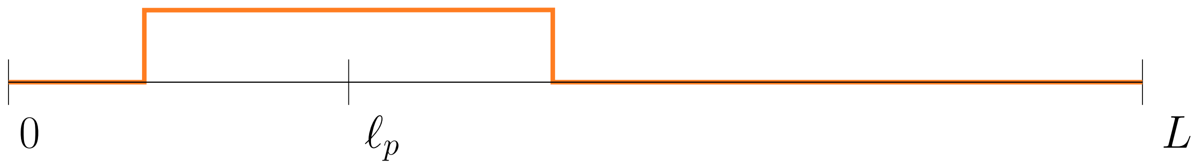
Modello meccanico

$$W_{out}(w,\omega) = \int_0^L (b \cdot w + c \cdot \omega)\, d\xi + \mathsf{t}_0 \cdot w(0) + \mathsf{t}_L \cdot w(L) + \mathsf{m}_0 \cdot \omega(0) + \mathsf{m}_L \cdot \omega(L),$$

$$W_b(w,\omega) = -\int_0^L (z \cdot w + t \cdot w' + \mathfrak{z} \cdot \omega + m \cdot \omega')\, d\xi,$$

$$W_a(w_a) = -\int_0^L H\,(z_a \cdot w_a + t_a \cdot w_a')\, d\xi,$$

$$W_s(w_s) = -\int_0^L H\,(z_s \cdot w_s + t_s \cdot w_s')\, d\xi,$$



$$H(\xi) = \begin{cases} 1 & \xi \in I_p \\ 0 & \xi \notin I_p \end{cases} \qquad I_p := [\ell_p - L_p/2, \ell_p + L_p/2] \subset [0, L]$$

Modello meccanico

$$W_{out}(w,\omega) = \int_0^L (b \cdot w + c \cdot \omega)\, d\xi + \mathsf{t}_0 \cdot w(0) + \mathsf{t}_L \cdot w(L) + \mathsf{m}_0 \cdot \omega(0) + \mathsf{m}_L \cdot \omega(L),$$

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$$W_a(w_a) = -\int_0^L H\,(z_a \cdot w_a + t_a \cdot w_a')\, d\xi,$$

$$W_s(w_s) = -\int_0^L H\,(z_s \cdot w_s + t_s \cdot w_s')\, d\xi,$$

$$w_a := w + \omega \times \frac{h}{2}\, a_2, \quad w_s := w - \omega \times \frac{h}{2}\, a_2,$$

Modello meccanico-elettrico

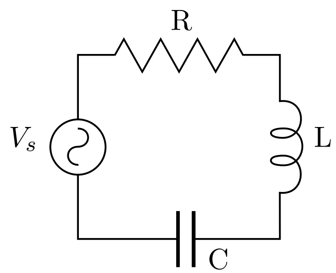
$$W_{out}(w, \omega) = \int_0^L (b \cdot w + c \cdot \omega)\, d\xi + \mathsf{t}_0 \cdot w(0) + \mathsf{t}_L \cdot w(L) + \mathsf{m}_0 \cdot \omega(0) + \mathsf{m}_L \cdot \omega(L),$$

$$W_b(w, \omega) = -\int_0^L (z \cdot w + t \cdot w' + \mathfrak{z} \cdot \omega + m \cdot \omega')\, d\xi,$$

$$W_a(w_a) = -\int_0^L H\,(z_a \cdot w_a + t_a \cdot w_a')\, d\xi,$$

$$W_s(w_s) = -\int_0^L H\,(z_s \cdot w_s + t_s \cdot w_s')\, d\xi,$$

$$W_c(I_s) = \left(\mathrm{L}\,\ddot{Q}_s + \mathrm{R}\,\dot{Q}_s + V_s\right) I_s.$$



