COMSOL CONFERENCE GRENOBLE 23-24 OCT 2007

Traction on the Retina Induced by Saccadic Eye Movements in the Presence of Posterior Vitreous Detachment

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Grenoble, 24th October 2007

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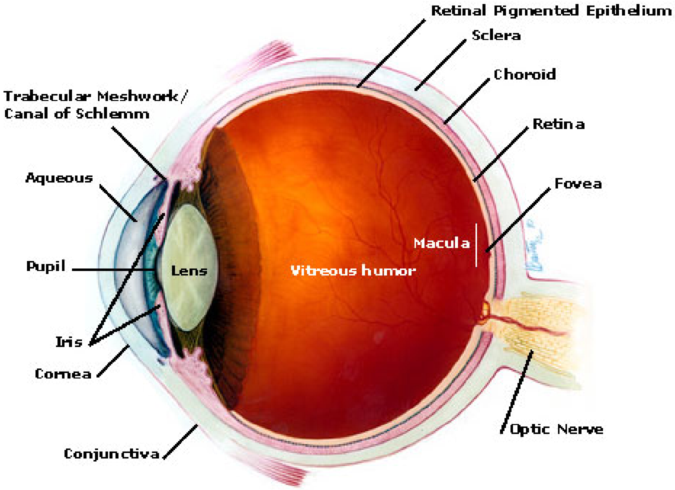
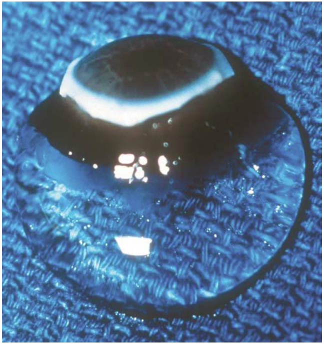
Physiology of the human eye



The vitreous body: structure and composition

The mature vitreous body has got a
transparent, gel-like consistence



It contains:

- 99% water
- 0,9% low molecular weight solutes
- 0,1% macromolecules such as collagen and HA
- few hyalocytes (vitreous cells)

Aging of the vitreous

With aging, substantial alterations take place in the vitreous body and it undergoes a process of

Liquefaction

- Hyaluronic acid may dissociate from collagen fibrils and be redistributed from the gel to the liquid vitreous forming pools (synchysis);
- Collagen fibrils are no more separated by HA hydrated molecules and aggregate together into fibers (syneresis).

With age, there is a weakening of the vitreo-retinal adhesion, mostly due to biochemical alterations at the interface.

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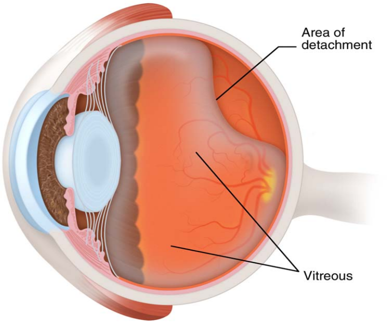
The decrease of vitreo-retinal adhesion, in conjunction with liquefaction of the vitreous body, leads to

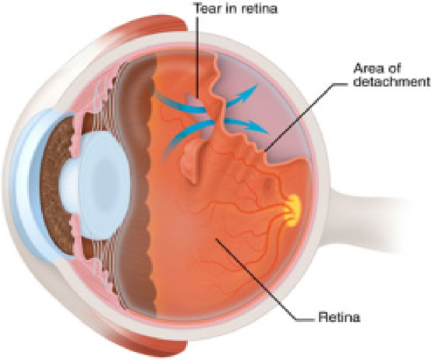
POSTERIOR VITREOUS DETACHMENT

Posterior Vitreous Detachment (PVD)



It typically has no clinical consequences and leaves the sight abilities unchanged;

At the adhesion points between the cortex and the retina, during eye movements vitreous fibers may exert so high tractions as to generate tears on the retina.



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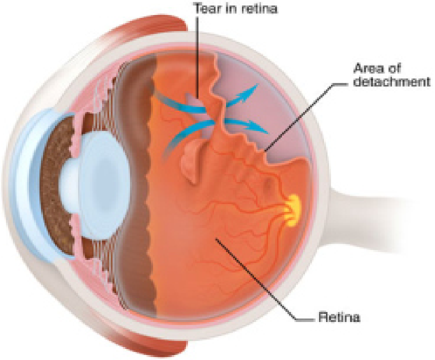


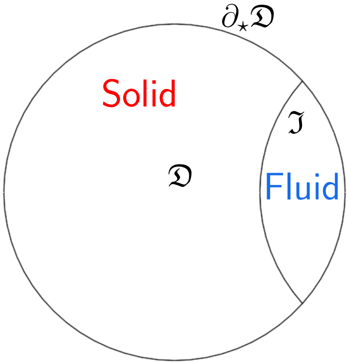
Fluid vitreous may then flow into the sub-retinal space, thus triggering a rhegmatogenous retinal detachment.

