How the vitreous motion can generate severe tractions on the retina

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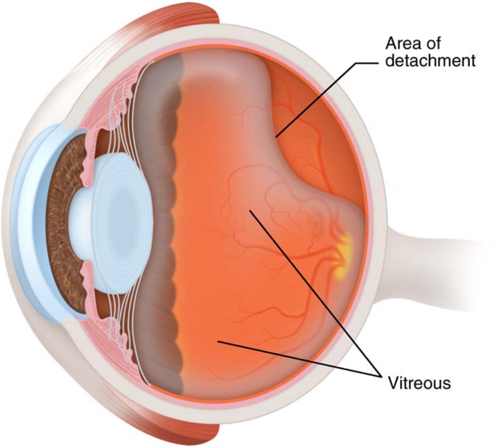
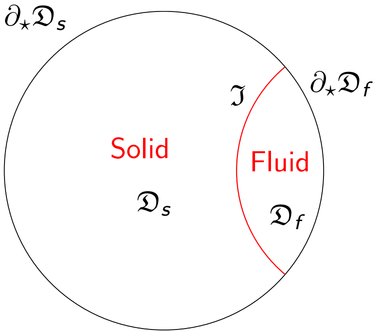
Posterior vitreous detachment



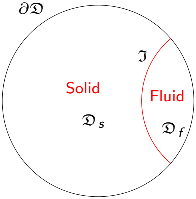


Vitreous chamber after PVD

Balance Principle (solid & fluid)

For any test velocity field **w**

$$\int_{\mathfrak{D}} \mathbf{b} \cdot \mathbf{w} \, dV + \int_{\partial \mathfrak{D}} \mathbf{t} \cdot \mathbf{w} \, dA - \int_{\mathfrak{D}} \mathbf{T} \cdot \nabla \mathbf{w} \, dV = 0$$



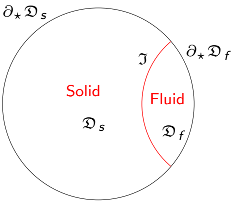
b bulk force per unit volume on $\mathfrak{D}$
t traction per unit area on $\partial \mathfrak{D}$
T Cauchy stress tensor

Balance Principle (solid & fluid)

For any test velocity field $\mathbf{w}$

$$\int_{\mathfrak{D}} \mathbf{b} \cdot \mathbf{w}\, dV + \int_{\partial \mathfrak{D}} \mathbf{t} \cdot \mathbf{w}\, dA - \int_{\mathfrak{D}} \mathbf{T} \cdot \nabla \mathbf{w}\, dV - \int_{\mathfrak{I}} \mathbf{t}^{*} \cdot [\![\mathbf{w}]\!]\, dA = 0$$

$\mathbf{b}$	bulk force per unit volume on $\mathfrak{D}$
$\mathbf{t}$	traction per unit area on $\partial\mathfrak{D}$
$\mathbf{T}$	Cauchy stress tensor
$\mathbf{t}^{*}$	interface stress

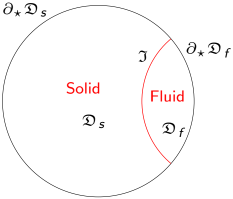


Balance Principle (fluid)

For any test velocity field $\mathbf{w}$

$$\int_{\mathfrak{D}_f} \mathbf{b}_f \cdot \mathbf{w}_f \, dV + \int_{\partial_\star \mathfrak{D}_f} \mathbf{t}_f \cdot \mathbf{w}_f \, dA + \int_{\mathfrak{I}} \mathbf{t}_f \cdot \mathbf{w}_f \, dA - \int_{\mathfrak{D}_f} \mathbf{T}_f \cdot \nabla \mathbf{w}_f \, dV = 0$$

$\mathbf{b}_f$	bulk force per unit volume on $\mathfrak{D}_f$
$\mathbf{t}_f$	traction per unit area on $\partial \mathfrak{D}_f$
$\mathbf{T}_f$	Cauchy stress tensor

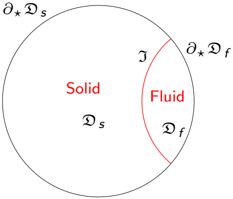


Balance Principle (solid)

For any test velocity field $\mathbf{w}$

$$\int_{\mathfrak{D}_s} \mathbf{b}_s \cdot \mathbf{w}_s \, dV + \int_{\partial_\star \mathfrak{D}_s} \mathbf{t}_s \cdot \mathbf{w}_s \, dA + \int_{\mathfrak{I}} \mathbf{t}_s \cdot \mathbf{w}_s \, dA - \int_{\mathfrak{D}_s} \mathbf{T}_s \cdot \nabla \mathbf{w}_s \, dV = 0$$

