# A Model of Posterior Vitreous Detachment and Generation of Tractions on the Retina

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# Posterior Vitreous Detachment



#### Types of Posterior Vitreous Detachment

(from Kakehashi et al.,1997)

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Posterior Vitreous Detachment

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## Rhegmatogenous retinal detachment

- Synchysis (liquefaction)
- Syneresis (dehydration and shrinkage)
- Weakening of vitreoretinal adhesion
- Fast eye-rotation generated oscillations
- Quasi-static shrinkage of the vitreous

How to best evaluate the presence or absence of vitreous traction and how to quantitate the degree of vitreous traction is presently not known [Sebag, 1989]

#### The mechanical model



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# Balance principle (solid & fluid)

For any test velocity field w

$$\int_{\mathfrak{D}} \mathbf{b} \cdot \mathbf{w} \, dV + \int_{\partial \mathfrak{D}} \mathbf{t} \cdot \mathbf{w} \, dA - \int_{\mathfrak{D}} \mathbf{T} \cdot \nabla \mathbf{w} \, dV - \int_{\mathfrak{I}} \boldsymbol{\tau} \cdot \llbracket \mathbf{w} \rrbracket \, dA = 0$$

- $\boldsymbol{b} \quad \text{bulk force per unit volume in } \mathfrak{D}$
- **t** traction per unit area on  $\partial \mathfrak{D}$
- T Cauchy stress tensor
- au interface stress



# Balance principle (fluid)

For any test velocity field w

$$\int_{\mathfrak{D}_f} \mathbf{b}_f \cdot \mathbf{w}_f \, dV + \int_{\partial_\star \mathfrak{D}_f} \mathbf{t}_f^\star \cdot \mathbf{w}_f \, dA + \int_{\mathfrak{I}} \mathbf{t}_f \cdot \mathbf{w}_f \, dA$$
$$- \int_{\mathfrak{D}_f} \mathbf{T}_f \cdot \nabla \mathbf{w}_f \, dV = 0$$

- $\mathbf{b}_f$  bulk force per unit volume in  $\mathfrak{D}_f$
- $\mathbf{t}_f^\star$  traction per unit area on  $\partial_\star \mathfrak{D}_f$
- $\mathbf{t}_f$  traction per unit area on  $\mathfrak{I}$
- **T**<sub>f</sub> Cauchy stress tensor



For any test velocity field w

$$\int_{\mathfrak{D}_s} \mathbf{b}_s \cdot \mathbf{w}_s \, dV + \int_{\partial_\star \mathfrak{D}_s} \mathbf{t}_s^\star \cdot \mathbf{w}_s \, dA + \int_{\mathfrak{I}} \mathbf{t}_s \cdot \mathbf{w}_s \, dA$$
$$- \int_{\mathfrak{D}_s} \mathbf{T}_s \cdot \nabla \mathbf{w}_s \, dV - \int_{\mathfrak{I}} \mathbf{N}_m \cdot \nabla_m \, \mathbf{w}_s \, dA + \int_{\partial \mathfrak{I}} \mathbf{f}_m^\star \cdot \mathbf{w}_s \, d\ell = 0$$

- $\mathbf{b}_s$  bulk force per unit volume in  $\mathfrak{D}_s$
- $\mathbf{t}^{\star}_{s}$  traction per unit area on  $\partial_{\star}\mathfrak{D}_{s}$
- $\mathbf{t}_s$  traction per unit area on  $\Im$
- **T**<sub>s</sub> Cauchy stress tensor
- $\mathbf{N}_m$  Membrane stress tensor
- $\mathbf{f}_m^\star$  traction on  $\partial \mathfrak{I}$



Incompressibility

$$\det \mathbf{F} = 1$$
$$\operatorname{div} \mathbf{v_f} = 0$$

Viscoelastic solid

$$\mathbf{T}_{s} = \hat{\mathbf{T}}_{s}(\mathbf{F}) - p_{s}\mathbf{I} + 2\mu_{s} \operatorname{sym} \nabla \mathbf{v}_{s}$$

Newtonian fluid

$$\mathbf{T}_f = -p_f \mathbf{I} + 2\mu_f \operatorname{sym} \nabla \mathbf{v}_f$$