...How to best ***evaluate*** *the presence or absence of vitreous traction and how to* ***quantitate*** *the degree of vitreous traction is presently not known...*

(J.Sebag, The Vitreous, 1989)

The mathematical model

We account for a **2D plain strain** problem and we consider a configuration of the rigid vitreous chamber after PVD.



The Balance Principle of Working

For a Cauchy-continuum in any shape $\mathfrak{D}$

$$\int_{\mathfrak{D}} \mathbf{b} \cdot \mathbf{w}\, dV + \int_{\partial\mathfrak{D}} \mathbf{t} \cdot \mathbf{w}\, dA - \int_{\mathfrak{D}} \mathbf{T} \cdot \nabla\mathbf{w}\, dV = 0\,,$$

for any test velocity field $\mathbf{w}$.

Bulk force $\mathbf{b}$ measured by unit volume on $\mathfrak{D}$;

Surface force $\mathbf{t}$ exerted on the body through its boundary $\partial\mathfrak{D}$;

Cauchy stress tensor $\mathbf{T}$.

It can be restated by pulling-back all fields to a fixed paragon shape $\mathcal{D}$

$$\int_{\mathcal{D}} \mathsf{b} \cdot \mathsf{w} \, dV + \int_{\partial\mathcal{D}} \mathsf{t} \cdot \mathsf{w} \, dA - \int_{\mathcal{D}} \mathsf{S} \cdot \nabla \mathsf{w} \, dV = 0 \,,$$

for any test velocity field w.

Piola-Kirchhoff stress tensor $\mathsf{S} := \mathbf{T}\,\mathsf{F}^{-\mathsf{T}} \det \mathsf{F}$;

Deformation gradient F.

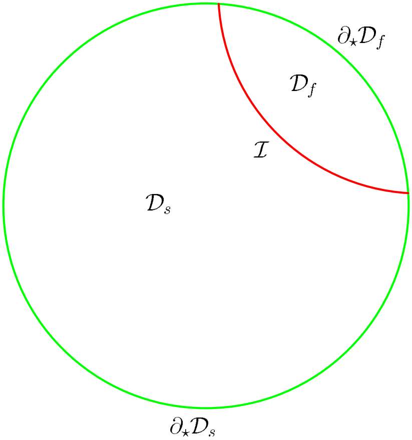
Looking at this composite continuum as a whole, we require

$$\int_{\mathfrak{D}} \mathbf{b}\cdot\mathbf{w}\, dV + \int_{\partial_\star \mathfrak{D}} \mathbf{t}\cdot\mathbf{w}\, dA - \int_{\mathfrak{D}} \mathbf{T}\cdot\nabla\mathbf{w}\, dV - \int_{\mathfrak{I}} \mathbf{t}^*\cdot[\![\mathbf{w}]\!]\, dA = 0\,,$$

for any test velocity field $\mathbf{w}$ allowed to be discontinuous at $\mathfrak{I}$.

$\mathbf{t}^*$ is the interface stress.

It is convenient to write the balance principles for fluid and solid parts separately.



The Solid

The gel-like vitreous is modeled as a homogeneous, isotropic, non-linear hyperelastic incompressible solid.

The Balance Principle for the solid yields

$$\int_{\mathfrak{D}_s} \mathbf{b}_s \cdot \mathbf{w}_s \, dV + \int_{\partial_\star \mathfrak{D}_s} \mathbf{t}_s \cdot \mathbf{w}_s \, dA + \int_{\mathfrak{I}} \mathbf{t}_s \cdot \mathbf{w}_s \, dA - \int_{\mathfrak{D}_s} \mathbf{T}_s \cdot \nabla \mathbf{w}_s \, dV = 0$$

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for any test velocity field $\mathbf{w}_s$ on $\mathfrak{D}_s$.