$\mathbf{b}_s$ bulk force per unit volume on $\mathfrak{D}_s$
$\mathbf{t}_s$ traction per unit area on $\partial \mathfrak{D}_s$
$\mathbf{T}_s$ Cauchy stress tensor

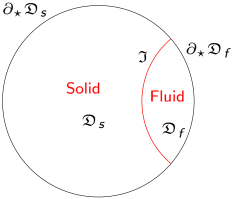


Balance Principle (solid & membrane)

For any test velocity field **w**

$$\int_{\mathfrak{D}_s} \mathbf{b}_s \cdot \mathbf{w}_s \, dV + \int_{\partial_\star \mathfrak{D}_s} \mathbf{t}_s \cdot \mathbf{w}_s \, dA + \int_{\mathfrak{I}} \mathbf{t}_s \cdot \mathbf{w}_s \, dA - \int_{\mathfrak{D}_s} \mathbf{T}_s \cdot \nabla \mathbf{w}_s \, dV - \int_{\mathfrak{I}} \mathbf{N}_s \cdot \nabla^* \mathbf{w}_s \, dA = 0$$

$\mathbf{b}_s$	bulk force per unit volume on $\mathfrak{D}_s$
$\mathbf{t}_s$	traction per unit area on $\partial_\star \mathfrak{D}_s$
$\mathbf{T}_s$	Cauchy stress tensor
$\mathbf{N}_s$	Membrane stress tensor



Balance Equations (solid & membrane)

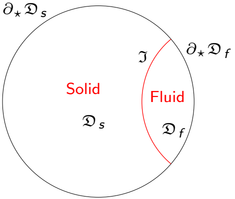
$$\operatorname{div} \mathbf{T}_s + \mathbf{b}_s = 0 \qquad \text{on} \quad \mathfrak{D}_s$$

$$\mathbf{t}_s = \mathbf{T}_s \, \mathbf{n} \qquad \text{on} \quad \partial_\star \mathfrak{D}_s$$

$$\operatorname{div}^* \mathbf{N}_s + \mathbf{t}_s - \mathbf{T}_s \, \mathbf{n} = 0 \qquad \text{on} \quad \mathfrak{I}$$

$$\mathbf{t}_{\partial \mathfrak{I}} = \mathbf{N}_s \, \mathbf{n}_{\partial \mathfrak{I}} \qquad \text{on} \quad \partial \mathfrak{I}$$

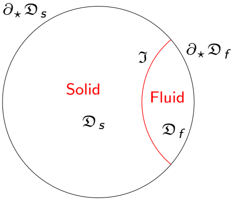
$\mathbf{t}_{\partial \mathfrak{I}}$ traction on $\partial \mathfrak{I}$
$\mathbf{N}_s$ membrane stress tensor



Boundary conditions (solid & fluid)

$$\mathbf{t}_s = \mathbf{T}_f \, \mathbf{n}_s \qquad \text{on} \quad \mathfrak{I}$$

$$\mathbf{v}_s = \mathbf{v}_f \qquad \text{on} \quad \mathfrak{I}$$



Material response

$$\mathbf{T}_s = -p_s \mathbf{I} + \hat{\mathbf{T}}_s(\mathbf{F}) + 2\mu_s \ \mathrm{sym}\, \nabla \mathbf{v_s}$$

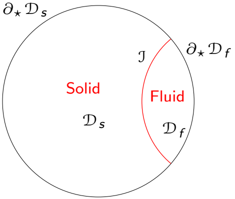
$$\mathbf{T}_f = -p_s \mathbf{I} + 2\mu_f \ \mathrm{sym}\, \nabla \mathbf{v_f}$$

