

A mechanical model for dynamical tractions on the retina in the presence of posterior vitreous detachment

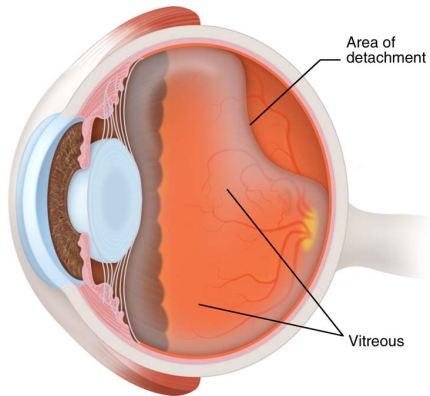
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L'Aquila, Italy

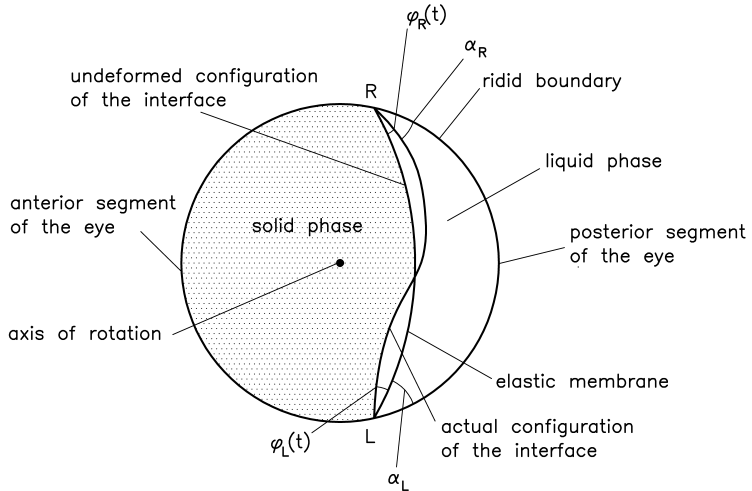
Euromech Colloquium 518 – Biomechanics of the Eye
Imperial College London, 26-28 July 2010

Joint work with:
Rodolfo Repetto, Alessandro Testa and Elisa Colangeli
[*Biomech Model Mechanobiol*, 2010]

Posterior vitreous detachment



Posterior vitreous detachment



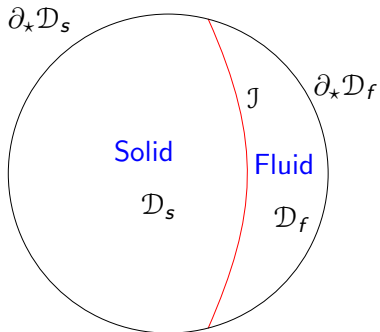
Posterior vitreous detachment

Tractions on the retina can be induced by

- eye-rotation generated oscillations
- quasi-static shrinkage of the vitreous

How to best evaluate the presence or absence of vitreous traction and how to quantitate the degree of vitreous traction is presently not known [Sebag-1989]

The mechanical model

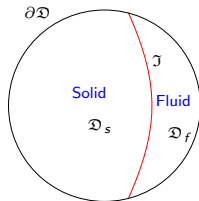


Balance principle (solid & fluid)

For any test velocity field \mathbf{w}

$$\int_{\mathcal{D}} \mathbf{b} \cdot \mathbf{w} dV + \int_{\partial\mathcal{D}} \mathbf{t} \cdot \mathbf{w} dA - \int_{\mathcal{D}} \mathbf{T} \cdot \text{grad } \mathbf{w} dV = 0$$

- b** bulk force per unit volume on \mathcal{D}
- t** traction per unit area on $\partial\mathcal{D}$
- T** Cauchy stress tensor

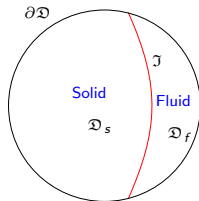


Balance principle (solid & fluid)

For any test velocity field \mathbf{w}

$$\int_{\mathcal{D}} \mathbf{b} \cdot \mathbf{w} dV + \int_{\partial\mathcal{D}} \mathbf{t} \cdot \mathbf{w} dA - \int_{\mathcal{D}} \mathbf{T} \cdot \text{grad } \mathbf{w} dV - \int_{\mathcal{I}} \boldsymbol{\tau} \cdot [\mathbf{w}] dA = 0$$

- b** bulk force per unit volume in \mathcal{D}
- t** traction per unit area on $\partial\mathcal{D}$
- T** Cauchy stress tensor
- $\boldsymbol{\tau}$ interface stress



Balance principle (fluid)

For any test velocity field \mathbf{w}

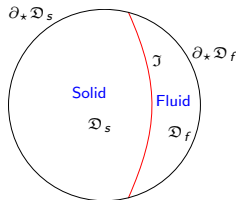
$$\int_{\mathcal{D}_f} \mathbf{b}_f \cdot \mathbf{w}_f dV + \int_{\partial_* \mathcal{D}_f} \mathbf{t}_f^* \cdot \mathbf{w}_f dA + \int_{\mathcal{I}} \mathbf{t}_f \cdot \mathbf{w}_f dA - \int_{\mathcal{D}_f} \mathbf{T}_f \cdot \text{grad } \mathbf{w}_f dV = 0$$

\mathbf{b}_f bulk force per unit volume in \mathcal{D}_f

\mathbf{t}_f^* traction per unit area on $\partial_* \mathcal{D}_f$

\mathbf{t}_f traction per unit area on \mathcal{I}

\mathbf{T}_f Cauchy stress tensor



Balance principle (solid)

For any test velocity field \mathbf{w}

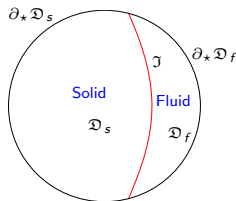
$$\int_{\mathcal{D}_s} \mathbf{b}_s \cdot \mathbf{w}_s dV + \int_{\partial_* \mathcal{D}_s} \mathbf{t}_s^* \cdot \mathbf{w}_s dA + \int_{\mathcal{I}} \mathbf{t}_s \cdot \mathbf{w}_s dA - \int_{\mathcal{D}_s} \mathbf{T}_s \cdot \text{grad } \mathbf{w}_s dV = 0$$

\mathbf{b}_s bulk force per unit volume in \mathcal{D}_s

\mathbf{t}_s^* traction per unit area on $\partial_* \mathcal{D}_s$

\mathbf{t}_s traction per unit area on \mathcal{I}

\mathbf{T}_s Cauchy stress tensor



Balance principle (solid & boundary membrane)

For any test velocity field \mathbf{w}

$$\int_{\mathcal{D}_s} \mathbf{b}_s \cdot \mathbf{w}_s dV + \int_{\partial_* \mathcal{D}_s} \mathbf{t}_s^* \cdot \mathbf{w}_s dA + \int_{\mathcal{I}} \mathbf{t}_s \cdot \mathbf{w}_s dA \\ - \int_{\mathcal{D}_s} \mathbf{T}_s \cdot \text{grad } \mathbf{w}_s dV - \int_{\mathcal{I}} \mathbf{N}_m \cdot \text{grad}_m \mathbf{w}_s dA + \int_{\partial \mathcal{I}} \mathbf{f}_m^* \cdot \mathbf{w}_s dl = 0$$

\mathbf{b}_s bulk force per unit volume in \mathcal{D}_s

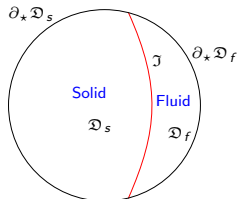
\mathbf{t}_s^* traction per unit area on $\partial_* \mathcal{D}_s$

\mathbf{t}_s traction per unit area on \mathcal{I}

\mathbf{T}_s Cauchy stress tensor

\mathbf{N}_m Membrane stress tensor

\mathbf{f}_m^* traction on $\partial \mathcal{I}$



Balance equations (solid & boundary membrane)

$$\operatorname{div} \mathbf{T}_s + \mathbf{b}_s = 0$$

$$\text{in } \mathcal{D}_s$$

$$\mathbf{t}_s^* = \mathbf{T}_s \mathbf{n}$$

$$\text{on } \partial_* \mathcal{D}_s$$

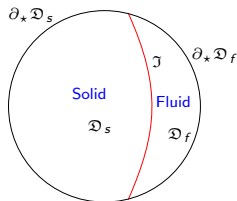
$$\operatorname{div}_m \mathbf{N}_m + \mathbf{t}_s = \mathbf{T}_s \mathbf{n}$$

$$\text{on } \mathcal{I}$$

$$\mathbf{f}_m^* = \mathbf{N}_m \mathbf{n}_m$$

$$\text{on } \partial \mathcal{I}$$

\mathbf{f}_m^* traction on $\partial \mathcal{I}$
 \mathbf{N}_m membrane stress tensor
 \mathbf{n}_m unit vector normal to $\partial \mathcal{I}$



Balance equations (fluid)

$$\operatorname{div} \mathbf{T}_f + \mathbf{b}_f = 0$$

in \mathcal{D}_f

$$\mathbf{t}_f^* = \mathbf{T}_f \mathbf{n}$$

on $\partial_* \mathcal{D}_f$

$$\mathbf{t}_f = \mathbf{T}_f \mathbf{n}$$

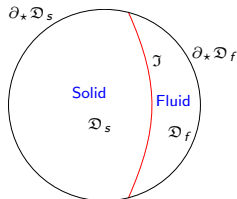
on \mathcal{I}

\mathbf{b}_f bulk force per unit volume in \mathcal{D}_f

\mathbf{t}_f^* traction per unit area on $\partial_* \mathcal{D}_f$

\mathbf{t}_f traction per unit area on \mathcal{I}

\mathbf{T}_f Cauchy stress tensor



Material characterization

Incompressibility

$$\det F = 1$$

$$\operatorname{div} \mathbf{v}_f = 0$$

Viscoelastic solid

$$\mathbf{T}_s = -p_s \mathbf{I} + \hat{\mathbf{T}}_s(F) + 2\mu_s \operatorname{sym} \operatorname{grad} \mathbf{v}_s$$

Newtonian fluid

$$\mathbf{T}_f = -p_f \mathbf{I} + 2\mu_f \operatorname{sym} \operatorname{grad} \mathbf{v}_f$$

Neo-Hookean response

$$\hat{\mathbf{T}}_s(F) = 2c_0 \operatorname{dev}(FF^T)$$

$$\mathbf{b}_s = -\rho_s \dot{\mathbf{v}}_s$$

$$\mathbf{b}_f = -\rho_f (\dot{\mathbf{v}}_f + (\text{grad } \mathbf{v}_f) \mathbf{v}_f)$$

Balance equation (fluid)

$$\operatorname{div} \mathbf{T}_f + \mathbf{b}_f = 0$$

$$\mathbf{b}_f = -\rho_f (\dot{\mathbf{v}}_f + (\operatorname{grad} \mathbf{v}_f) \mathbf{v}_f)$$

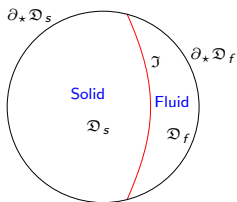
$$\mathbf{T}_f = -p_f \mathbf{I} + 2\mu_f \operatorname{sym} \operatorname{grad} \mathbf{v}_f$$

Navier-Stokes equation on \mathcal{D}_f

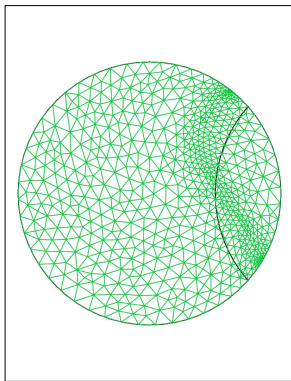
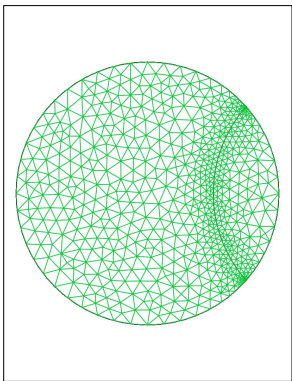
$$-\rho_f (\dot{\mathbf{v}}_f + (\operatorname{grad} \mathbf{v}_f) \mathbf{v}_f) - \operatorname{grad} p_f + \mu_f \Delta \mathbf{v}_f = 0$$

Boundary conditions (solid & fluid)

$$\begin{aligned}\boldsymbol{\tau} &= \mathbf{t}_f = -\mathbf{t}_s && \text{on } \mathcal{I} \\ \mathbf{v}_s &= \mathbf{v}_f && \text{on } \mathcal{I} \\ \mathbf{u}_s &= \bar{\mathbf{u}} && \text{on } \partial_* \mathcal{D}_s \\ \mathbf{v}_f &= \dot{\bar{\mathbf{u}}} && \text{on } \partial_* \mathcal{D}_f\end{aligned}$$



$$\gamma : \mathcal{D} \rightarrow \mathcal{D}$$



$$\gamma_s = \phi_s$$

$$\Delta \gamma_f = 0$$

Balance principle for the solid in the *reference shape*

For any test velocity field w

$$\int_{\mathcal{D}_s} b_s \cdot w_s \, dV + \int_{\partial_\star \mathcal{D}_s} t_s^\star \cdot w_s \, dA + \int_{\mathcal{J}} t_s \cdot w_s \, dA \\ - \int_{\mathcal{D}_s} S_s \cdot \text{grad} w_s \, dV - \int_{\mathcal{J}} N_m \cdot \text{grad}_m w_s \, dA + \int_{\partial \mathcal{J}} f_m^\star \cdot w_s \, d\ell = 0$$

b_s bulk force per unit volume in \mathcal{D}_s

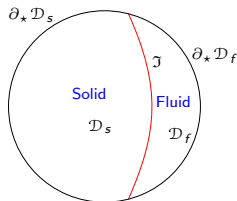
t_s^\star traction per unit area on $\partial_\star \mathcal{D}_s$

t_s traction per unit area on \mathcal{J}

S_s Piola stress tensor

N_m Membrane stress tensor

f_m^\star traction on $\partial \mathcal{J}$



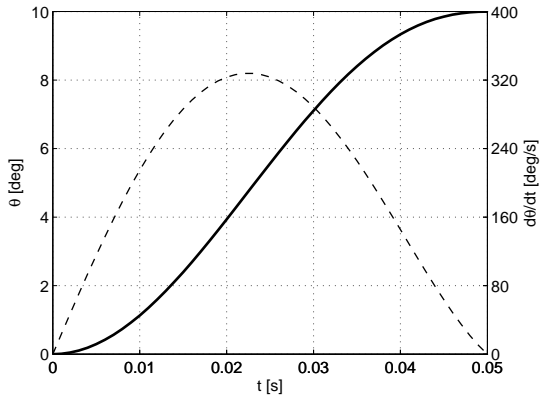
Balance equation for the fluid in the *reference shape*

$$\operatorname{div} S_f + \mathbf{b}_f = 0$$

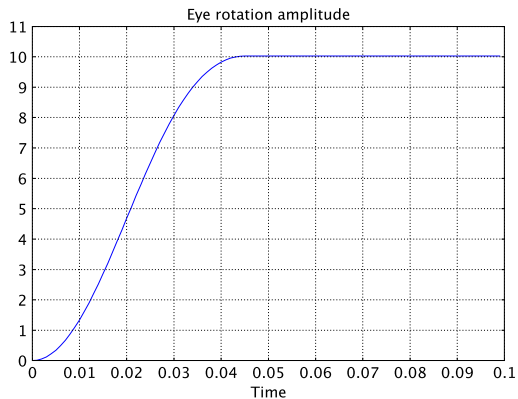
$$\mathbf{b}_f = -\rho_f (\dot{\mathbf{v}}_f + \operatorname{grad} \mathbf{v}_f \Gamma^{-1} (\mathbf{v}_f - \dot{\mathbf{u}}_\gamma)) \det \Gamma$$

$$S_f = (-p_f \mathbf{I} + 2\mu \operatorname{sym}(\operatorname{grad} \mathbf{v}_f \Gamma^{-1})) \Gamma^{-T} \det \Gamma$$