The incompressibility condition (enforced in the weak form) is

$$\int_{\mathcal{D}_s} (\det \mathsf{F} - 1) \, \tilde{p}_s \, dV = 0 \, ,$$

The Fluid

The liquefied vitreous is modeled as a Newtonian incompressible fluid. The Balance Principle for the fluid states

$$\int_{\mathfrak{D}_f} \mathbf{b}_f \cdot \mathbf{w}_f \, dV + \int_{\partial_\star \mathfrak{D}_f} \mathbf{t}_f \cdot \mathbf{w}_f \, dA + \int_{\mathfrak{I}} \mathbf{t}_f \cdot \mathbf{w}_f \, dA - \int_{\mathfrak{D}_f} \mathbf{T}_f \cdot \nabla \mathbf{w}_f \, dV = 0$$

for any test velocity field $\mathbf{w}_f$ on the actual shape $\mathfrak{D}_f$.

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The incompressibility condition (enforced in the weak form) is given by

$$\int_{\mathfrak{D}_f} (\text{div } \mathbf{v}_f) \, \tilde{p}_f \, dV = 0$$

The response function provides

$$\mathbf{T}_f = -p_f \mathbf{I} + 2\mu \; \text{sym} \, \nabla \mathbf{v}_f \, .$$

Interface

The interface condition consists in the continuity of the velocity field across $\mathfrak{I}$

$$\int_{\mathfrak{I}} (\mathbf{v}_f - \dot{\mathbf{u}}_s) \cdot \tilde{\mathbf{t}}^* \, dA = 0 \,,$$

where $\dot{\mathbf{u}}_s$ is the solid velocity field on $\mathfrak{D}_s$.

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In addition we deduce

$$\int_{\mathfrak{I}} \mathbf{t}_s \cdot \mathbf{w}_s \, dA = - \int_{\mathfrak{I}} \mathbf{t}^* \cdot \mathbf{w}_s \, dA \, ,$$

$$\int_{\mathfrak{I}} \mathbf{t}_f \cdot \mathbf{w}_f \, dA = \int_{\mathfrak{I}} \mathbf{t}^* \cdot \mathbf{w}_f \, dA \, .$$

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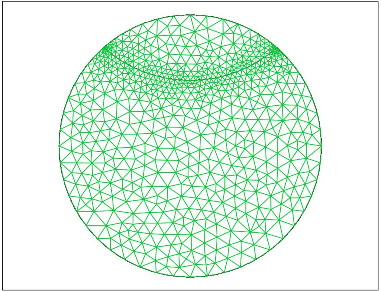
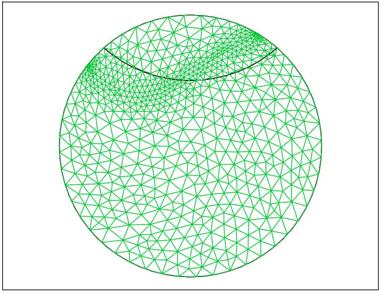
Summing up terms on $\mathfrak{I}$

$$\int_{\mathfrak{I}} \mathbf{t}^* \cdot (\mathbf{w}_f - \mathbf{w}_s) + (\mathbf{v}_f - \dot{\mathbf{u}}_s) \cdot \tilde{\mathbf{t}}^* \, dA = 0 \, .$$

Though the motion is confined inside a rigid container, the interface between solid and fluid phases is deformable.

We use the ALE formulation to rewrite the balance principle on $\mathcal{D}_f$ by means of a virtual grid deformation γ mapping $\mathcal{D}_f$ into $\mathfrak{D}_f$.





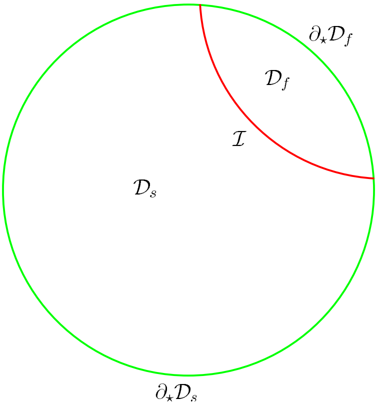
$\mathcal{D}$ $\gamma \longrightarrow$ $\mathfrak{D}$

Moving grid

The Moving Mesh Application Mode within Comsol allows us to avoid explicitly the pulling-back of the balance principle to $\mathcal{D}_f$, and extends γ from the boundary $\partial_\star \mathcal{D}_f \cup \mathcal{I}$ to the interior of the fluid domain.