Obiettività materiale

$$W_b(w,\omega) = -\int_0^L (z \cdot w + t \cdot w' + \mathfrak{z} \cdot \omega + m \cdot \omega')\, d\xi$$

$$W_a(w_a) = -\int_0^L H\,(z_a \cdot w_a + t_a \cdot w_a')\, d\xi$$

$$W_s(w_s) = -\int_0^L H\,(z_s \cdot w_s + t_s \cdot w_s')\, d\xi$$

Per ogni atto di moto test rigido

$$\omega(\xi) = \bar{\omega}$$

$$w(\xi) = \bar{w} + \bar{\omega} \times (x(\xi) - x_0)$$

sia

$$z \cdot w + t \cdot w' + \mathfrak{z} \cdot \omega + m \cdot \omega' = 0$$

$$z_a \cdot w_a + t_a \cdot w_a' = 0$$

$$z_s \cdot w_s + t_s \cdot w_s' = 0$$

Obiettività materiale

$$
\begin{aligned}
z = 0, &\quad \mathfrak{z} = -x' \times t, \\
z_a = 0, &\quad 0 = x'_a \times t_a, \\
z_s = 0, &\quad 0 = x'_s \times t_s.
\end{aligned}
$$

$$
W_b(w, \omega) = -\int_0^L (t \cdot w' - x' \times t \cdot \omega + m \cdot \omega')\, d\xi
$$

$$
W_a(w_a) = -\int_0^L (t_a H) \cdot w'_a d\xi
$$

$$
W_s(w_s) = -\int_0^L (t_s H) \cdot w'_s d\xi
$$

Equazioni di bilancio

$$W_{out} + W_b + W_a + W_s + W_c = 0, \quad \forall w, \omega \in C^1([0,L]), \forall I_s \in \mathbb{R}$$

Integrando per parti

$$\int_0^L (t_a H) \cdot w_a' \, d\xi = -\int_0^L (t_a H)' \cdot w_a \, d\xi$$

$$\int_0^L t \cdot w' \, d\xi = -\int_0^L t' \cdot w \, d\xi + t(L)w(L) - t(0)w(0)$$

si ottiene

$$\begin{aligned}
&\int_0^L (b + t' + (t_a H)' + (t_s H)') \cdot w \, d\xi \\
&+ \int_0^L \left(c + x' \times t + m' + \frac{h}{2} a_2 \times (t_a H)' - \frac{h}{2} a_2 \times (t_s H)' \right) \cdot \omega \, d\xi \\
&+ (\mathsf{t}_0 + t(0)) \cdot w(0) + (\mathsf{m}_0 + m(0)) \cdot \omega(0) \\
&+ (\mathsf{t}_L - t(L)) \cdot w(L) + (\mathsf{m}_L - m(L)) \cdot \omega(L) \\
&+ \left(\mathrm{L} \, \ddot{Q}_s + \mathrm{R} \, \dot{Q}_s + V_s \right) I_s = 0.
\end{aligned}$$

Equazioni di bilancio

$$b + t' + (t_a H)' + (t_s H)' = 0$$

$$c + x' \times t + m' + \frac{h}{2}\, a_2 \times (t_a H)' - \frac{h}{2}\, a_2 \times (t_s H)' = 0$$

$$\mathrm{L}\, \ddot{Q}_s + \mathrm{R}\, \dot{Q}_s + V_s = 0$$

$$v(0) = 0, \qquad t(L) = \mathsf{t}_L$$
$$\theta(0) = 0, \quad m(L) = \mathsf{m}_L$$

$$\begin{aligned} b &:= -\rho\, \ddot{x} \\ c &:= 0 \\ \rho &:= \rho_b + (\rho_a + \rho_s) H \\ \mathsf{t}_L &:= -\mu\, \ddot{x}(L) + \mu\, r(\ddot{\theta}(L)\, a_1 + \dot{\theta}(L)^2\, a_2) \\ \mathsf{m}_L &:= \mu\, r^2\, \ddot{\theta}(L)\, a_3 - \mu\, r\, a_2 \times \ddot{x}(L) \end{aligned}$$

μ massa dell'accelerometro

r eccentricità dell'accelerometro

Tensione interlaminare

$$b + t' + (t_a H)' + (t_s H)' = 0$$

$$x'_a \times t_a = 0$$
$$x'_s \times t_s = 0$$

$$\big(t_a(\xi)\, H(\xi)\big)' = t'_a(\xi)\, H(\xi) + t_a(\xi)\, \delta(\xi - \ell_p + L_p/2) - t_a(\xi)\, \delta(\xi - \ell_p - L_p/2).$$