Saccadic movement



Saccadic movement



Constitutive parameters

Nickerson et al. (2008)	G'	10 Pa
	G''	3.9 Pa
Swindle et al. (2008)	G'	3.46 ± 0.30 Pa
	G''	0.71 ± 0.12 Pa

(porcine samples)

Constitutive parameters

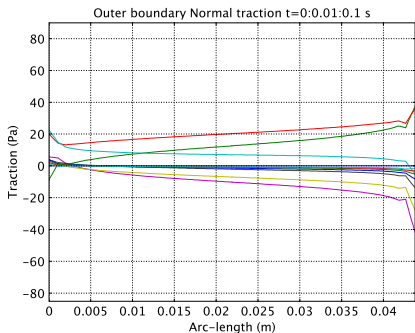
Radius $R = 0.012 \text{ m}$

Fluid viscosity $\mu_f = 10^{-3} \text{ Pa} \times \text{s}$

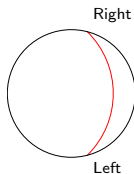
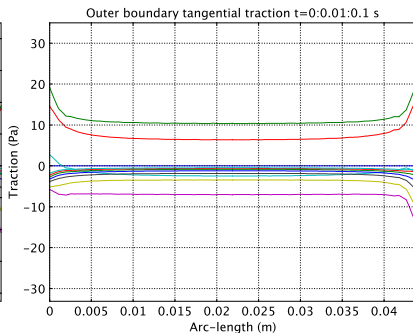
Mass density $\rho_s = \rho_f = 1000 \text{ kg/m}^3$

Boundary traction
for different values of the **solid elastic modulus**

Boundary traction (vs. solid elastic modulus)



[c3_case_003]

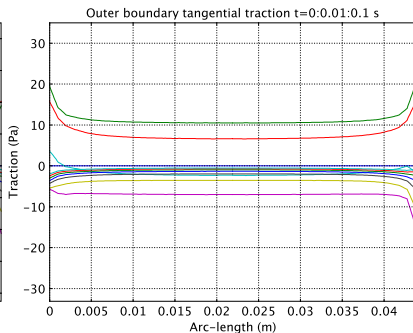
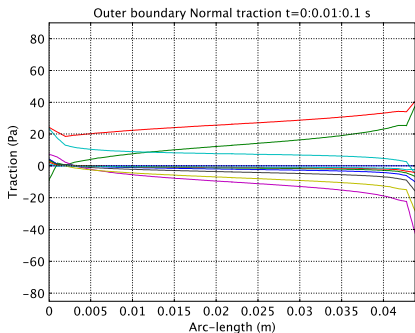


$$c_0 = 2 \quad \text{Pa}$$

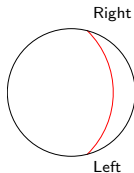
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. solid elastic modulus)



[c3_case_006]

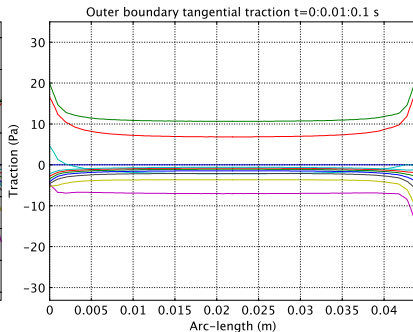
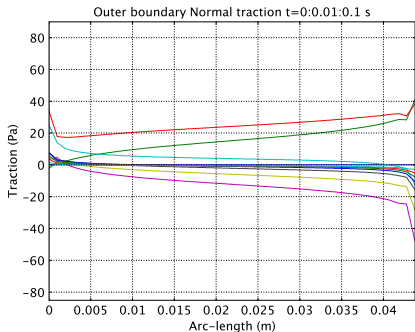


$$c_0 = 3 \quad \text{Pa}$$

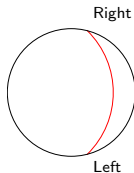
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. solid elastic modulus)



[c3_case_007]

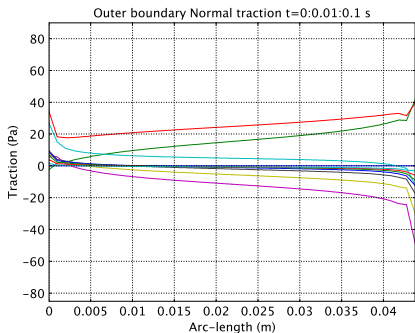


$$c_0 = 4 \quad \text{Pa}$$

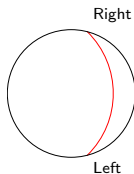
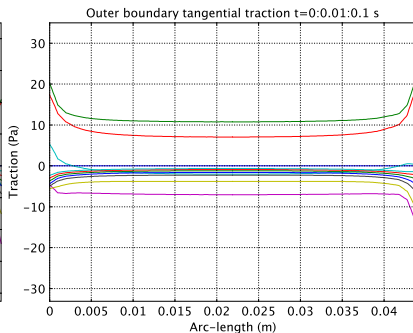
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. solid elastic modulus)



[c3_case_008]

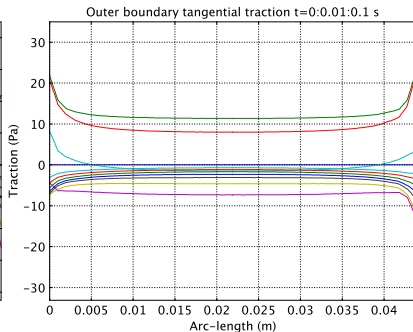
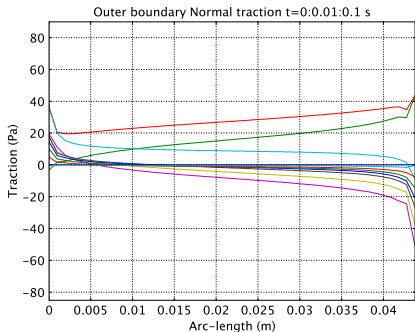


$$c_0 = 5 \quad \text{Pa}$$

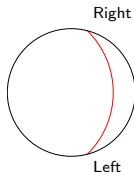
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. solid elastic modulus)



[c3_case_008_b]

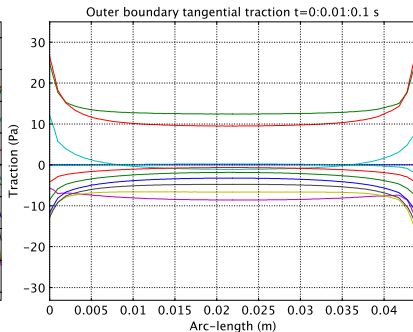
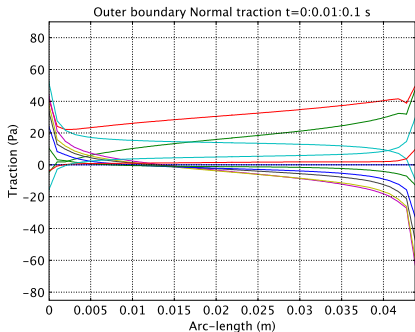


$$c_0 = 10 \text{ Pa}$$

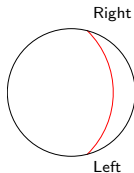
$$m_0 = 20 \text{ Pa}$$

$$\mu_s = 0.5 \text{ Pa s}$$

Boundary traction (vs. solid elastic modulus)



[c3_case_008_t]

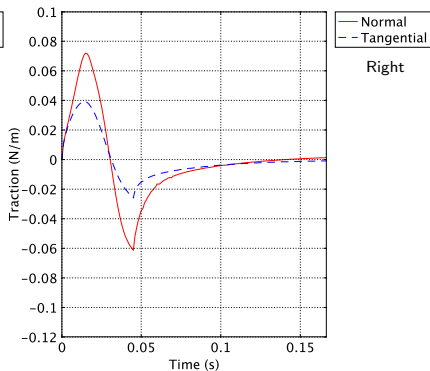
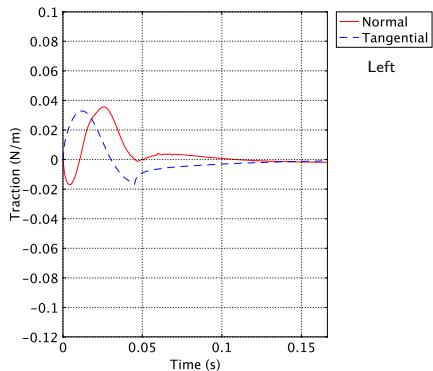


$$c_0 = 20 \quad \text{Pa}$$

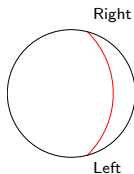
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. solid elastic modulus)



[c3_case_003]

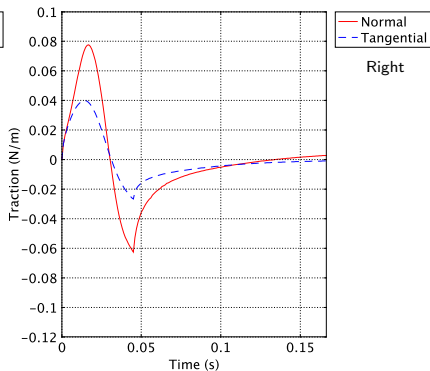
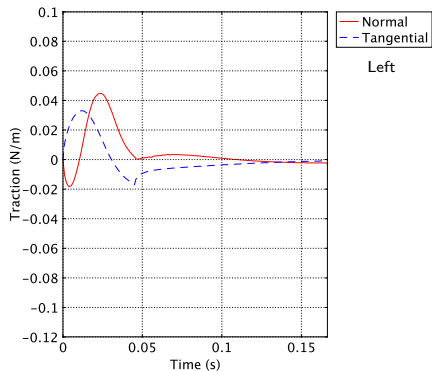


$$c_0 = 2 \quad \text{Pa}$$

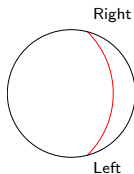
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. solid elastic modulus)



[c3_case_006]

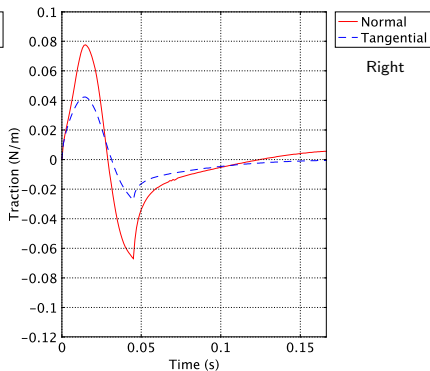
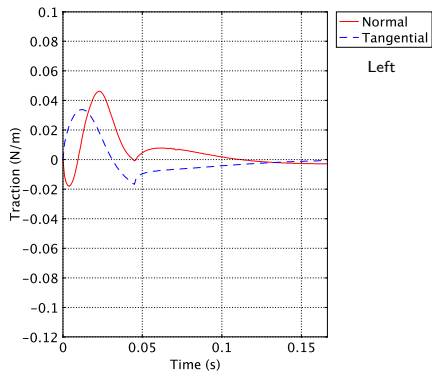


$$c_0 = 3 \quad \text{Pa}$$

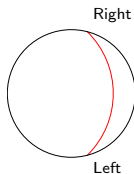
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. solid elastic modulus)



[c3.case_007]

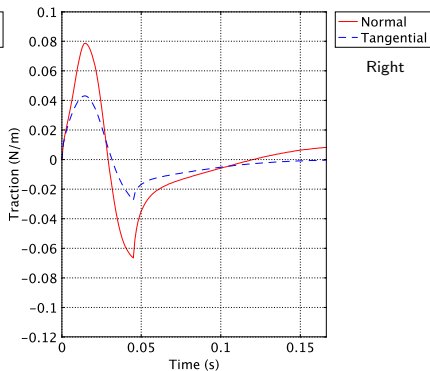
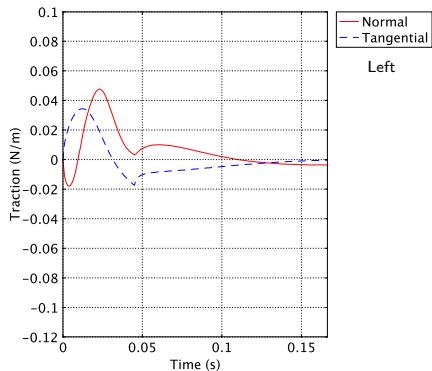


$$c_0 = 4 \quad \text{Pa}$$

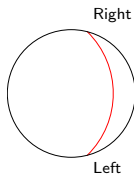
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. solid elastic modulus)



[c3.case_008]

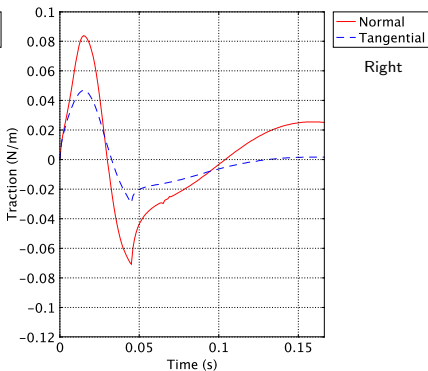
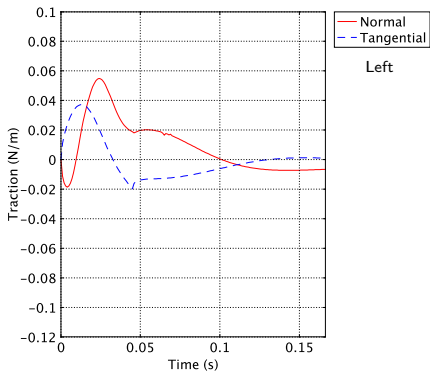


$$c_0 = 5 \quad \text{Pa}$$

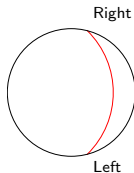
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. solid elastic modulus)



[c3_case_008_b]

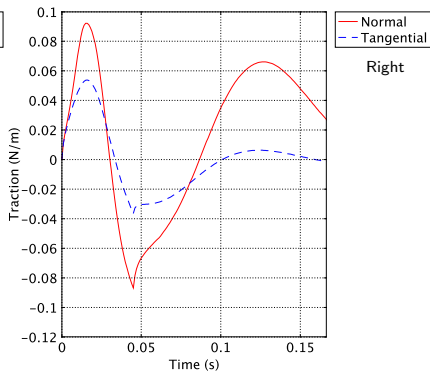
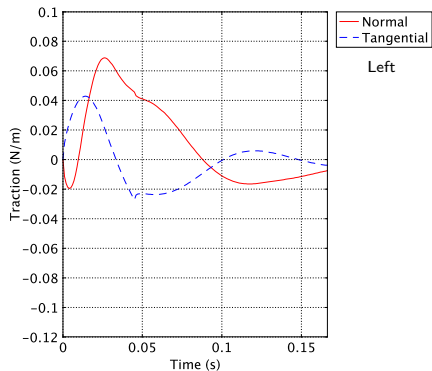


$$c_0 = 10 \text{ Pa}$$

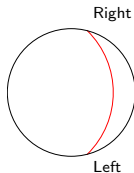
$$m_0 = 20 \text{ Pa}$$

$$\mu_s = 0.5 \text{ Pa s}$$

Boundary traction (vs. solid elastic modulus)



[c3.case_008.t]