- γ matches the deformation of the solid on $\mathfrak{D}_s$, $\partial_\star \mathfrak{D}_s$ and $\mathfrak{I}$;

- On the fluid side, γ matches the motion of $\partial_\star \mathfrak{D}_f$.



Boundary conditions

On the outermost boundary $\partial_\star \mathcal{D}$ we assign a saccadic motion as a rotation leaving the center of the eye x_0 fixed,

$$\mathsf{u}_\gamma = (\mathsf{R} - \mathsf{I})\,(\mathsf{x} - \mathsf{x}_0)\ .$$

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A no-slip condition is imposed in the weak form on $\partial_\star \mathcal{D}_f$

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$$\int_{\partial_\star \mathcal{D}_f} (\mathrm{v}_f - \dot{\mathrm{u}}_\gamma) \cdot \tilde{\mathrm{t}}_f \, dA = 0 \,,$$

while on $\partial_\star \mathcal{D}_s$

$$\int_{\partial_\star \mathcal{D}_s} (\mathrm{u}_s - \mathrm{u}_\gamma) \cdot \tilde{\mathrm{t}}_s \, dA = 0 \,.$$

Gel-like vitreous

The Mooney-Rivlin strain energy function for a plane deformation is given by

$$W(\mathsf{F}) = (c_{10} + c_{01})(I_1 - 3) \ ,$$

where $I_1 = \mathsf{F}\cdot\mathsf{F}$ is the first invariant of $\mathsf{C} := \mathsf{F}^{\mathsf{T}}\mathsf{F}$.

The response for the Piola-Kirchhoff stress tensor is

$$\mathsf{S}_s = 2\,(c_{10} + c_{01})\,\mathsf{F}$$

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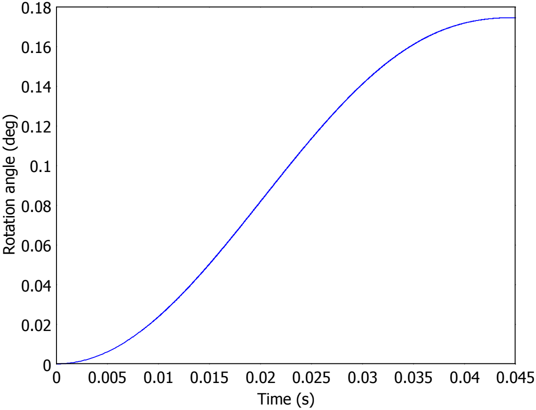
Combining the material response (deviatoric) with the reactive stress (spherical) we obtain the Cauchy stress tensor

$$\mathbf{T}_s = -p_s\mathbf{I} + 2\,(c_{10} + c_{01})\,\mathsf{F}\,\mathsf{F}^T\,.$$

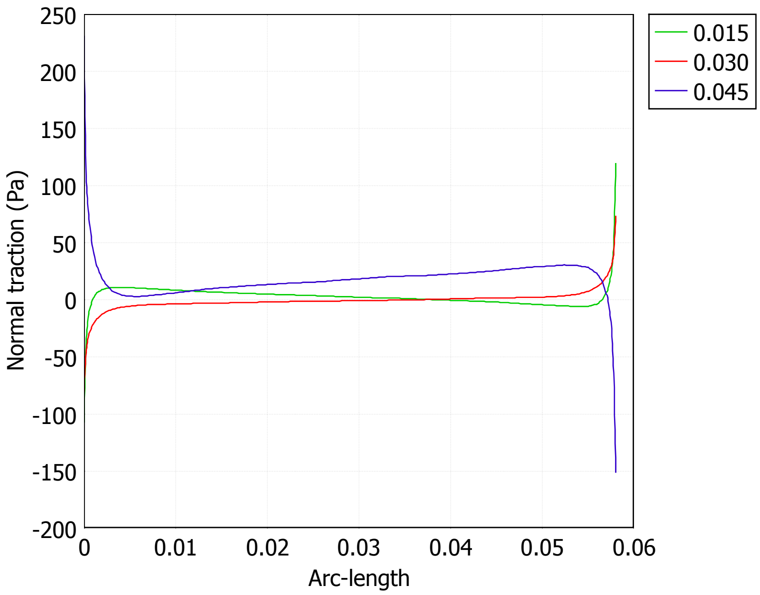
Numerical Simulations

A saccadic motion is a single rapid rotation of the eye which is usually approximated by a 5-degree polynomial law.