Tensione interlaminare

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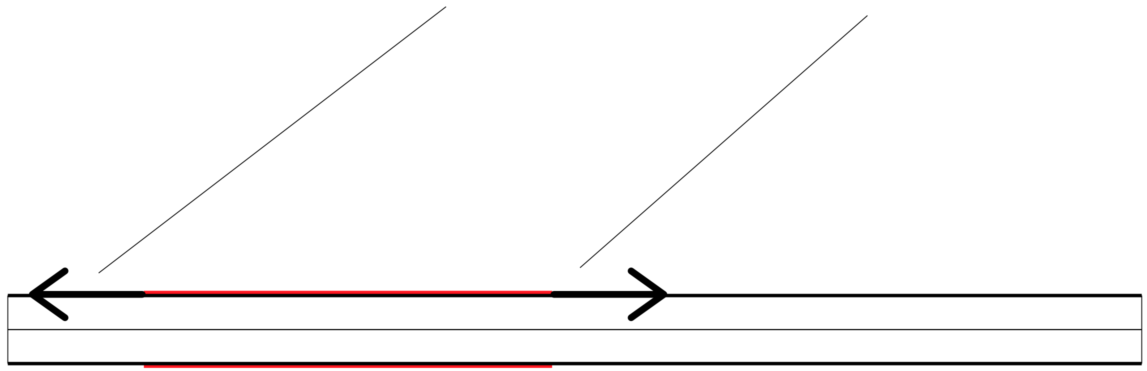

Tensione interlaminare

$$b + t' + (t_a H)' + (t_s H)' = 0$$

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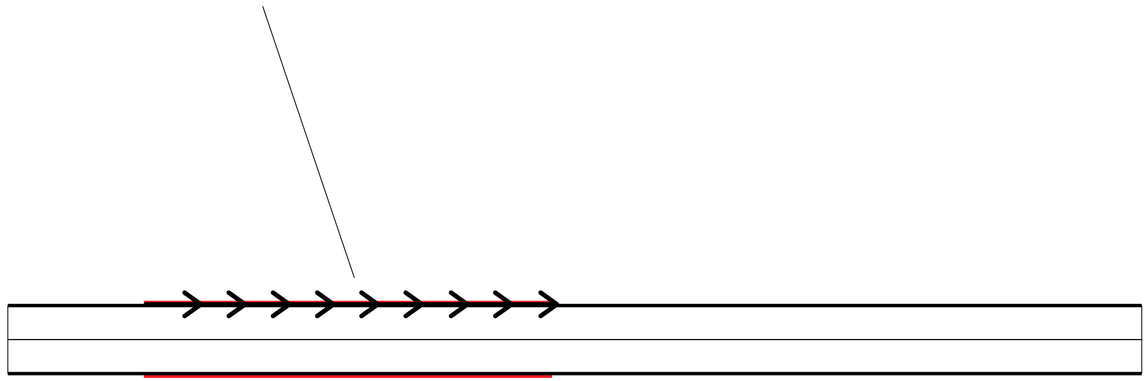
$$\big(t_a(\xi)\, H(\xi)\big)' = t'_a(\xi)\, H(\xi) + t_a(\xi)\, \delta(\xi - \ell_p + L_p/2) - t_a(\xi)\, \delta(\xi - \ell_p - L_p/2).$$



Caratterizzazione costitutiva delle lamine piezoelettriche

$$t_s = N_s\, a_1$$
$$t_a = N_a\, a_1$$

attuatore

$$\begin{pmatrix} N_a \\ Q_a \end{pmatrix} = \begin{pmatrix} \bar{C}_{11}^E & \bar{e}_{31} \\ \bar{e}_{31} & -C \end{pmatrix} \begin{pmatrix} \Delta_a \\ V_a \end{pmatrix}$$

smorzatore

$$\begin{pmatrix} N_s \\ V_s \end{pmatrix} = \begin{pmatrix} \bar{C}_{11}^D & -\bar{\nu}_{31} \\ -\bar{\nu}_{31} & 1/C \end{pmatrix} \begin{pmatrix} \Delta_s \\ Q_s \end{pmatrix}$$