Incompressible Mooney-Rivlin material (plane strain) response

$$\hat{\mathbf{T}}(\mathbf{F}) = 2 c_0 \operatorname{dev}(\mathbf{F}\mathbf{F}^{\mathsf{T}})$$

# Boundary conditions (solid & fluid)





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Moving grid



$$\gamma_s = \phi_s$$
  
 $\Delta \mathbf{u}_f = \mathbf{0}$ 

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For any test velocity field  ${\boldsymbol w}$ 

$$\int_{\mathcal{D}_{s}} \mathbf{b}_{s} \cdot \mathbf{w}_{s} \, dV + \int_{\partial_{\star} \mathcal{D}_{s}} \mathbf{t}_{s}^{\star} \cdot \mathbf{w}_{s} \, dA + \int_{\mathcal{I}} \mathbf{t}_{s} \cdot \mathbf{w}_{s} \, dA$$
$$- \int_{\mathcal{D}_{s}} \mathbf{S}_{s} \cdot \nabla \mathbf{w}_{s} \, dV - \int_{\mathcal{I}} \mathbf{N}_{m} \cdot \nabla_{m} \, \mathbf{w}_{s} \, dA + \int_{\partial \mathcal{I}} \mathbf{f}_{m}^{\star} \cdot \mathbf{w}_{s} \, d\ell = 0$$

- $\mathbf{b}_s$  bulk force per unit volume in  $\mathcal{D}_s$
- $\mathbf{t}^{\star}_{s}$  traction per unit area on  $\partial_{\star}\mathcal{D}_{s}$
- $\mathbf{t}_s$  traction per unit area on  $\mathcal{I}$
- **S**<sub>s</sub> Piola stress tensor
- **N**<sub>m</sub> Membrane stress tensor
- $\mathbf{f}_m^\star$  traction on  $\partial \mathcal{I}$



For any test velocity field  $\boldsymbol{w}$ 

$$\int_{\mathcal{D}_f} \mathbf{b}_f \cdot \mathbf{w}_f \, dV + \int_{\partial_\star \mathcal{D}_f} \mathbf{t}_f^\star \cdot \mathbf{w}_f \, dA + \int_{\mathcal{I}} \mathbf{t}_f \cdot \mathbf{w}_f \, dA$$
$$- \int_{\mathcal{D}_f} \mathbf{S}_f \cdot \nabla \mathbf{w}_f \, dV = 0$$

$$\begin{split} \mathbf{b}_{f} &= -\rho_{f} \left( \dot{\mathbf{v}}_{f} + \nabla \mathbf{v}_{f} \mathbf{\Gamma}^{-1} (\mathbf{v}_{f} - \dot{\mathbf{u}}_{\gamma}) \right) \operatorname{det} \mathbf{\Gamma} \\ \mathbf{S}_{f} &= \left( -p_{f} \mathbf{I} + 2\mu \operatorname{sym}(\nabla \mathbf{v}_{f} \mathbf{\Gamma}^{-1}) \right) \mathbf{\Gamma}^{-\mathsf{T}} \operatorname{det} \mathbf{\Gamma} \end{split}$$

For any regular test scalar field  $\tilde{p}_s$  on  $\mathcal{D}_s$ 

$$\int_{\mathcal{D}_s} (\det \mathbf{F} - 1) \, \tilde{p}_s \, dV = 0$$

For any regular test scalar field  $\tilde{p}_f$  on  $\mathcal{D}_f$ 

$$\int_{\mathcal{D}_f} \operatorname{tr}(\nabla \mathbf{v}_f \mathbf{\Gamma}^{-1}) \, \tilde{p}_f \, dV = 0$$

Saccadic rotation

$$\mathbf{u}_{\gamma}(\mathbf{x}) = (\mathbf{R} - \mathbf{I}) \left(\mathbf{x} - \mathbf{x}_{0}
ight)$$

Perfect adhesion condition

$$\int_{\partial_{\star}\mathcal{D}_{f}}\left(\mathbf{u}_{s}-\mathbf{u}_{\gamma}\right)\cdot\tilde{\mathbf{t}}_{s}^{\star}\,dA=0$$