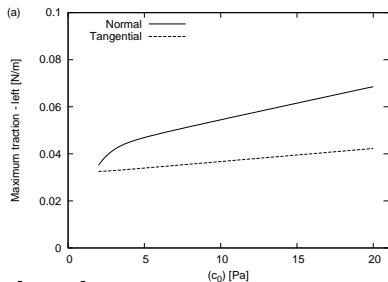


$$c_0 = 20 \quad \text{Pa}$$

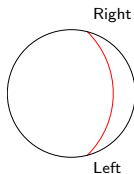
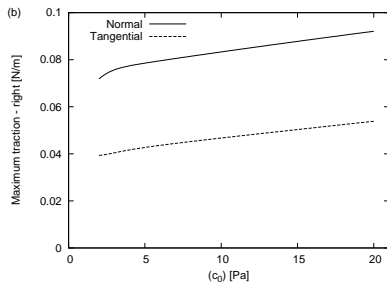
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. solid elastic modulus)



[c3_case]



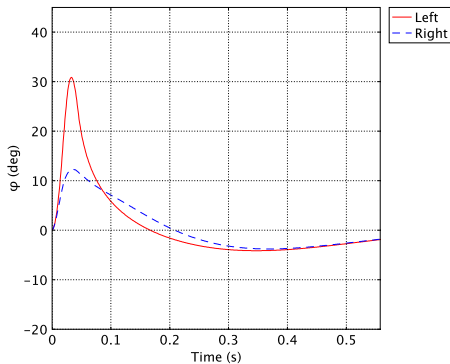
$$c_0 = 2 \div 20 \quad \text{Pa}$$

$$m_0 = 20 \quad \text{Pa}$$

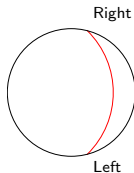
$$\mu_s = 0.5 \quad \text{Pa s}$$

Membrane oscillations
for different values of the **solid elastic modulus**

Membrane oscillations (vs. solid elastic modulus)

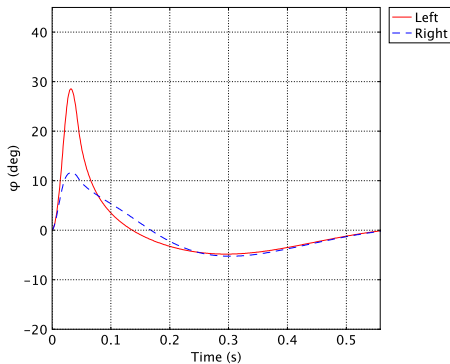


[c3_case_003]

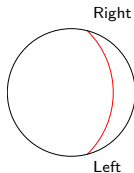


$$c_0 = 2 \quad \text{Pa}$$
$$m_0 = 20 \quad \text{Pa}$$
$$\mu_s = 0.5 \quad \text{Pa s}$$

Membrane oscillations (vs. solid elastic modulus)

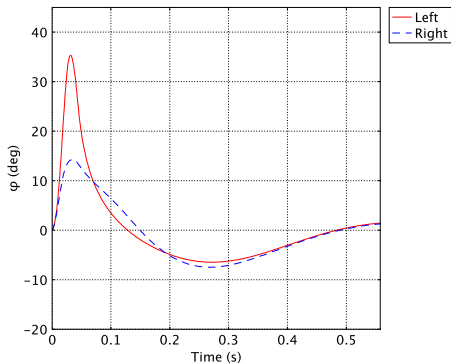


[c3_case_006]

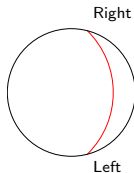


$$c_0 = 3 \quad \text{Pa}$$
$$m_0 = 20 \quad \text{Pa}$$
$$\mu_s = 0.5 \quad \text{Pa s}$$

Membrane oscillations (vs. solid elastic modulus)

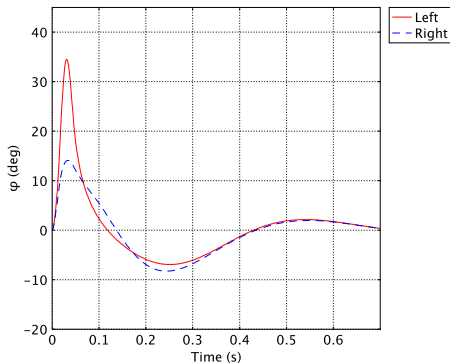


[c3_case_007]

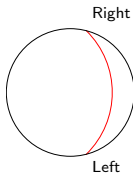


$$c_0 = 4 \quad \text{Pa}$$
$$m_0 = 20 \quad \text{Pa}$$
$$\mu_s = 0.5 \quad \text{Pa s}$$

Membrane oscillations (vs. solid elastic modulus)



[c3_case_008]

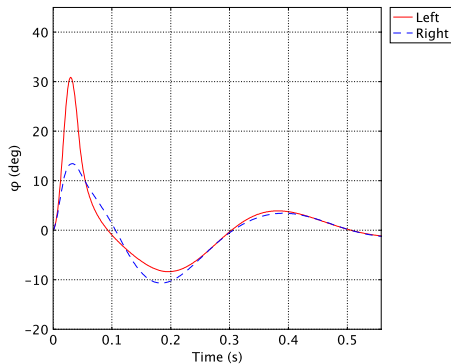


$$c_0 = 5 \quad \text{Pa}$$

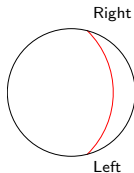
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Membrane oscillations (vs. solid elastic modulus)

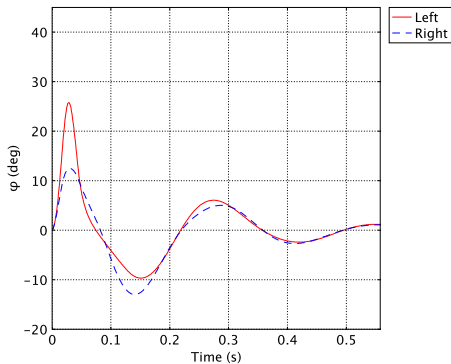


[c3_case_008_b]

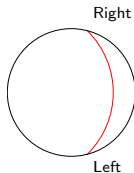


$$c_0 = 10 \text{ Pa}$$
$$m_0 = 20 \text{ Pa}$$
$$\mu_s = 0.5 \text{ Pa s}$$

Membrane oscillations (vs. solid elastic modulus)



[c3_case_008.t]



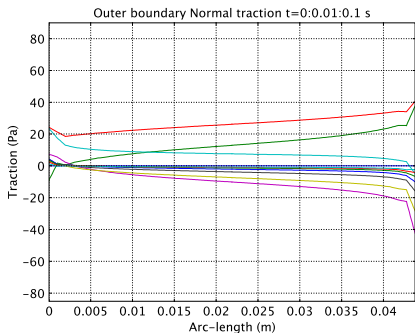
$$c_0 = 20 \text{ Pa}$$

$$m_0 = 20 \text{ Pa}$$

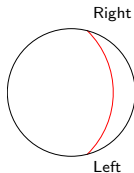
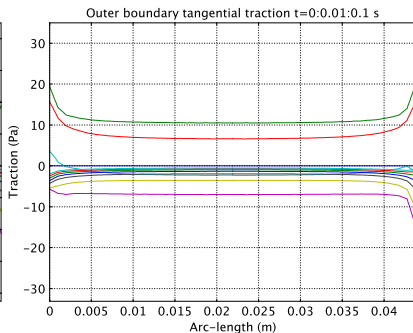
$$\mu_s = 0.5 \text{ Pa s}$$

Boundary traction
for different values of the **solid viscosity**

Boundary traction (vs. solid viscosity)



[c3_case_006]

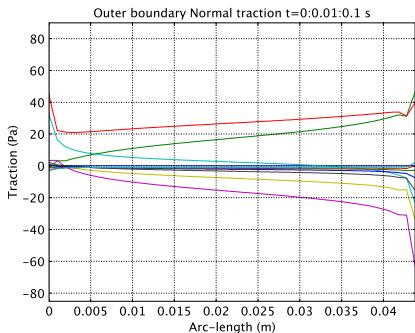


$$c_0 = 3 \quad \text{Pa}$$

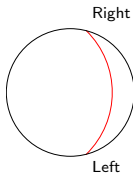
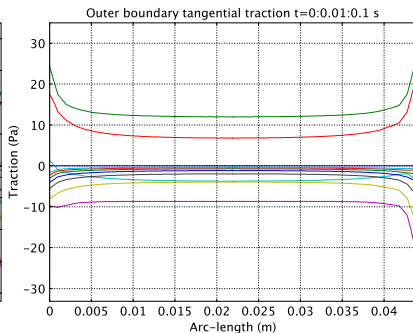
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.50 \quad \text{Pa s}$$

Boundary traction (vs. solid viscosity)



[c3_case_010]

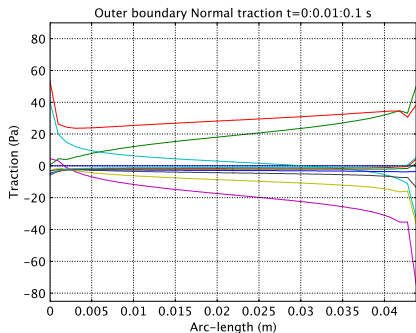


$$c_0 = 3 \quad \text{Pa}$$

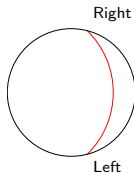
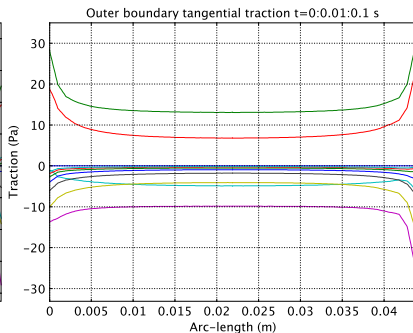
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.75 \quad \text{Pa s}$$

Boundary traction (vs. solid viscosity)



[c3_case_011]

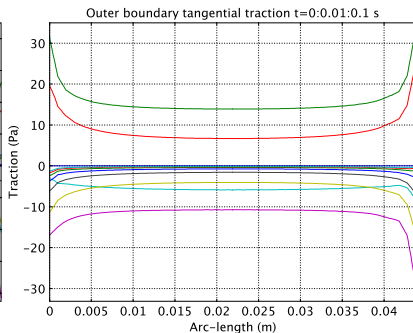
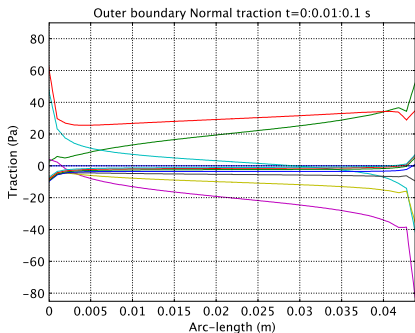


$$c_0 = 3 \quad \text{Pa}$$

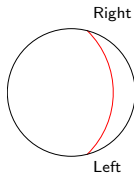
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 1.00 \quad \text{Pa s}$$

Boundary traction (vs. solid viscosity)



[c3_case_011_b]

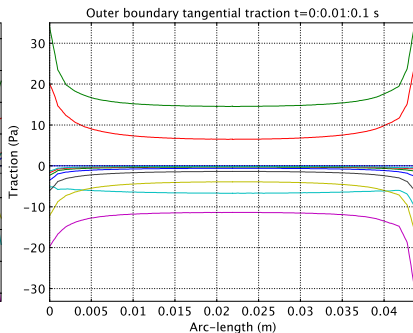
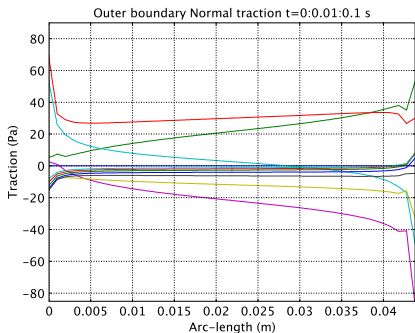


$$c_0 = 3 \quad \text{Pa}$$

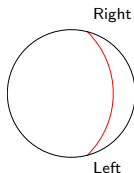
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 1.25 \quad \text{Pa s}$$

Boundary traction (vs. solid viscosity)



[c3_case_011- τ]

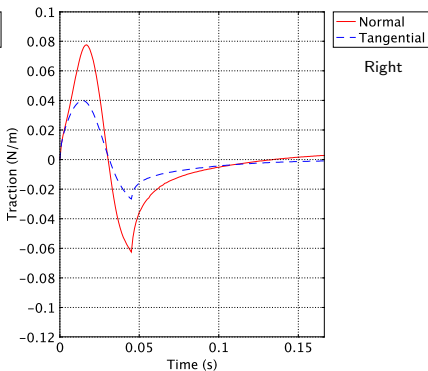
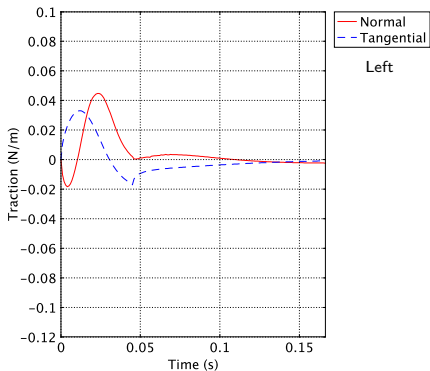


$$c_0 = 3 \quad \text{Pa}$$

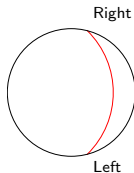
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 1.50 \quad \text{Pa s}$$

Boundary traction (vs. solid viscosity)



[c3_case_006]

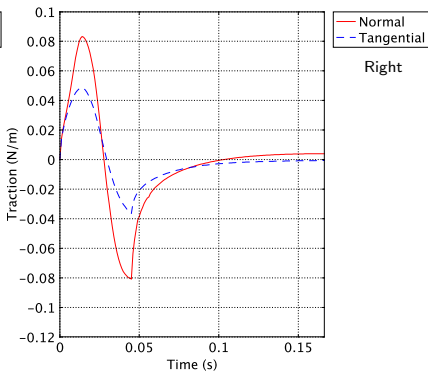
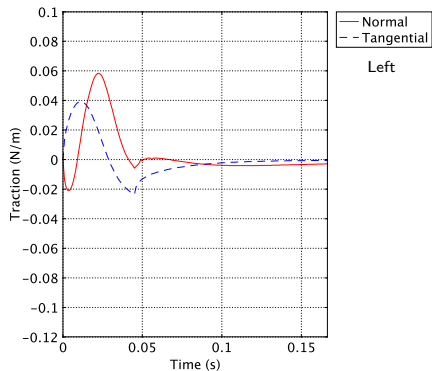


$$c_0 = 3 \quad \text{Pa}$$

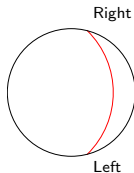
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.50 \quad \text{Pa s}$$

Boundary traction (vs. solid viscosity)



[c3.case_010]

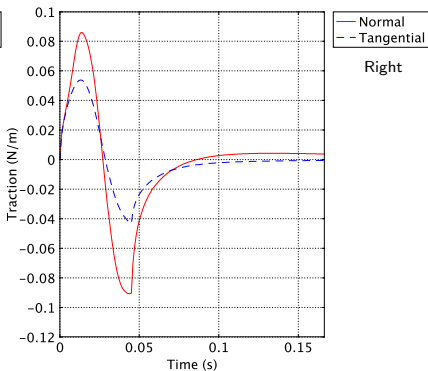
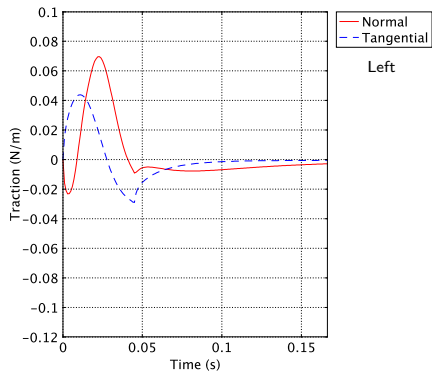


$$c_0 = 3 \quad \text{Pa}$$

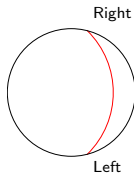
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.75 \quad \text{Pa s}$$