$$\Delta_s = -\Delta_a = \frac{h}{2}\left(\theta(\ell_p + L_p/2) - \theta(\ell_p - L_p/2)\right)$$

$$\bar{C}_{11}^D = 1.28 \times 10^7 \text{ N/m},$$
$$\bar{C}_{11}^E = 8.62 \times 10^6 \text{ N/m},$$
$$\bar{\nu}_{31} = -3.77 \times 10^7 \text{ N/nF},$$
$$\bar{e}_{31} = -0.157 \text{ N/V},$$
$$C = 48.6 \text{ nF}.$$

Equazioni lineari di bilancio

$$T' - \rho\,\ddot{v} = 0$$

$$M' + T - \frac{h}{2}(N_a H)' + \frac{h}{2}(N_s H)' = 0$$

$$v(0) = 0, \quad T(L) = \mu\,\ddot{v}(L),$$
$$\theta(0) = 0, \quad M(L) = \mu\, r^2\,\ddot{\theta}(L),$$

Equazioni lineari di bilancio

$$T' - \rho\, \ddot{v} = 0$$

$$M' + T - \frac{h}{2}(N_a H)' + \frac{h}{2}(N_s H)' = 0$$

$$\begin{aligned} v(0) &= 0, \quad T(L) = \mu\, \ddot{v}(L), \\ \theta(0) &= 0, \quad M(L) = \mu\, r^2\, \ddot{\theta}(L), \end{aligned}$$

$$M'' - \frac{h}{2}(N_a H)'' + \frac{h}{2}(N_s H)'' + \rho\, \ddot{v} = 0,$$

$$\begin{aligned} v(0) &= 0, \quad -M'(L) = \mu\, \ddot{v}(L), \\ \theta(0) &= 0, \quad\quad M(L) = \mu\, r^2\, \ddot{\theta}(L). \end{aligned}$$

Equazioni lineari di bilancio

$$T' - \rho\,\ddot{v} = 0$$

$$M' + T - \frac{h}{2}(N_a H)' + \frac{h}{2}(N_s H)' = 0$$

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(N_a, N_s, costanti)

$$M'' - \frac{h}{2}\,N_a H'' + \frac{h}{2}\,N_s H'' + \rho\,\ddot{v} = 0$$

Equazioni del moto

$(\theta = v')$

$$YJ\,v'''' + \frac{h^2}{4}\,(\bar{C}_{11}^E + \bar{C}_{11}^D)\Big(v'(\ell_p + L_p/2) - v'(\ell_p - L_p/2)\,\Big)H'' + \rho\ddot{v} = \frac{h}{2}\,\bar{\nu}_{31}Q_sH'' + \frac{h}{2}\,\bar{e}_{31}V_aH''$$

$$v(0) = 0, \quad -YJ\,v'''(L) = \mu\,\ddot{v}(L)$$

$$v'(0) = 0, \qquad YJ\,v''(L) = \mu\,r^2\,\ddot{v}'(L)$$

$$\mathrm{L}\,\ddot{Q}_s + \mathrm{R}\,\dot{Q}_s + \frac{Q_s}{C} - \bar{\nu}_{31}\frac{h}{2}\,\big(v'(\ell_p + L_p/2) - v'(\ell_p - L_p/2)\big) = 0$$

Equazioni modali

$$v(\xi, t) = \sum_{i=1}^{N} \Phi_i(\xi) X_i(t),$$

$$\ddot{X}_i + 2\omega_i\zeta_i\dot{X}_i + \omega_i^2 X_i + q_i\, Q_s = p_i V_a,$$

$$\mathrm{L}\, \ddot{Q}_s + \mathrm{R}\, \dot{Q}_s + \frac{Q_s}{C} + \sum_{i=1}^{N} q_i\, \|\Phi_i\|\, X_i = 0,$$