No-slip condition

$$\int_{\partial_{\star}\mathcal{D}_{f}} (\mathbf{v}_{f} - \dot{\mathbf{u}}_{\gamma}) \cdot \tilde{\mathbf{t}}_{f}^{\star} \, dA = 0$$

# Interface conditions (weak form)

Velocity field continuity

$$\int_{\mathfrak{I}} (\mathbf{v}_f - \dot{\mathbf{u}}_s) \cdot \tilde{\boldsymbol{\tau}} \, dA = 0$$

From the interface integrals

$$\int_{\mathfrak{I}} \mathbf{t}_{s} \cdot \mathbf{w}_{s} \, dA + \int_{\mathfrak{I}} \mathbf{t}_{f} \cdot \mathbf{w}_{f} \, dA$$

where

$$\mathbf{t}_f = -\mathbf{t}_s = \boldsymbol{\tau}$$

Summarizing

$$\int_{\Im} \boldsymbol{\tau} \cdot (\mathbf{w}_f - \mathbf{w}_s) + (\mathbf{v}_f - \dot{\mathbf{u}}_s) \cdot \tilde{\boldsymbol{\tau}} \, dA$$

# Saccadic movement



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  $\begin{array}{ll} {\rm Radius} & R=0.012\,{\rm m} \\ \\ {\rm Fluid\ viscosity} & \mu_f=10^{-3}\,{\rm Pa}\times{\rm s} \\ \\ {\rm Mass\ density} & \rho_s=\rho_f=1000\,{\rm kg/m^3} \end{array}$ 

























$$\mu_{s}=0.5$$
 Pas













$$\mu_{s}=0.5$$
 Pas












Preceding results from:

R. Repetto, A. Tatone, A. Testa, E. Colangeli, *Biomech Model Mechanobiol*, 2010

# A shrinkage and adhesion model

With ageing the vitreous humor undergoes the processes of synchysis (liquefaction) and syncresis (dehydration and shrinkage)



Age-related vitreous liquefaction and PVD. Pockets of liquid appear within the central vitreous that gradually coalesce. There is a concurrent weakening of postoral vitreoretinal adhesion. Eventually, this can progress to PVD, where the liquid vitreous dissects the residual cortical gel away from the ILL on the inner surface of the retina as far anteriorly as the posterior border of the vitreous base



Schematic representation of the cooperation between two networks responsible for the gel structure of the vitreous. A network of collagen fibrils maintains the gel state and provides the vitreous with tensile strength. A network of hyaluronan fills the spaces between these collagen fibrils and provides a swelling pressure to inflate the gel.



(a) The collagen fibrils (thick grey lines) form an extended network of small bundles. Within each bundle, the collagen fibrils are both connected together and spaced apart by type IX collagen chains.

(b) With ageing the loss of the type IX collagen from the fibril surfaces combined with an increased surface exposure of type II collagen results in collagen fibrillar aggregation.



Diagram representing the postbasal vitreoretinal junction. Weakening of the adhesion at this interface predisposes to posterior vitreous detachment. Vitreoretinal adhesion may be dependent upon intermediary molecules acting as a 'molecular glue' and linking the cortical vitreous collagen fibrils to components of ILL. It is possible that opticin, because it binds to both vitreous collagen fibrils and HS proteoglycans in the ILL, contributes towards this 'molecular glue'.

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## Kröner-Lee decomposition of the deformation gradient $abla \mathbf{p}$



For any test velocity field  $(\mathbf{w}, \mathbf{V})$ 

$$\int_{\partial \mathcal{D}} \mathbf{t} \cdot \mathbf{w} \, dA - \int_{\mathcal{D}} \mathbf{S} \cdot \nabla \mathbf{w} \, dV + \int_{\mathcal{D}} (\mathbf{Q} - \mathbf{A}) \cdot \mathbf{V} \, dV = 0$$