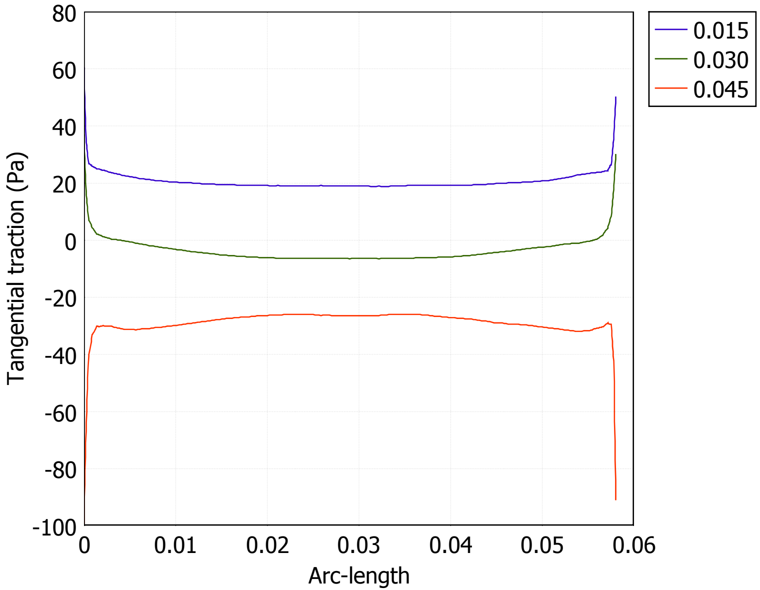


- Radius of the circular domain $R = 0.012\,\mathrm{m}$;
- Material constants for the solid $c_{01} = c_{10} = 50\,\mathrm{Pa}$;
- Fluid viscosity $\mu = 10^{-3}\,\mathrm{Pa\,s}$;
- Solid and fluid mass densities $\rho_s = \rho_f = 1000\,\mathrm{kg/m^3}$.

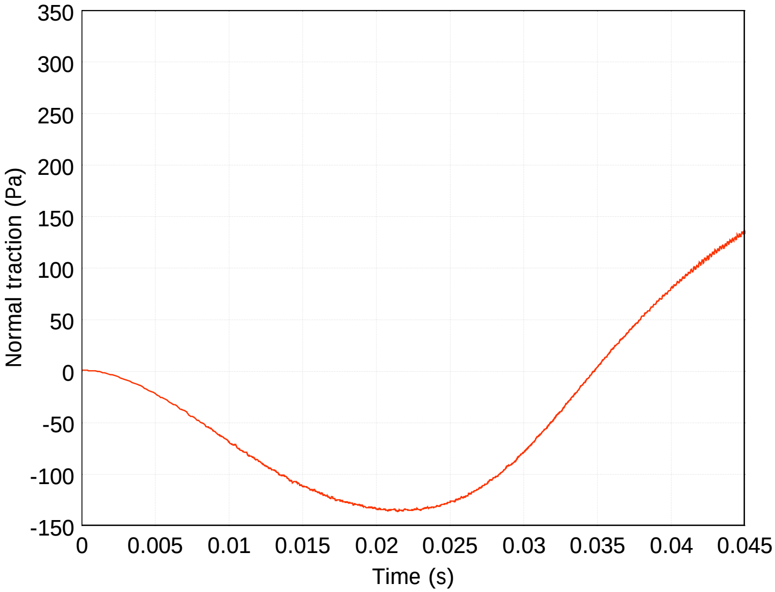
Results displayed are for a saccade of amplitude $A = 10^\circ$ and duration $T = 0.045\,\mathrm{s}$.



250
200
150
100
50
0
-50
-100
-150
-200
Normal traction (Pa)
0
0.01
0.02
0.03
0.04
0.05
0.06
Arc-length
0.015
0.030
0.045



Tangential traction (Pa)
80
60
40
20
0
-20
-40
-60
-80
-100
0
0.01
0.02
0.03
0.04
0.05
0.06
Arc-length
0.015
0.030
0.045



350
300
250
200
150
100
50
0
-50
-100
-150
Normal traction (Pa)
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
Time (s)

Future developments

- Build a 3D model;

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- Build a 3D model;
- Evaluate the influence of adhesion points on traction;

Future developments

- Build a 3D model;
- Evaluate the influence of adhesion points on traction;
- Perform a sensitivity analysis based on the variation of physical parameters.

Thanks for your attention