Boundary traction (vs. solid viscosity)



[c3_case_011]

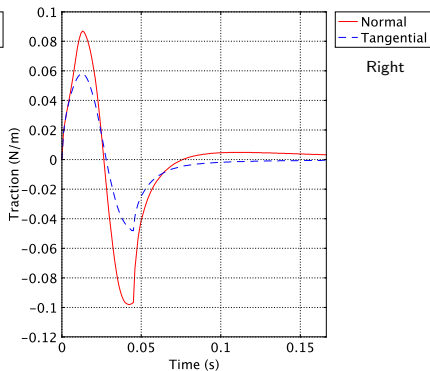
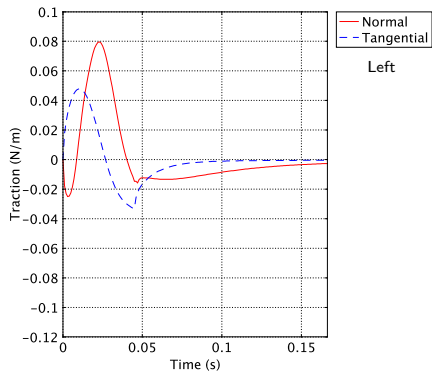


$$c_0 = 3 \quad \text{Pa}$$

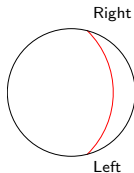
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 1.00 \quad \text{Pa s}$$

Boundary traction (vs. solid viscosity)



[a2.case_011.b]

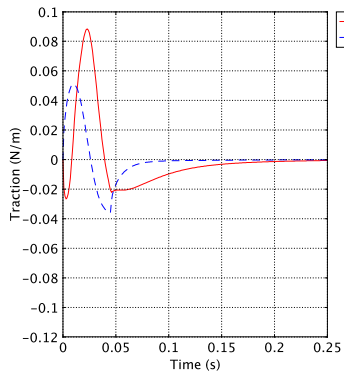


$$c_0 = 3 \quad \text{Pa}$$

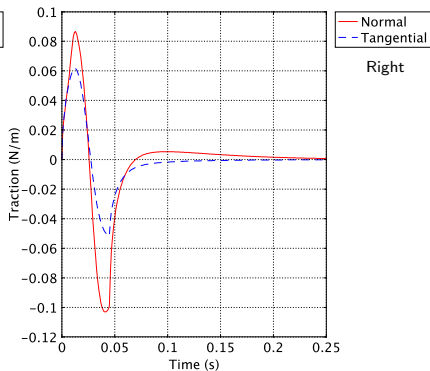
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 1.25 \quad \text{Pa s}$$

Boundary traction (vs. solid viscosity)

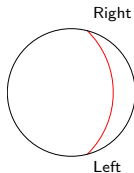


Left



Right

[a2.case_011.t]



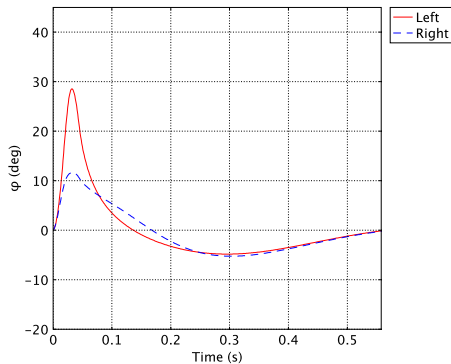
$$c_0 = 3 \quad \text{Pa}$$

$$m_0 = 20 \quad \text{Pa}$$

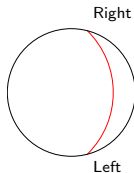
$$\mu_s = 1.50 \quad \text{Pa s}$$

Membrane oscillations
for different values of the **solid viscosity**

Membrane oscillations (vs. solid viscosity)



[c3_case_006]

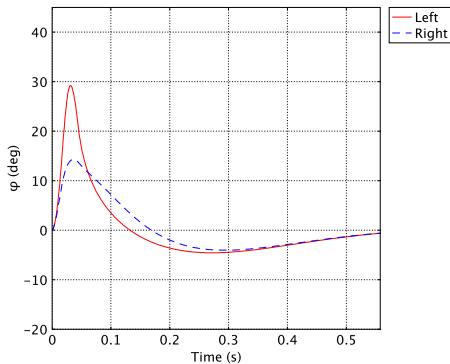


$$c_0 = 3 \quad \text{Pa}$$

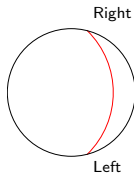
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.50 \quad \text{Pa s}$$

Membrane oscillations (vs. solid viscosity)



[c3_case_010]

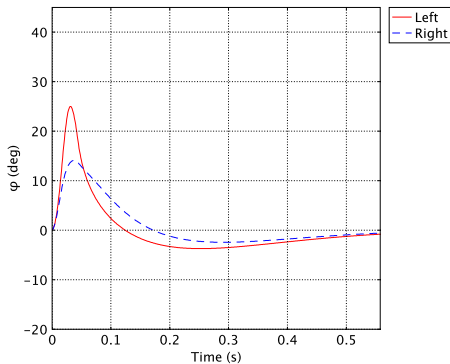


$$c_0 = 3 \quad \text{Pa}$$

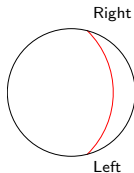
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.75 \quad \text{Pa s}$$

Membrane oscillations (vs. solid viscosity)



[c3.case_011]

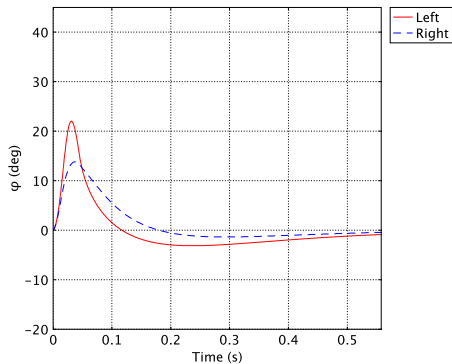


$$c_0 = 3 \quad \text{Pa}$$

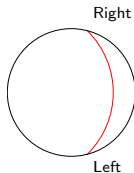
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 1.00 \quad \text{Pa s}$$

Membrane oscillations (vs. solid viscosity)



[c3.case_011.b]

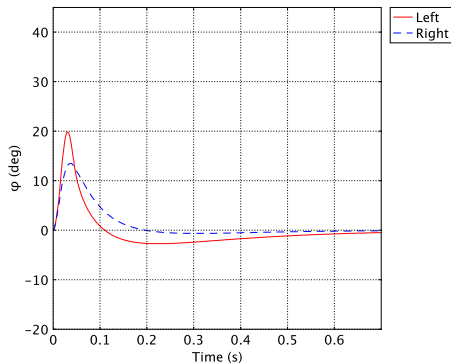


$$c_0 = 3 \quad \text{Pa}$$

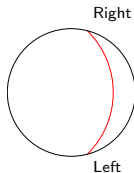
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 1.25 \quad \text{Pa s}$$

Membrane oscillations (vs. solid viscosity)



[c3.case_011.t]



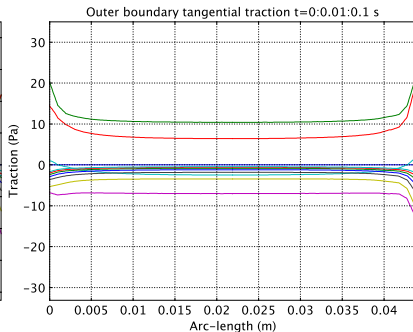
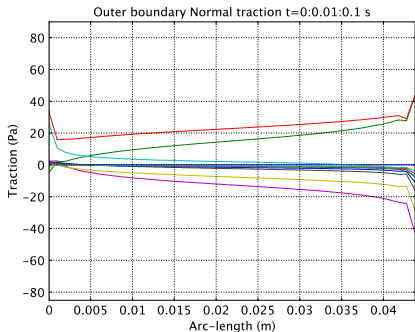
$$c_0 = 3 \quad \text{Pa}$$

$$m_0 = 20 \quad \text{Pa}$$

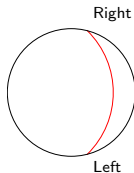
$$\mu_s = 1.50 \quad \text{Pa s}$$

Boundary traction
for different values of the **membrane elastic modulus**

Boundary traction (vs. membrane elastic modulus)



[c3_case_001]

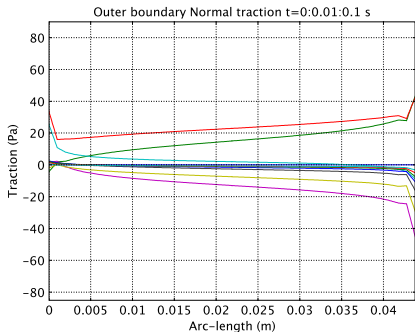


$$c_0 = 2 \quad \text{Pa}$$

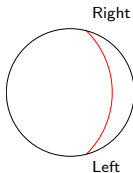
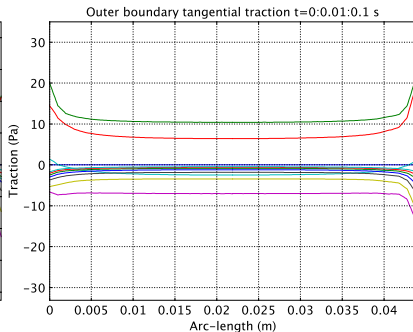
$$m_0 = 2 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. membrane elastic modulus)



[c3_case_001_b]

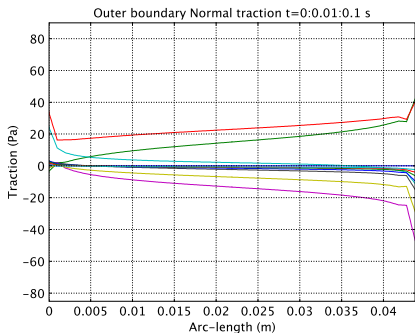


$$c_0 = 2 \quad \text{Pa}$$

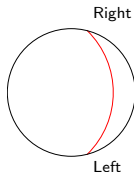
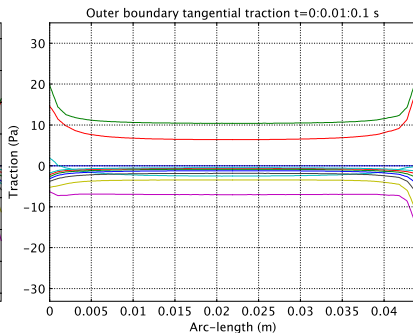
$$m_0 = 5 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. membrane elastic modulus)



[c3_case_002]

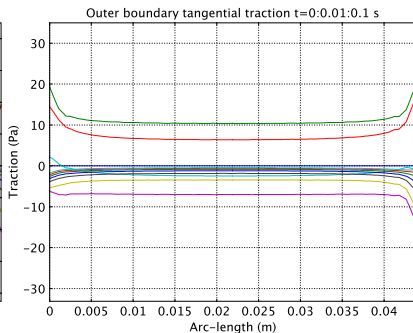
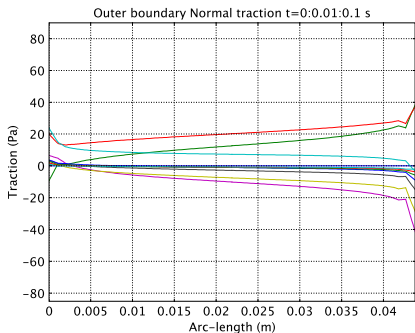


$$c_0 = 2 \quad \text{Pa}$$

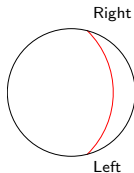
$$m_0 = 10 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. membrane elastic modulus)



[c3_case_002_b]

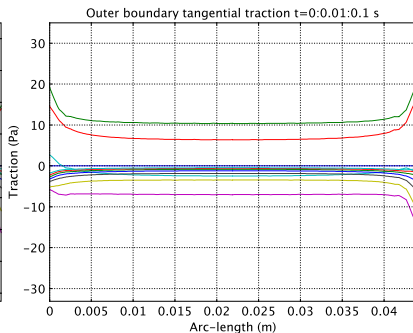
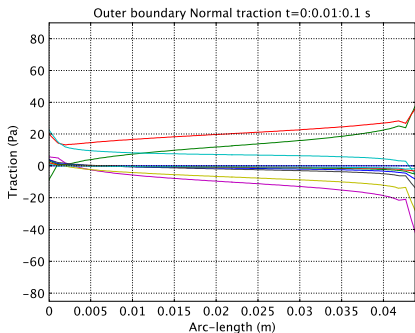


$$c_0 = 2 \quad \text{Pa}$$

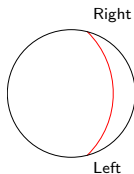
$$m_0 = 15 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. membrane elastic modulus)



[c3_case_003]

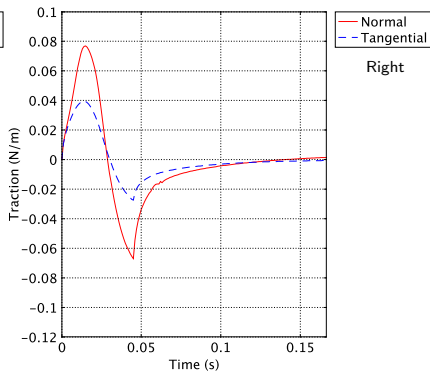
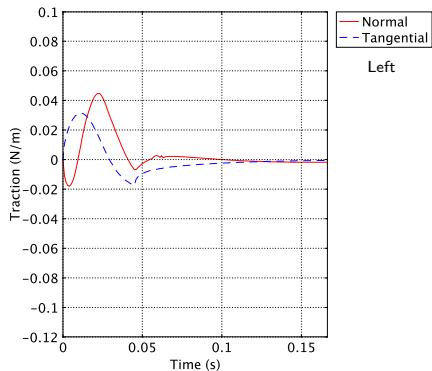


$$c_0 = 2 \quad \text{Pa}$$

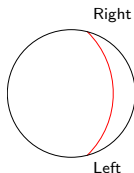
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. membrane elastic modulus)



[c3_case_001]

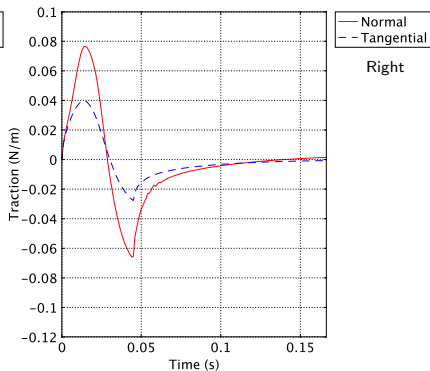
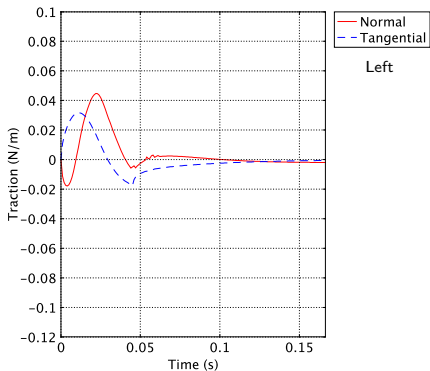


$$c_0 = 2 \quad \text{Pa}$$

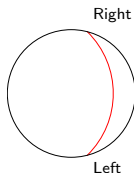
$$m_0 = 2 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. membrane elastic modulus)



[c3_case_001.b]