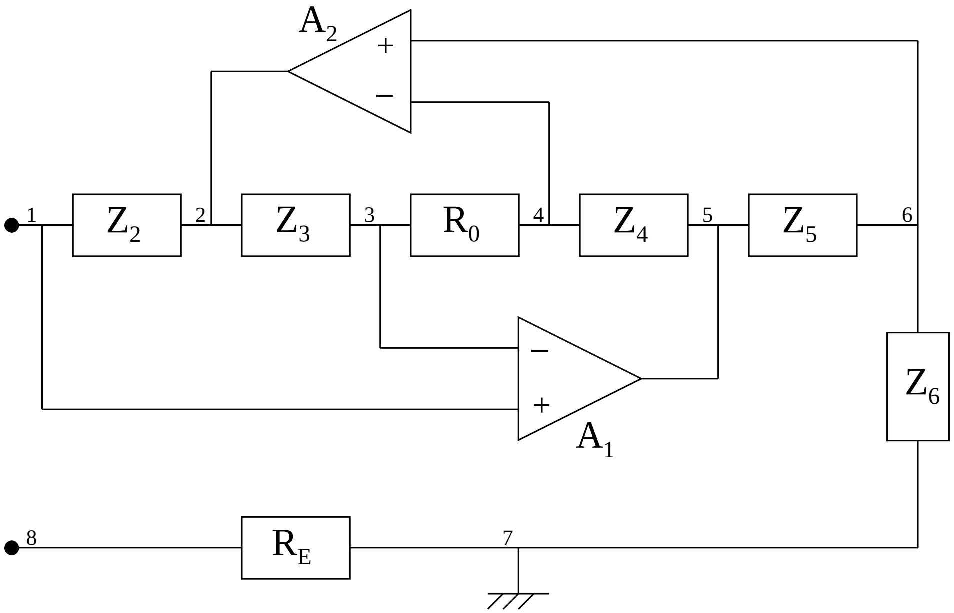
$$\|\Phi_i\| := \int_0^L \rho\, \Phi_i^2(\xi)\, d\xi + \mu\, \Phi_i(L)^2 + \mu\, r^2 \Phi_i'(L)^2,$$

$$q_i := -\frac{1}{\|\Phi_i\|} \int_0^L \frac{h}{2}\, \bar{\nu}_{31} \Phi_i(\xi) H''(\xi)\, d\xi = -\frac{h}{2}\, \bar{\nu}_{31}\, \frac{\Phi_i'(\ell_p + L_p/2) - \Phi_i'(\ell_p - L_p/2)}{\|\Phi_i\|},$$

$$p_i := \frac{1}{\|\Phi_i\|} \int_0^L \frac{h}{2}\, \bar{e}_{31} \Phi_i(\xi) H''(\xi)\, d\xi = \frac{h}{2}\, \bar{e}_{31} \frac{\Phi_i'(\ell_p + L_p/2) - \Phi_i'(\ell_p - L_p/2)}{\|\Phi_i\|}.$$

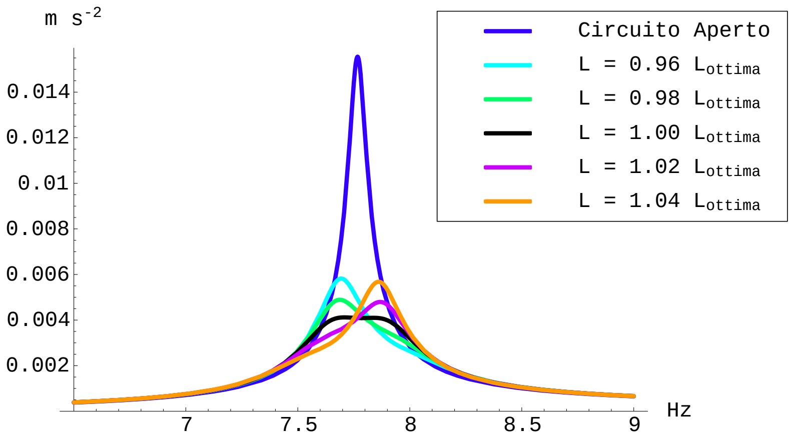
Barra di alluminio	Lamina Piezoelettrica
$L = 0.511$ m	$L_p = 4.597 \times 10^{-2}$ m $\ell_p = 8.4 \times 10^{-2}$ m
$h = 3.21 \times 10^{-3}$ m	$h_p = 2.54 \times 10^{-4}$ m
$J = 7.028 \times 10^{-11}$ m^4	$Y_{11}^E = 6.9 \times 10^{10}$ N/m^2
$Y = 67.6$ GPa	$Y_{33}^E = 5.5 \times 10^{10}$ N/m^2
$\rho_b = 0.2236$ Kg/m	$\rho_p = 2 \times 0.0616$ Kg/m