- t traction per unit reference area
- **S** reference Piola stress tensor
- A inner remodeling couple per unit reference volume
- **Q** outer remodeling couple per unit reference volume

$$\begin{aligned} &\operatorname{div} \mathbf{S} + \mathbf{b} = 0 & & \operatorname{in} & \mathcal{D} \\ & & \mathbf{t} = \mathbf{S} \, \mathbf{n} & & & \operatorname{on} & \partial \mathcal{D} \\ & & & & \mathbf{Q} - \mathbf{A} = 0 & & & \operatorname{in} & \mathcal{D} \end{aligned}$$

Incompressibility

$$\det \mathbf{F} = 1$$

Spherical shrinkage

$$\mathbf{G} = g\mathbf{I}$$

Viscoelastic solid

$$\mathbf{T} = \hat{\mathbf{T}}(\mathbf{F}) - \rho \mathbf{I} + 2\mu \operatorname{sym} \nabla \mathbf{v}$$

Incompressible Mooney-Rivlin material (plane strain) response

$$\hat{\mathsf{T}}(\mathsf{F}) = 2 c_0 \operatorname{dev}(\mathsf{F}\mathsf{F}^{\mathsf{T}})$$

Eshelby tensor

$$\mathbf{A} = -J(\mathbf{F}^{\mathsf{T}} \mathbb{S} - \varphi(\mathbf{F})\mathbf{I})$$



Diagram representing the postbasal vitreoretinal junction. Weakening of the adhesion at this interface predisposes to posterior vitreous detachment. Vitreoretinal adhesion may be dependent upon intermediary molecules acting as a "molecular glue" and linking the cortical vitreous collagen fibrils to components of ILL. It is possible that opticin, because it binds to both vitreous collagen fibrils and HS proteoglycans in the ILL, contributes towards this "molecular glue".

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Boundary traction

$$\mathbf{t} = \mathbf{t}_{ad} + \mathbf{t}_{rep}$$

Adhesive force

$$\mathbf{t}_{ad}(\mathbf{x},t) = -k_{ad} \frac{d_{ad}(\mathbf{x},t)}{u_0} e^{-\left(\frac{d_{ad}(\mathbf{x},t)}{u_0}\right)^2} \frac{\mathbf{u}(\mathbf{x},t)}{d_{ad}(\mathbf{x},t)}$$

$$d_{ad}(\mathbf{x},t) = \|\mathbf{u}(\mathbf{x},t)\|$$

Adhesion force potential

$$\phi_{ad}(d_{ad}(\mathbf{x},t)) = \frac{1}{2} k_{ad} u_0 \left( e^{-\left(\frac{d_{ad}(\mathbf{x},t)}{u_0}\right)^2} - 1 \right)$$

Repulsive force

$$\mathbf{t}_{rep}(\mathbf{x},t) = k_{rep} \left( \left( \frac{d_0}{d_{rep}(\mathbf{x},t)} \right)^2 - \left( \frac{d_0}{d_{rep}(\mathbf{x},t)} \right)^3 \right) \frac{\mathbf{r}(\mathbf{x},t)}{\|\mathbf{r}(\mathbf{x},t)\|}$$

$$\mathbf{r}(\mathbf{x}, t) = \mathbf{p}(\mathbf{x}, t) - \mathbf{x}_0$$
$$d_{rep}(\mathbf{x}, t) = R - \|\mathbf{r}(\mathbf{x}, t)\| + d_0$$

Repulsive force potential

$$\phi_{rep}(d_{rep}(\mathbf{x},t)) = rac{1}{2} k_{rep} \ d_0 \left(1 - rac{d_0}{d_{rep}(\mathbf{x},t)}
ight)^2$$



- Adhesive force **t**<sub>ad</sub> (red line)
- Repulsive force **t**<sub>rep</sub> (blue line)



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- Adhesive force potentials  $\phi_{ad}$  (red line)
- Repulsive force potentials  $\phi_{rep}$  (blue line)

# Case-02 (Strong adhesion) Slip pulses









Strain energy 1.8 x10<sup>-3</sup> 1.6 1.4 1.2 1 0.8 0.6 0.4 0.2 0 0 2 4 8 10 12 14 16 18 20 6 t

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Strain energy

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- The shrinkage process induces an additional contribution to the traction, which is independent of the eye movements;
- The PVD shapes obtained through the simulations look close to the observed PVD shapes, even with a simple spherical shrinkage.