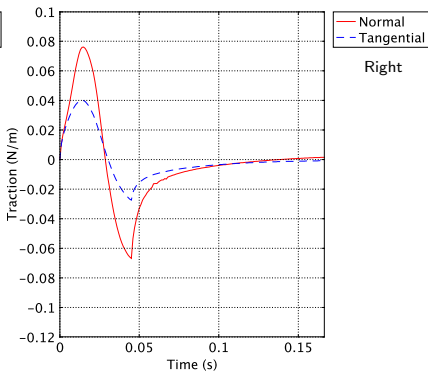
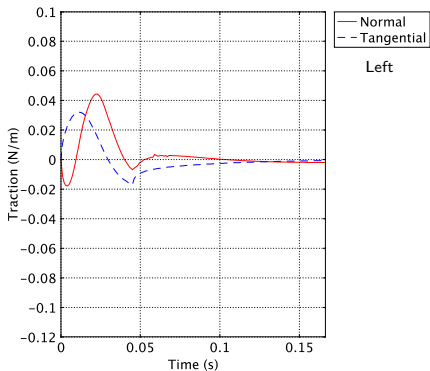


$$c_0 = 2 \quad \text{Pa}$$

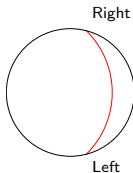
$$m_0 = 5 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. membrane elastic modulus)



[c3_case_002]

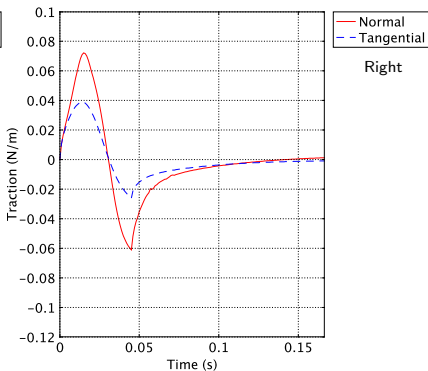
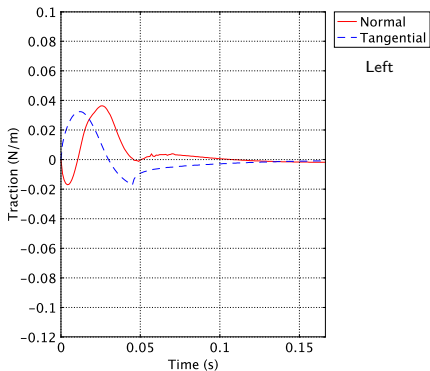


$$c_0 = 2 \quad \text{Pa}$$

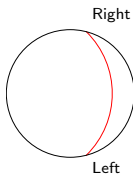
$$m_0 = 10 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. membrane elastic modulus)



[c3_case_002.b]

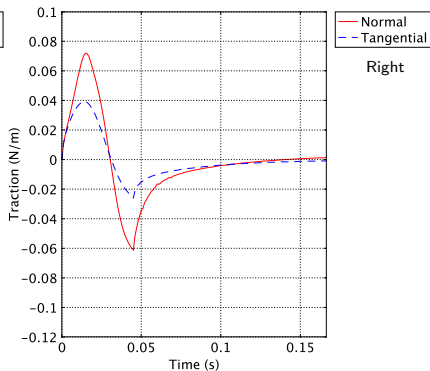
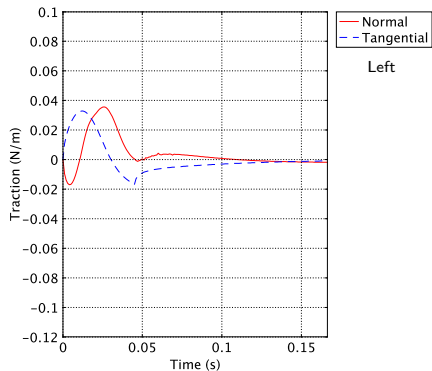


$$c_0 = 2 \quad \text{Pa}$$

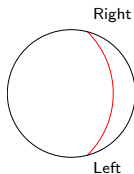
$$m_0 = 15 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. membrane elastic modulus)



[c3_case_003]

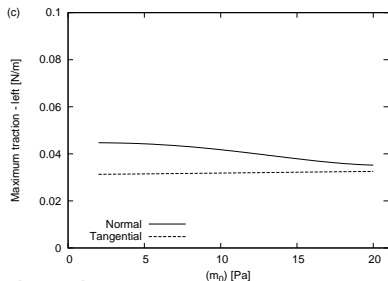


$$c_0 = 2 \quad \text{Pa}$$

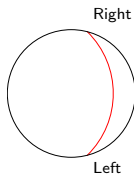
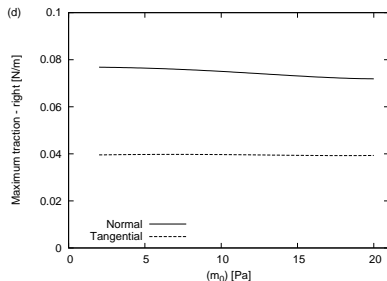
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Boundary traction (vs. membrane elastic modulus)



[c3_case]



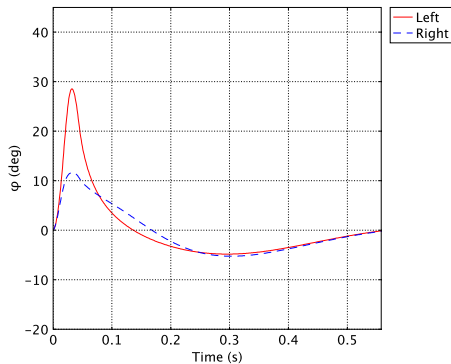
$$c_0 = 2 \quad \text{Pa}$$

$$m_0 = 2 \div 20 \quad \text{Pa}$$

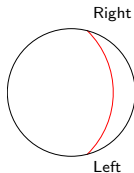
$$\mu_s = 0.5 \quad \text{Pa s}$$

Membrane oscillations
for different values of the **solid viscosity**

Membrane oscillations (vs. solid viscosity)



[c3_case_006]

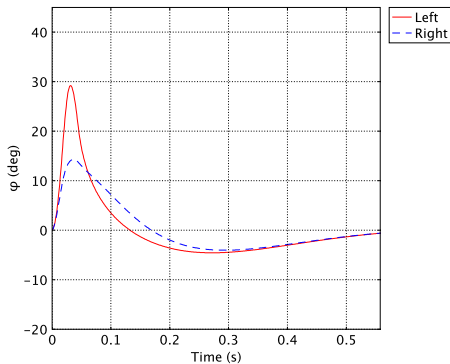


$$c_0 = 3 \quad \text{Pa}$$

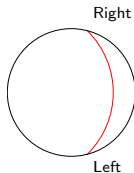
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.50 \quad \text{Pa s}$$

Membrane oscillations (vs. solid viscosity)



[c3_case_010]

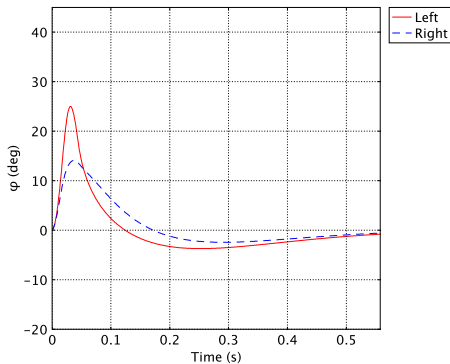


$$c_0 = 3 \quad \text{Pa}$$

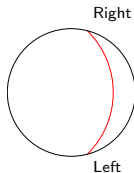
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.75 \quad \text{Pa s}$$

Membrane oscillations (vs. solid viscosity)



[c3.case_011]

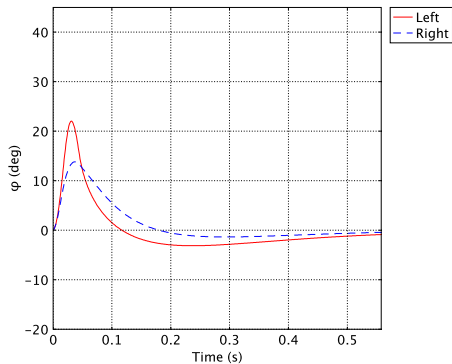


$$c_0 = 3 \quad \text{Pa}$$

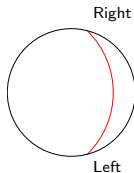
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 1.00 \quad \text{Pa s}$$

Membrane oscillations (vs. solid viscosity)



[c3.case_011.b]

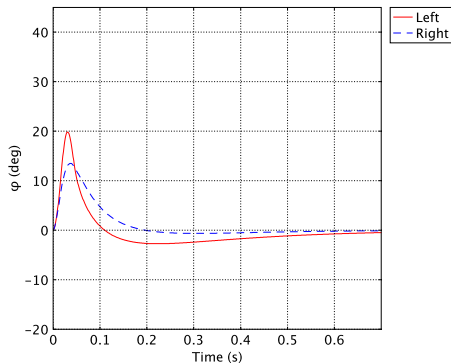


$$c_0 = 3 \quad \text{Pa}$$

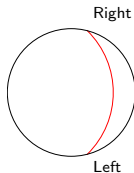
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 1.25 \quad \text{Pa s}$$

Membrane oscillations (vs. solid viscosity)



[c3.case_011.t]



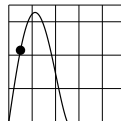
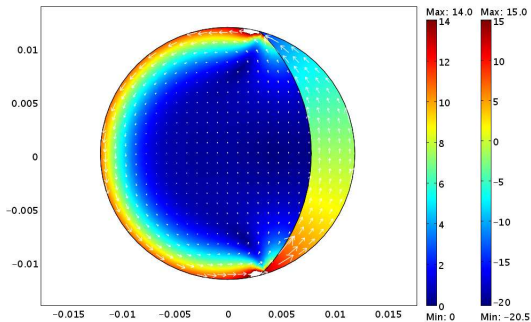
$$c_0 = 3 \quad \text{Pa}$$

$$m_0 = 20 \quad \text{Pa}$$

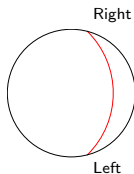
$$\mu_s = 1.50 \quad \text{Pa s}$$

Stress and velocity field in the solid

Deviatoric stress and velocity field in the solid



[c3_case_008]

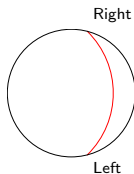
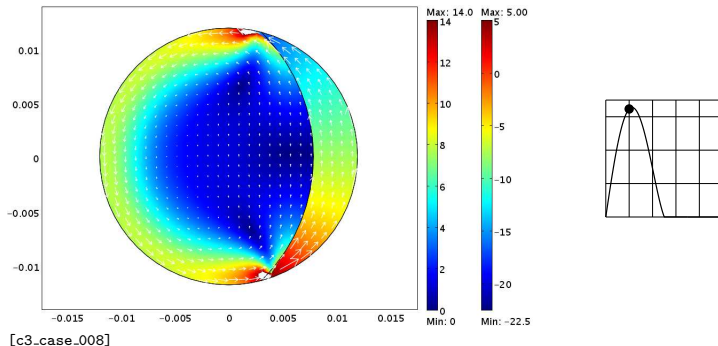


$$c_0 = 5 \quad \text{Pa}$$

$$m_0 = 20 \quad \text{Pa}$$

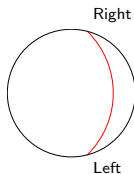
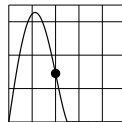
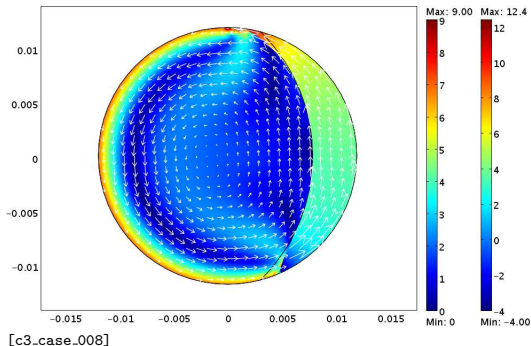
$$\mu_s = 0.5 \quad \text{Pa s}$$

Deviatoric stress and velocity field in the solid



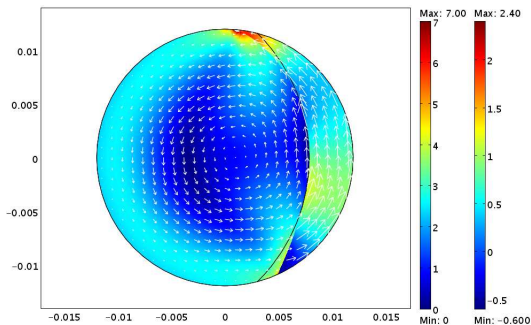
$$c_0 = 5 \quad \text{Pa}$$
$$m_0 = 20 \quad \text{Pa}$$
$$\mu_s = 0.5 \quad \text{Pa s}$$

Deviatoric stress and velocity field in the solid

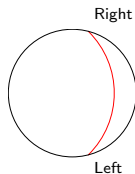


$$c_0 = 5 \quad \text{Pa}$$
$$m_0 = 20 \quad \text{Pa}$$
$$\mu_s = 0.5 \quad \text{Pa s}$$

Deviatoric stress and velocity field in the solid

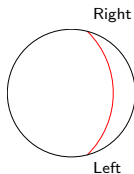
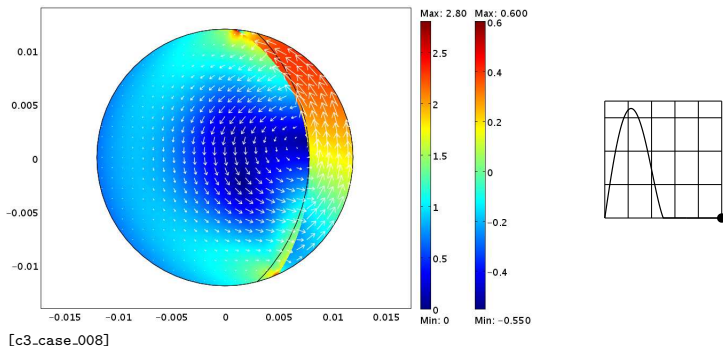


[c3_case_008]



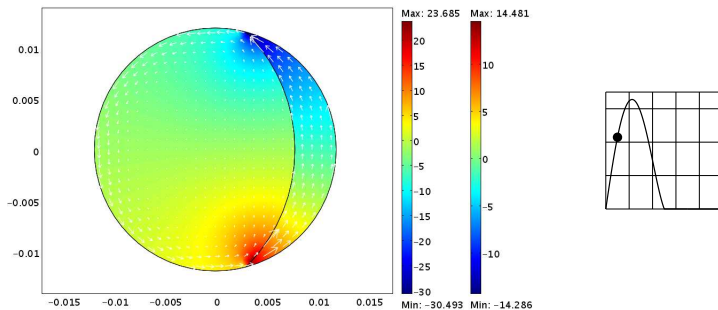
$$\begin{aligned}c_0 &= 5 \quad \text{Pa} \\ m_0 &= 20 \quad \text{Pa} \\ \mu_s &= 0.5 \quad \text{Pa s}\end{aligned}$$

Deviatoric stress and velocity field in the solid

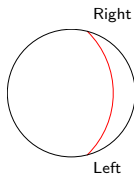


$$c_0 = 5 \quad \text{Pa}$$
$$m_0 = 20 \quad \text{Pa}$$
$$\mu_s = 0.5 \quad \text{Pa s}$$

Pressure and velocity field in the solid

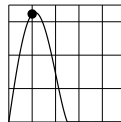
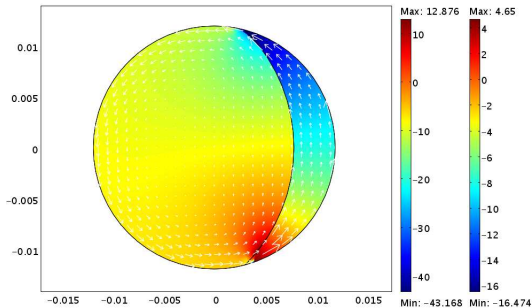