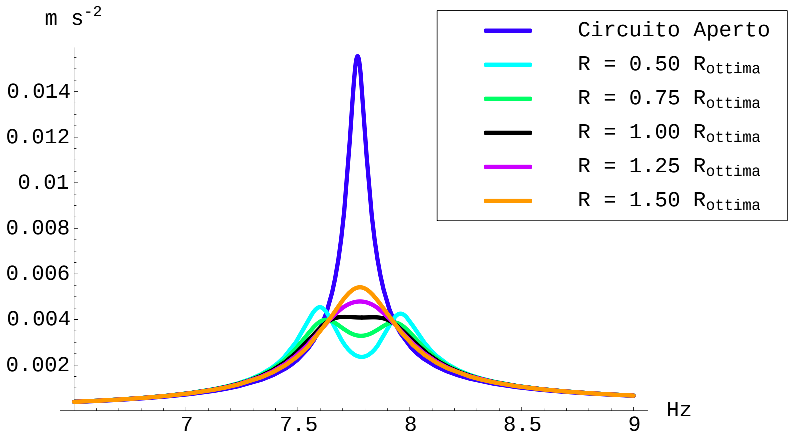
Giratore di Antoniou modificato



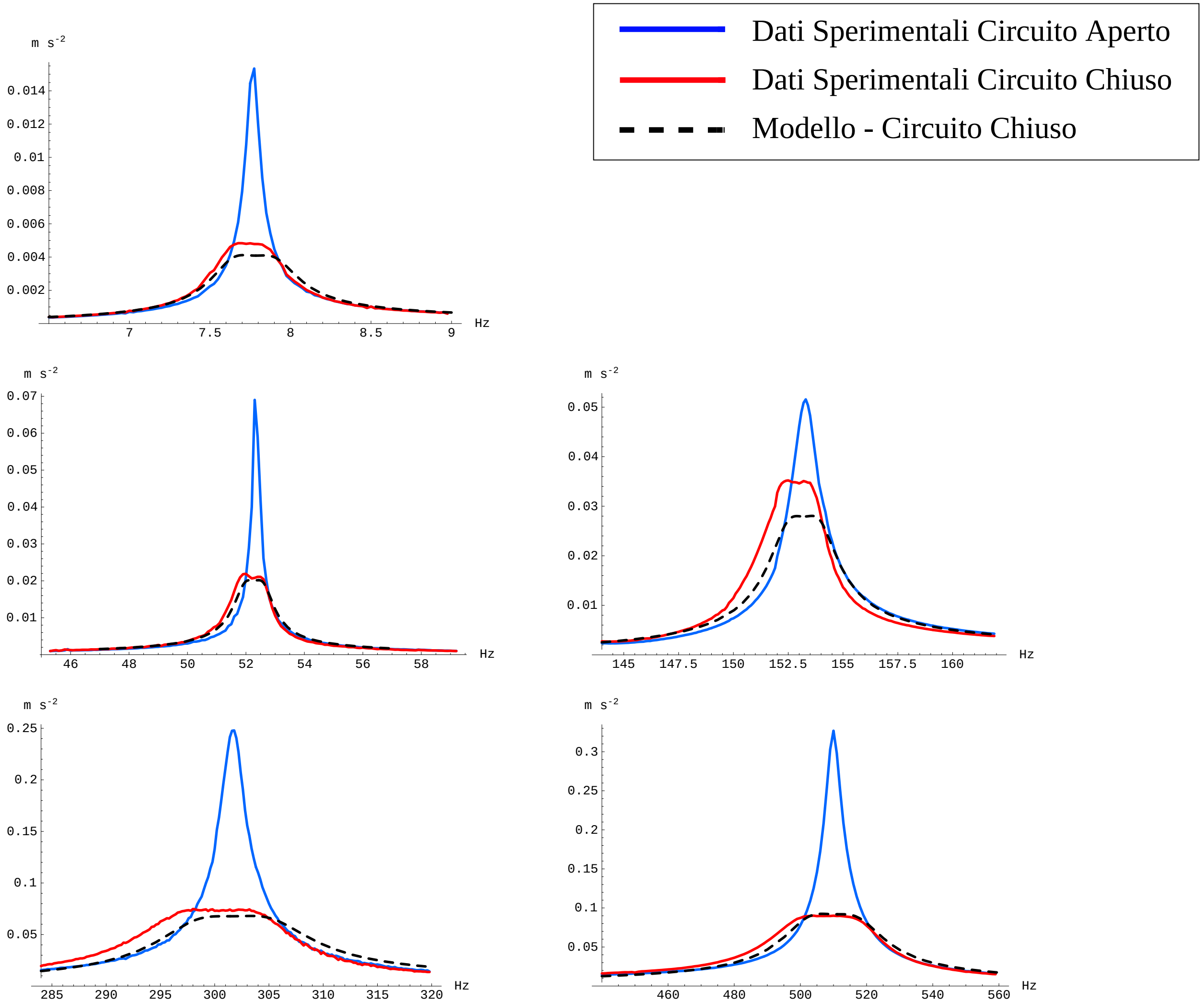
$$Z_{in} = \frac{Z_2\ Z_4\ Z_6}{Z_3\ Z_5} - R_0\ \frac{Z_2}{Z_3} + R_E$$