[c3_case_008]

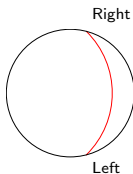


$$c_0 = 5 \quad \text{Pa}$$
$$m_0 = 20 \quad \text{Pa}$$
$$\mu_s = 0.5 \quad \text{Pa s}$$

Pressure and velocity field in the solid

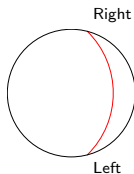
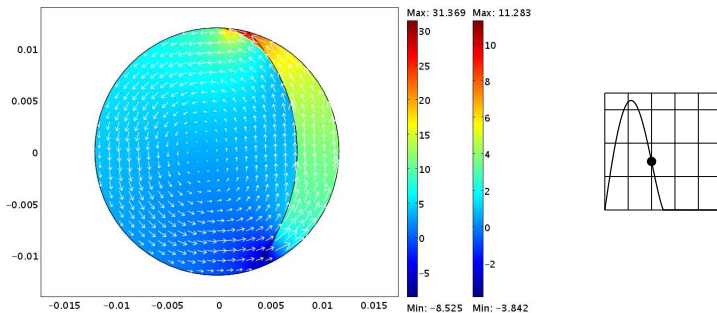


[c3_case_008]



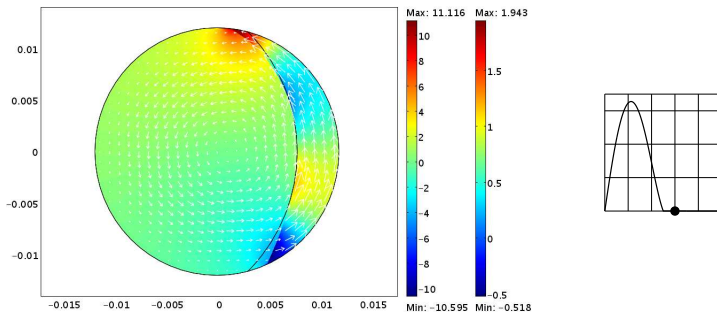
$$\begin{aligned}c_0 &= 5 \quad \text{Pa} \\m_0 &= 20 \quad \text{Pa} \\ \mu_s &= 0.5 \quad \text{Pa s}\end{aligned}$$

Pressure and velocity field in the solid

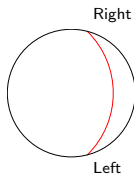


$$c_0 = 5 \quad \text{Pa}$$
$$m_0 = 20 \quad \text{Pa}$$
$$\mu_s = 0.5 \quad \text{Pa s}$$

Pressure and velocity field in the solid

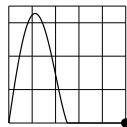
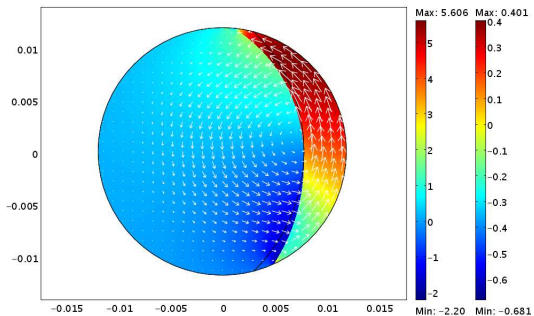


[c3_case_008]

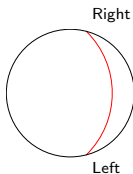


$$c_0 = 5 \quad \text{Pa}$$
$$m_0 = 20 \quad \text{Pa}$$
$$\mu_s = 0.5 \quad \text{Pa s}$$

Pressure and velocity field in the solid



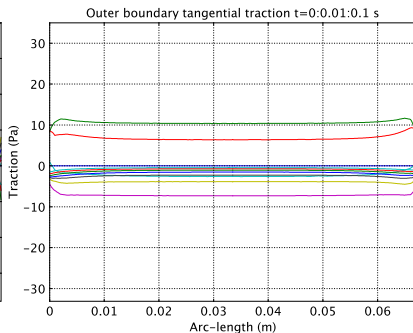
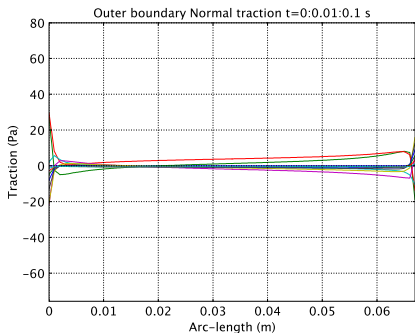
[c3_case_008]



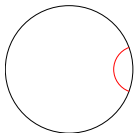
$$c_0 = 5 \quad \text{Pa}$$
$$m_0 = 20 \quad \text{Pa}$$
$$\mu_s = 0.5 \quad \text{Pa s}$$

Comparing different PVD conformations

Different PVD conformations



[a1_case_008]

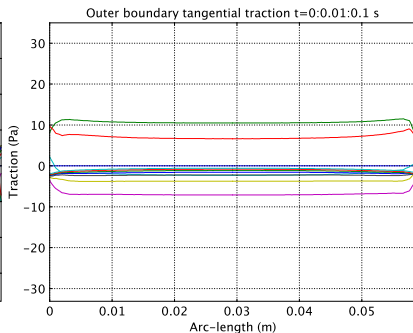
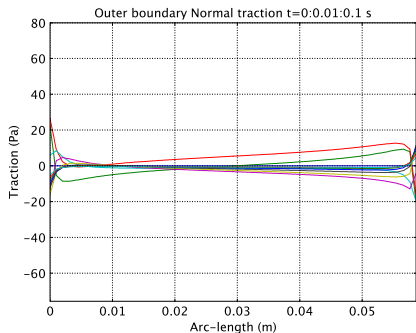


$$c_0 = 5 \quad \text{Pa}$$

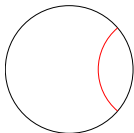
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[a2_case_008]

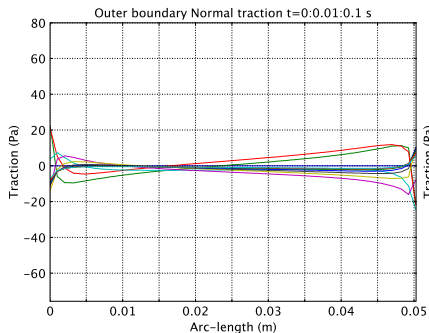


$$c_0 = 5 \quad \text{Pa}$$

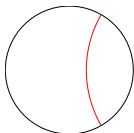
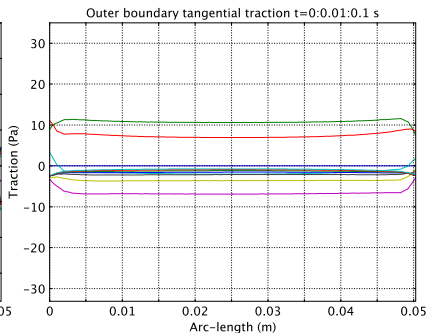
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[a3_case_008]

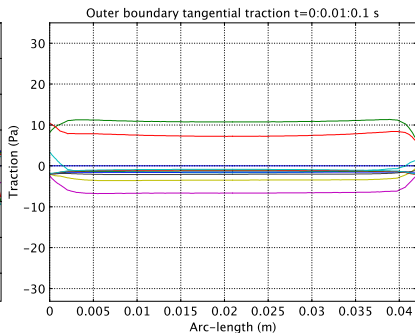
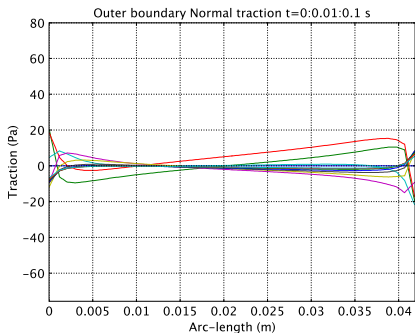


$$c_0 = 5 \quad \text{Pa}$$

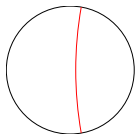
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[a4_case_008]

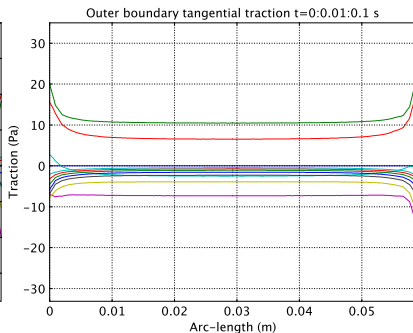
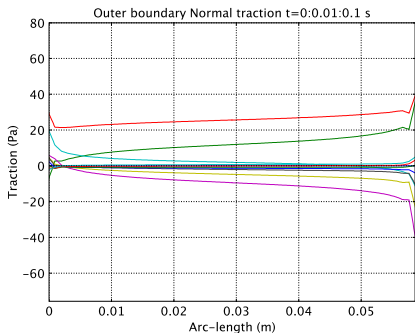


$$c_0 = 5 \quad \text{Pa}$$

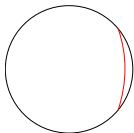
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[b5_case_008]

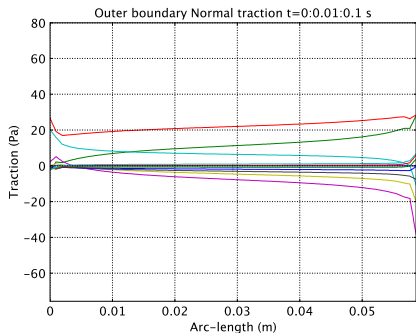


$$c_0 = 5 \quad \text{Pa}$$

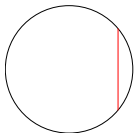
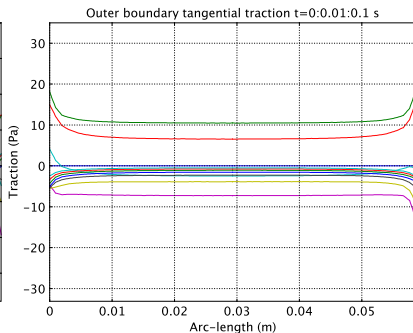
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[b6_case_008]

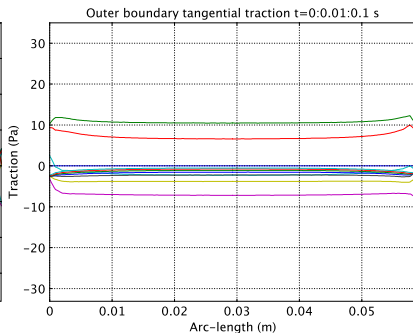
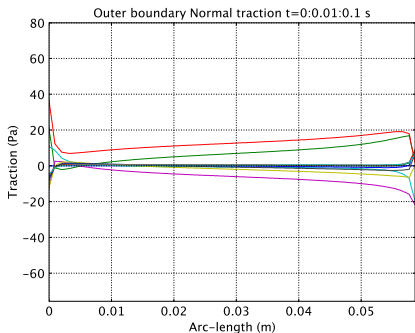


$$c_0 = 5 \quad \text{Pa}$$

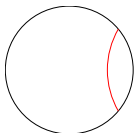
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[b7_case_008]

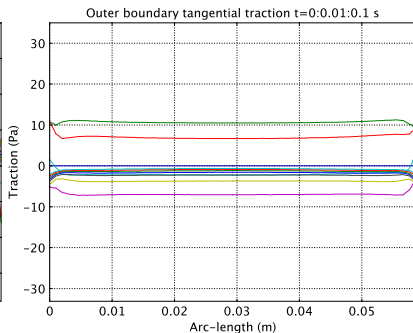
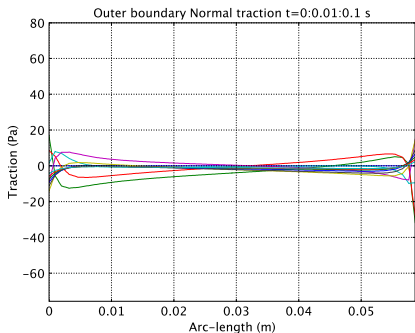


$$c_0 = 5 \quad \text{Pa}$$

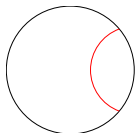
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[b8_case_008]

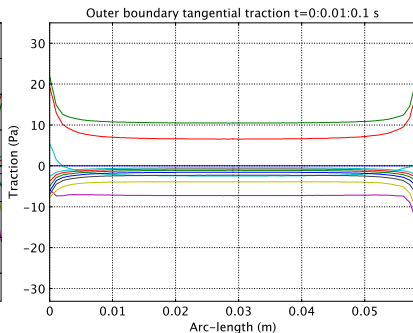
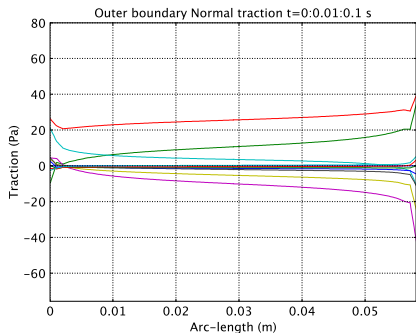


$$c_0 = 5 \quad \text{Pa}$$

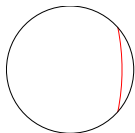
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[c1_case_008]

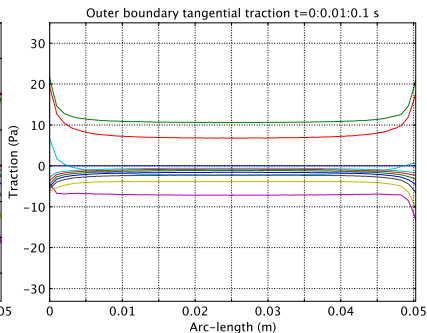
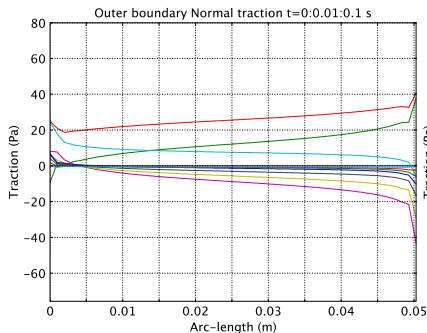


$$c_0 = 5 \quad \text{Pa}$$

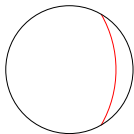
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[c2_case_008]

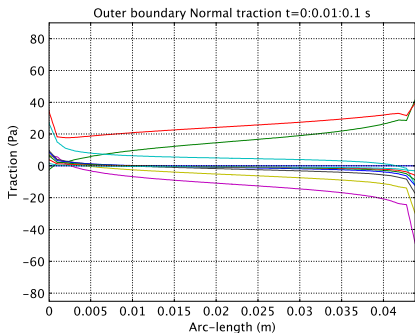


$$c_0 = 5 \quad \text{Pa}$$

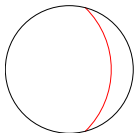
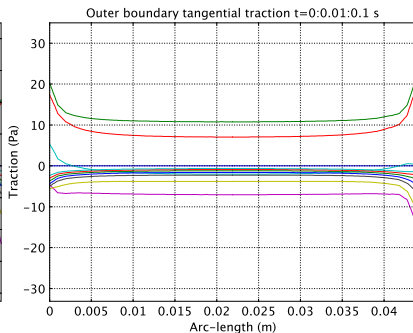
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[c3_case_008]

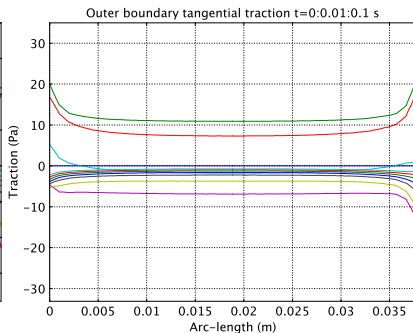
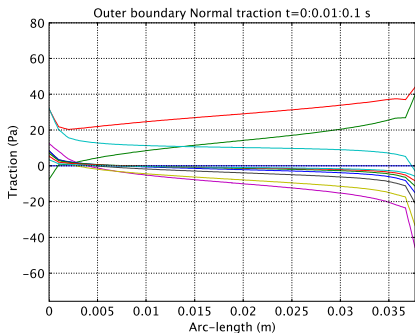


$$c_0 = 5 \quad \text{Pa}$$

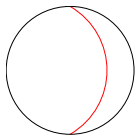
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[c4_case_008]

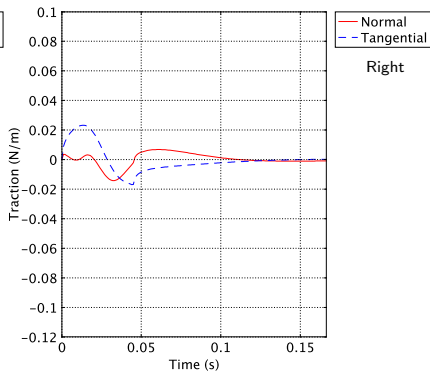
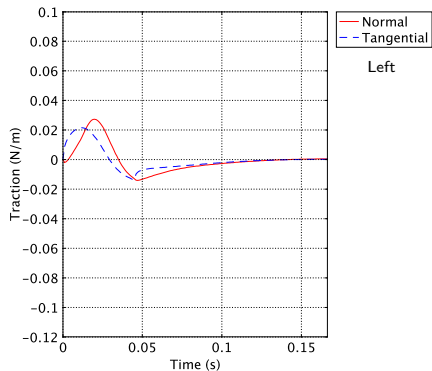


$$c_0 = 5 \quad \text{Pa}$$

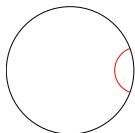
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[a1_case_008]

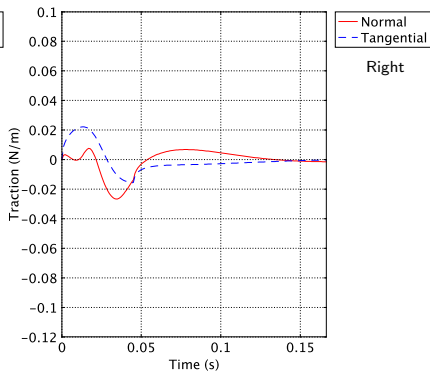
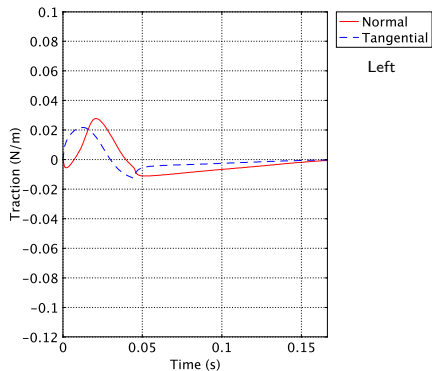


$$c_0 = 5 \quad \text{Pa}$$

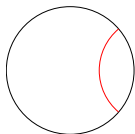
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[a2_case_008]

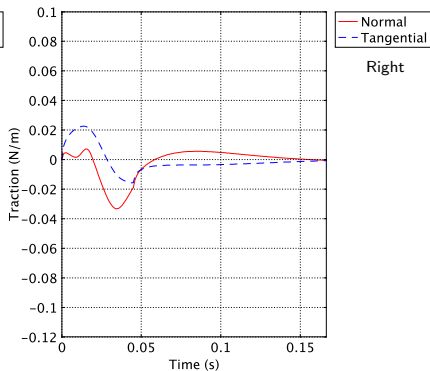
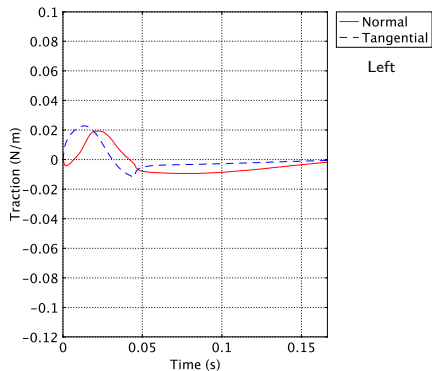


$$c_0 = 5 \quad \text{Pa}$$

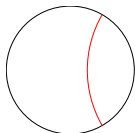
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[a3_case_008]

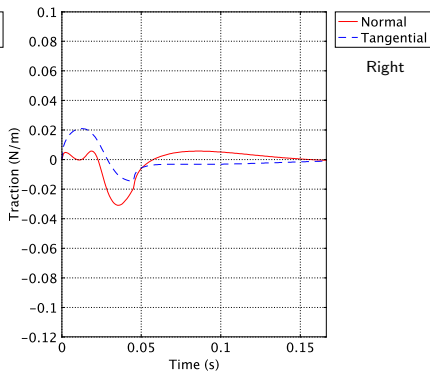
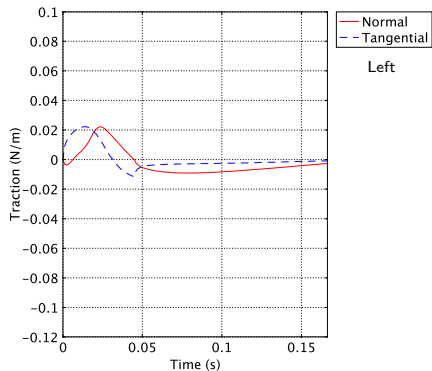


$$c_0 = 5 \quad \text{Pa}$$

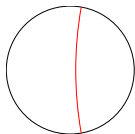
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[a4_case_008]

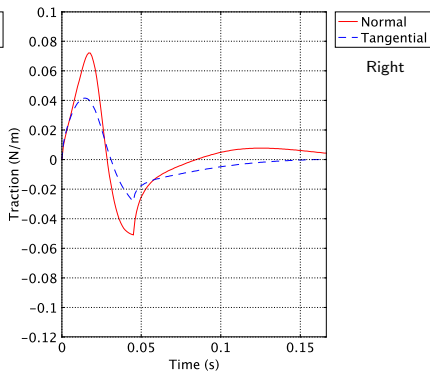
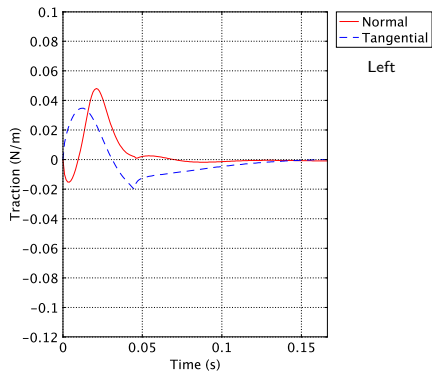


$$c_0 = 5 \quad \text{Pa}$$

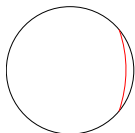
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[b5_case_008]

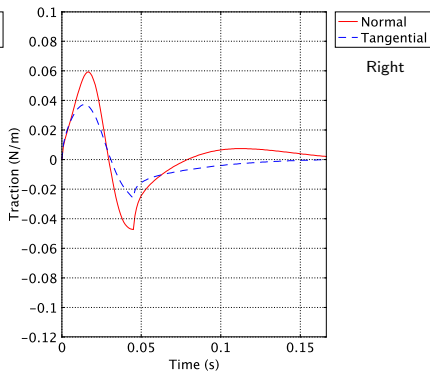
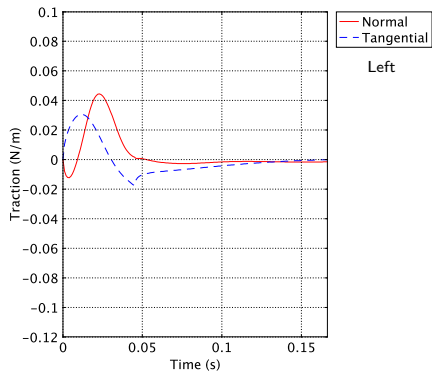


$$c_0 = 5 \quad \text{Pa}$$

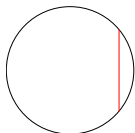
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[b6_case_008]

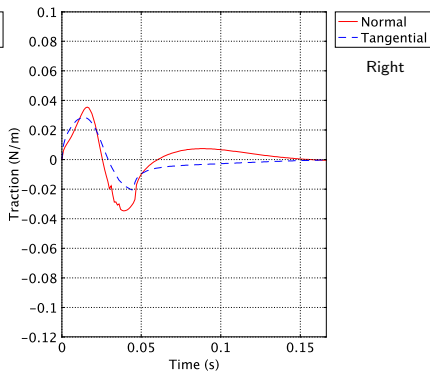
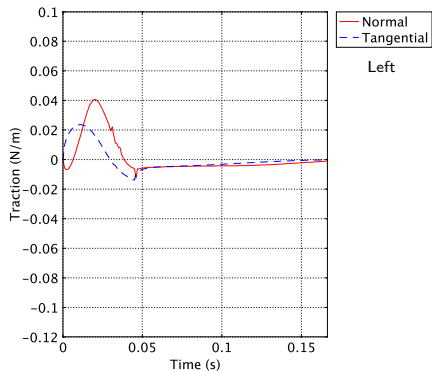


$$c_0 = 5 \quad \text{Pa}$$

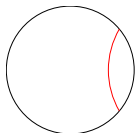
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[b7_case_008]

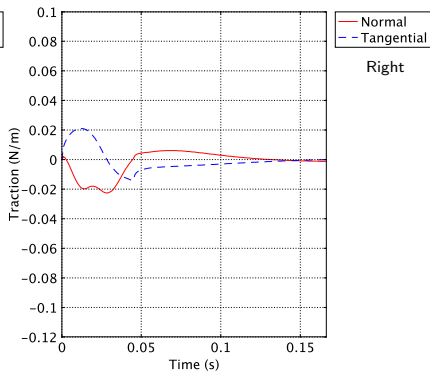
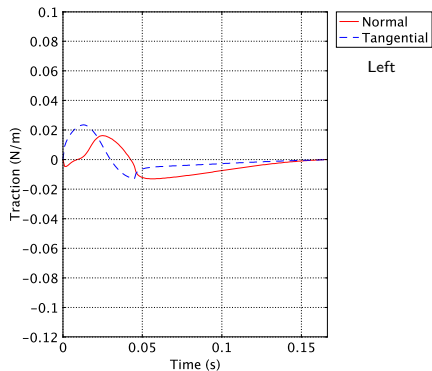


$$c_0 = 5 \quad \text{Pa}$$

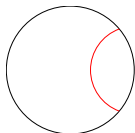
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[b8_case_008]

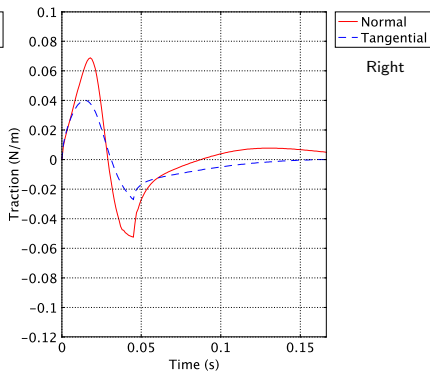
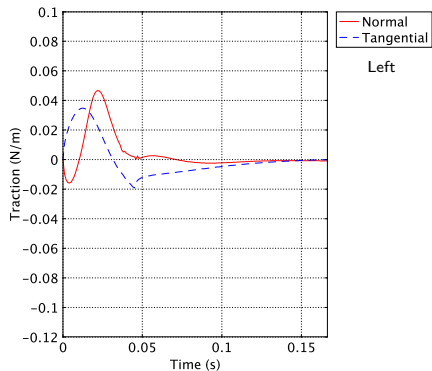


$$c_0 = 5 \quad \text{Pa}$$

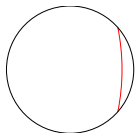
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[c1.case_008]

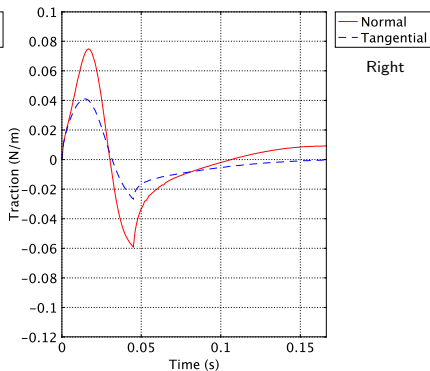
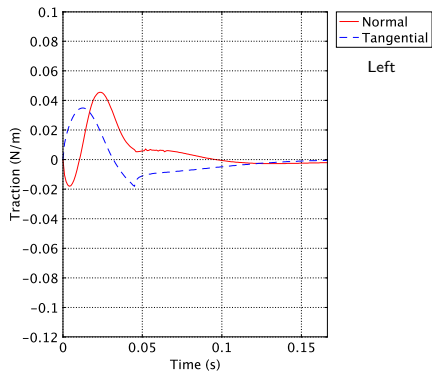


$$c_0 = 5 \quad \text{Pa}$$

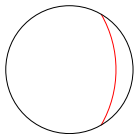
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[c2.case_008]

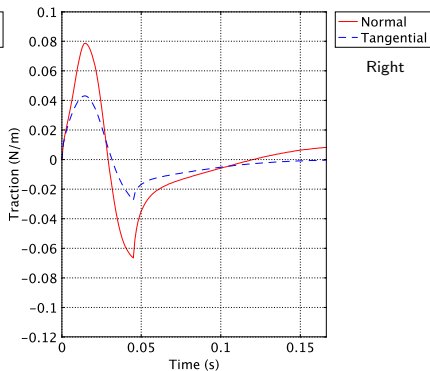
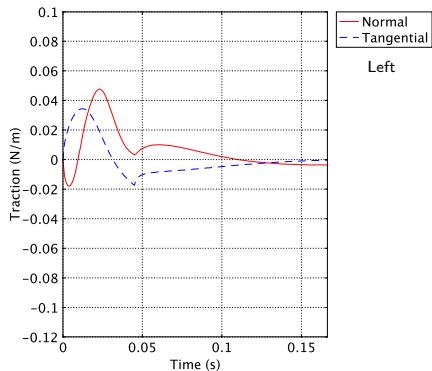


$$c_0 = 5 \quad \text{Pa}$$

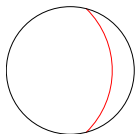
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[c3.case_008]

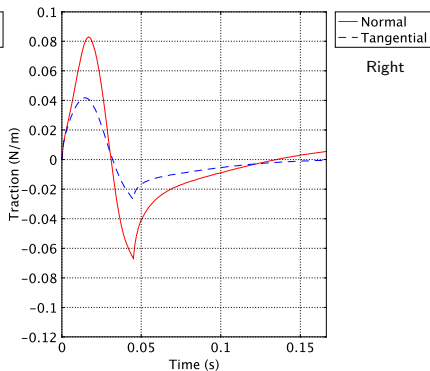
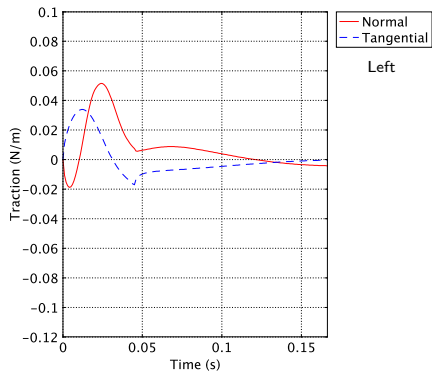


$$c_0 = 5 \quad \text{Pa}$$

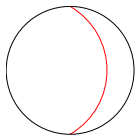
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[c4_case_008]