Giratore	1	2	3	4	5
frequenza (Hz)	7.775	52.3	153.3	301.8	510
C (nF)	48.6	48.6	48.6	48.6	48.6
Operazionali A_1-A_2	TL081 $\pm$15 Volt				
V_a(Volt)	0.7	0.1	0.1	0.01	0.05
Z_2 $(K\Omega)$	46.7	3	3	3	3
Z_3 $(K\Omega)$; Z_4 $(K\Omega)$	0.995	0.995	0.995	0.995	0.995
Z_5 (μF)	10	0.88	0.88	0.0221	0.0221
Z_6 $(K\Omega)$	17.52	70.6	8.52	93	33.4
R_E $(K\Omega)$	3.88	0	0	0.0681	0.173
R_0 $(K\Omega)$	0	0.418	0.1961	0	0
$L = 1/(\omega^2 C)$ (Henry)	8622	190.6	22.18	5.722	2.004
L_{ottima} (Henry)	**8562**	**190.1**	**22.13**	**5.694**	**2.002**
L_{ottima}/L	0.9930	0.9975	0.9979	0.9951	0.9993
R_{ottima} $(K\Omega)$	**37.908**	**2.098**	**0.4657**	**0.6728**	**0.5073**
Attenuazione della risposta	69 %	70 %	32 %	71 %	73 %


m s-2
0.014
0.012
0.01
0.008
0.006
0.004
0.002
7
7.5
8
8.5
9
Hz
Circuito Aperto
L = 0.96 Lottima
L = 0.98 Lottima
L = 1.00 Lottima
L = 1.02 Lottima
L = 1.04 Lottima


m s-2
0.014
0.012
0.01
0.008
0.006
0.004
0.002
7
7.5
8
8.5
9
Hz
Circuito Aperto
R = 0.50 Rottima
R = 0.75 Rottima
R = 1.00 Rottima
R = 1.25 Rottima
R = 1.50 Rottima

Confronto tra risposta calcolata e risposta misurata



Conclusioni

- sensibile attenuazione dell'ampiezza delle oscillazioni
- buon accordo tra risposta calcolata e risposta misurata

Problemi

- saturazione degli operazionali del giratore
- risposta del modello poco accurata per frequenze alte

Propositi

- utilizzare nuova elettronica (giratore basato su *current-conveyors*)
- adottare nuovo modello costitutivo per le lamine piezoelettriche

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