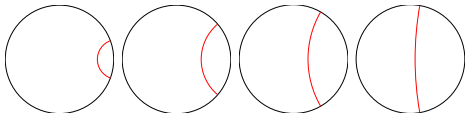
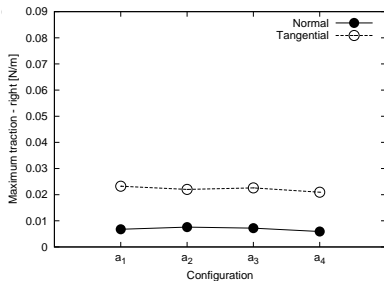
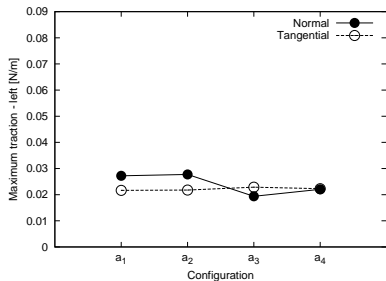


$$c_0 = 5 \quad \text{Pa}$$

$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations

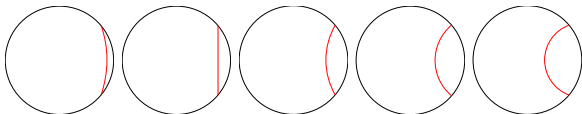
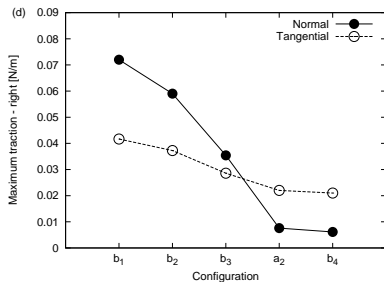
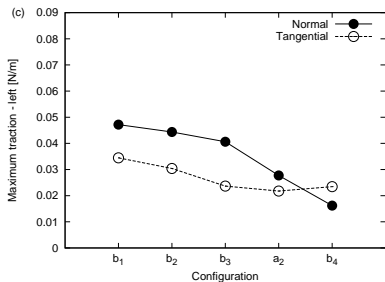


$$c_0 = 5 \quad \text{Pa}$$

$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations

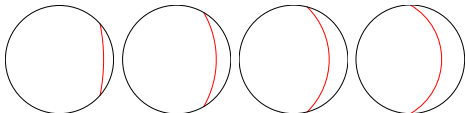
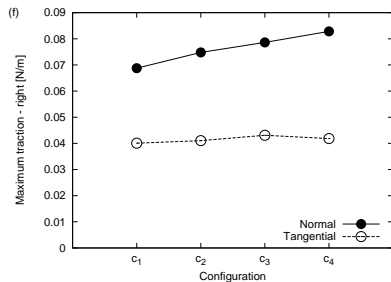
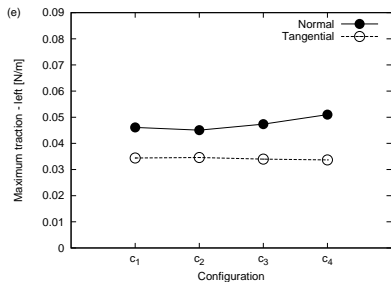


$$c_0 = 5 \quad \text{Pa}$$

$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations

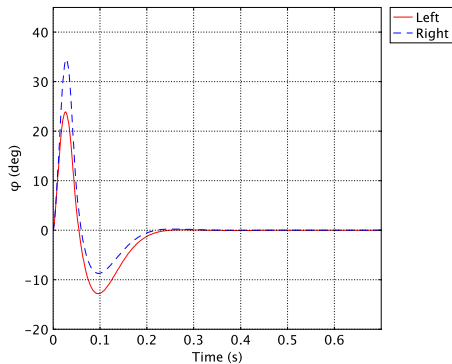


$$c_0 = 5 \quad \text{Pa}$$

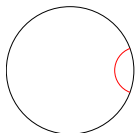
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations

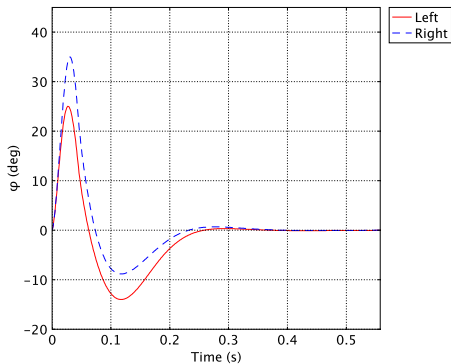


[a1_case_008]

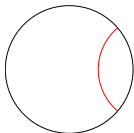


$$\begin{aligned}c_0 &= 5 \quad \text{Pa} \\m_0 &= 20 \quad \text{Pa} \\\mu_s &= 0.5 \quad \text{Pa s}\end{aligned}$$

Different PVD conformations

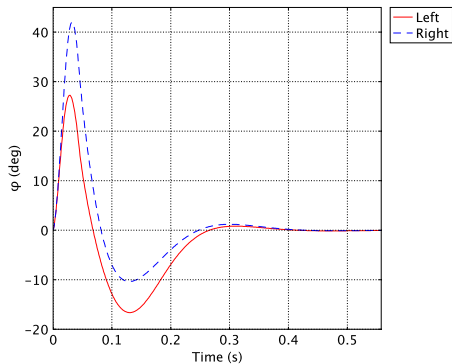


[a2_case_008]

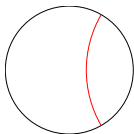


$$\begin{aligned}c_0 &= 5 \quad \text{Pa} \\ m_0 &= 20 \quad \text{Pa} \\ \mu_s &= 0.5 \quad \text{Pa s}\end{aligned}$$

Different PVD conformations



[a3_case_008]

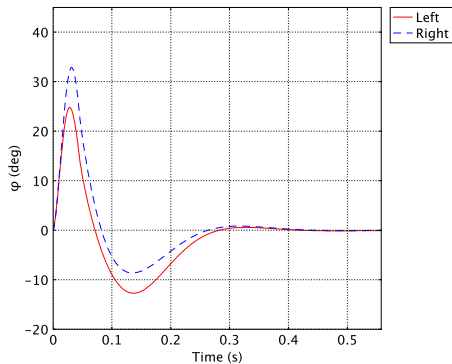


$$c_0 = 5 \quad \text{Pa}$$

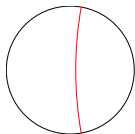
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[a4_case_008]

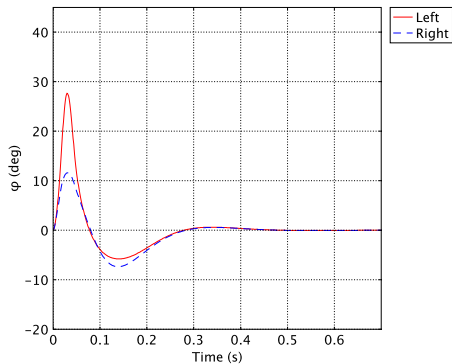


$$c_0 = 5 \quad \text{Pa}$$

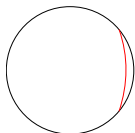
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations

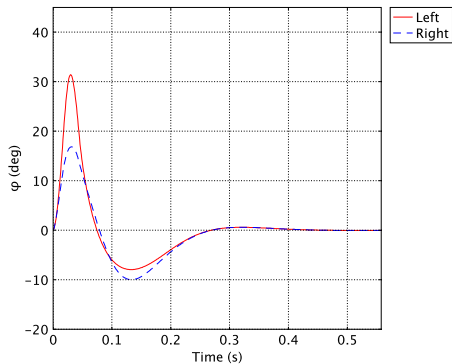


[b5_case_008]

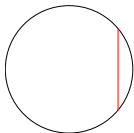


$$\begin{aligned}c_0 &= 5 \quad \text{Pa} \\m_0 &= 20 \quad \text{Pa} \\\mu_s &= 0.5 \quad \text{Pa s}\end{aligned}$$

Different PVD conformations



[b6_case_008]

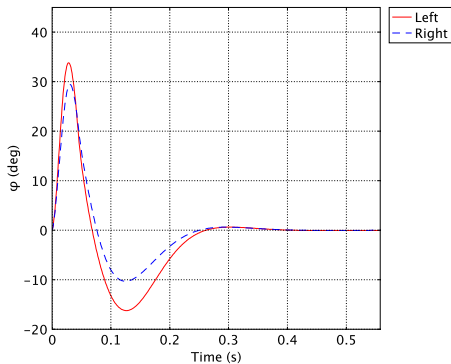


$$c_0 = 5 \quad \text{Pa}$$

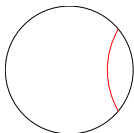
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[b7_case_008]

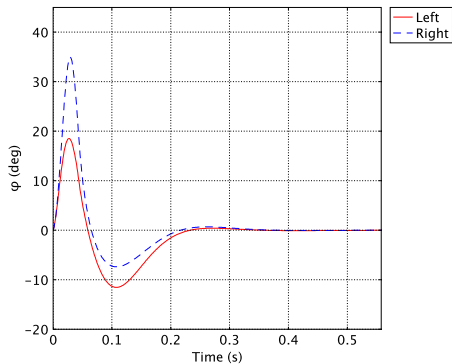


$$c_0 = 5 \quad \text{Pa}$$

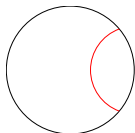
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations

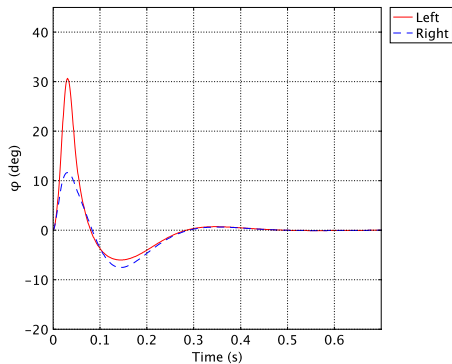


[b8_case_008]

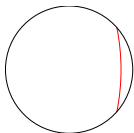


$$\begin{aligned}c_0 &= 5 \quad \text{Pa} \\ m_0 &= 20 \quad \text{Pa} \\ \mu_s &= 0.5 \quad \text{Pa s}\end{aligned}$$

Different PVD conformations

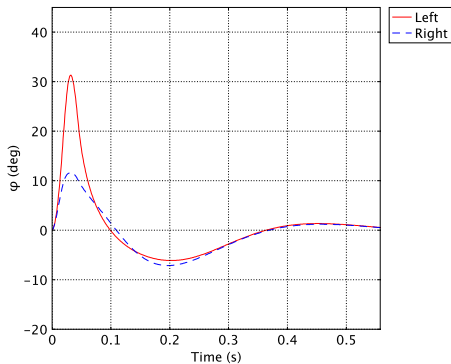


[c1_case_008]

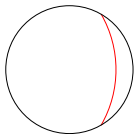


$$\begin{aligned}c_0 &= 5 \quad \text{Pa} \\ m_0 &= 20 \quad \text{Pa} \\ \mu_s &= 0.5 \quad \text{Pa s}\end{aligned}$$

Different PVD conformations



[c2_case_008]

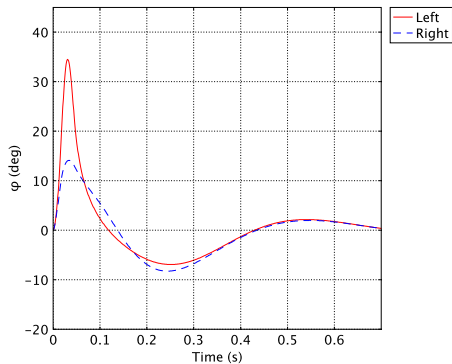


$$c_0 = 5 \quad \text{Pa}$$

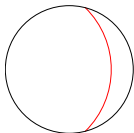
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[c3_case_008]

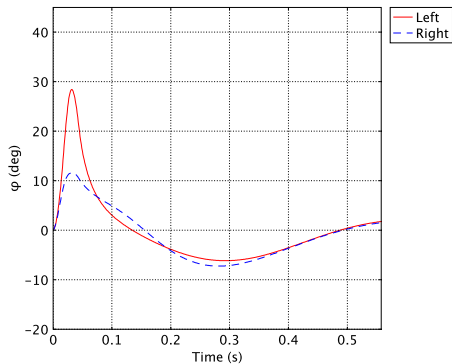


$$c_0 = 5 \quad \text{Pa}$$

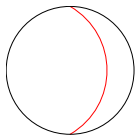
$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$

Different PVD conformations



[c4_case_008]

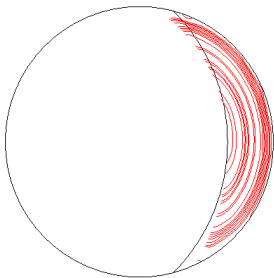


$$\begin{aligned}c_0 &= 5 \quad \text{Pa} \\ m_0 &= 20 \quad \text{Pa} \\ \mu_s &= 0.5 \quad \text{Pa s}\end{aligned}$$

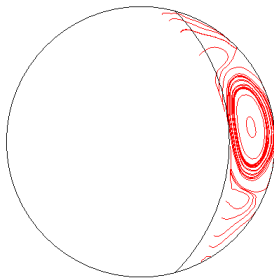
Velocity and pressure in the fluid part of the vitreous

Fluid velocity field

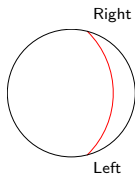
Streamlines of the fluid flow at time=0.09 s



Streamlines of the fluid flow at time=3.7 s



[c3_case..008]



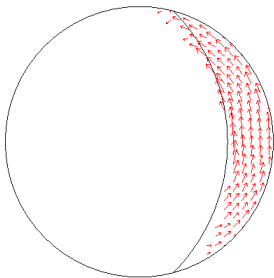
$$c_0 = 5 \quad \text{Pa}$$

$$m_0 = 20 \quad \text{Pa}$$

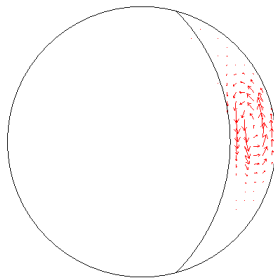
$$\mu_s = 0.5 \quad \text{Pa s}$$

Fluid velocity field

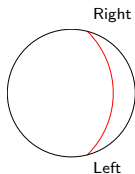
Fluid velocity field at time=0.09 s



Fluid velocity field at time=3.7 s



[c3_case_008]



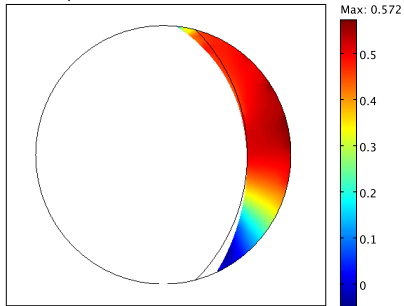
$$c_0 = 5 \quad \text{Pa}$$

$$m_0 = 20 \quad \text{Pa}$$

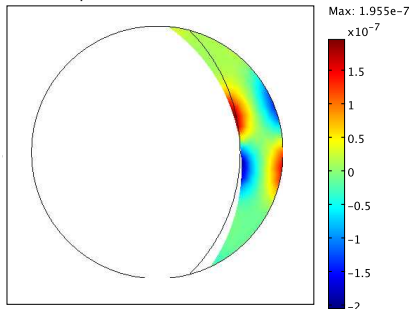
$$\mu_s = 0.5 \quad \text{Pa s}$$

Fluid stress field

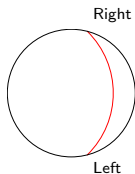
Fluid pressure field at time=0.09 s



Fluid pressure field at time=3.7 s



[c3_case..008]



$$c_0 = 5 \quad \text{Pa}$$

$$m_0 = 20 \quad \text{Pa}$$

$$\mu_s = 0.5 \quad \text{Pa s}$$