Balance Principle on the (solid) *paragon shape*

For any test velocity field w

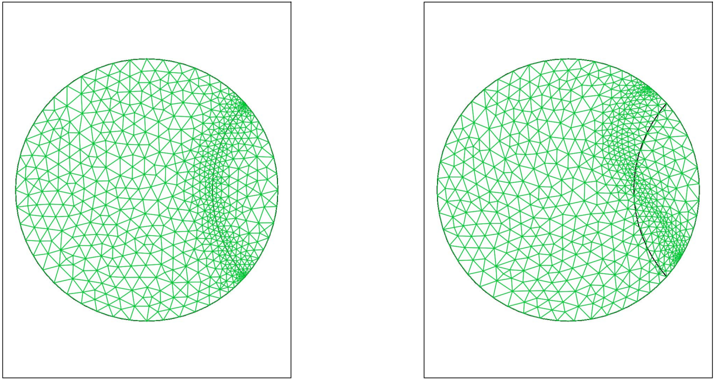
$$\int_{\mathcal{D}_s} \mathsf{b}_s \cdot \mathsf{w}_s \, dV + \int_{\partial_\star \mathcal{D}_s} \mathsf{t}_s \cdot \mathsf{w}_s \, dA + \int_{\mathcal{J}} \mathsf{t}_s \cdot \mathsf{w}_s \, dA - \int_{\mathcal{D}_s} \mathsf{S}_s \cdot \nabla \mathsf{w}_s \, dV - \int_{\mathcal{J}} \mathsf{N}_s \cdot \nabla^* * \mathsf{w}_s \, dA = 0$$

b_s	bulk force per unit volume on $\mathcal{D}_s$
t_s	traction per unit area on $\partial_\star \mathcal{D}_s$
S_s	Piola Kirchhoff stress tensor
N_s	Membrane Piola Kirchhoff stress tensor



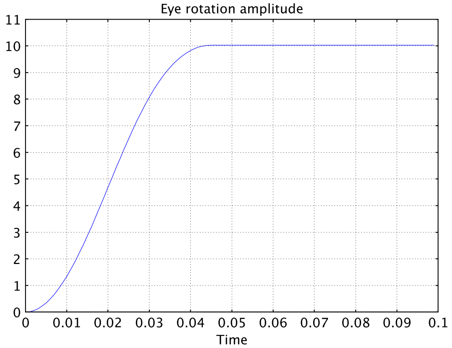
Moving grid

$$\gamma : \mathcal{D} \to \mathfrak{D}$$



$$\gamma_s = \phi_s$$
$$\Delta \gamma_f = 0$$

Saccadic movement



Constitutive parameters

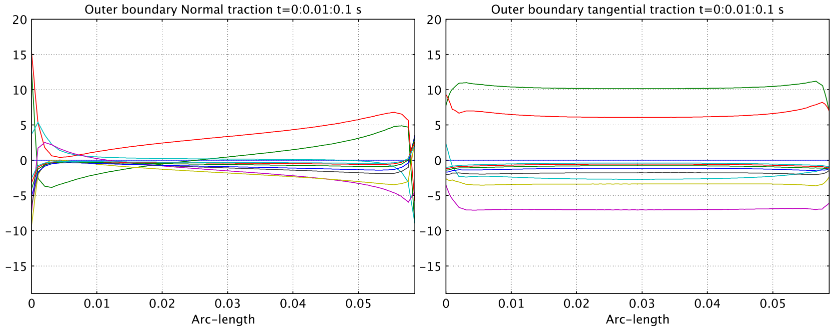
Author	Parameter	
Bettelheim et al. (1976)	Dynamic extensional moduli (bovine)	3-4 Pa
Zimmermann (1980)	Elastic shear modulus G (human)	0.05 Pa
Buchsbaum et al. (1984)	Dynamic shear storage G' and loss G'' (human)	$\ll$ 1.5 Pa
	Dynamic shear storage G' and loss G'' (porcine)	1-1.5 Pa
Lee et al. (1992)	Internal elastic shear modulus G_k (human)	1.2 Pa
	Internal viscosity η_k	0.489 $Pa\,s$
	Instantaneous elastic compliance $J_m = 1/G_m$ (human)	0.3 Pa^{-1}
Nickerson et al. (2008)	Dynamic shear storage G' (bovine) - Initial value	32 Pa
	Dynamic shear storage G' (bovine) - Final value	7 Pa
	Dynamic shear loss G'' (bovine) - Initial value	17 Pa
	Dynamic shear loss G'' (bovine) - Final value	2.2 Pa
	Dynamic shear storage G' (porcine) - Initial value	10 Pa
	Dynamic shear storage G' (porcine) - Final value	2.8 Pa
	Dynamic shear loss G'' (porcine) - Initial value	3.9 Pa
	Dynamic shear loss G'' (porcine) - Final value	0.7 Pa

Eye radius and constitutive parameters

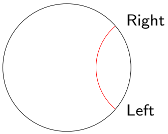
Radius	$R = 0.012\,\mathrm{m}$
Fluid viscosity	$\mu = 10^{-3}\,\mathrm{Pa\,s}$
Mass density	$\rho_s = \rho_f = 1000\,\mathrm{kg/m^3}$

Boundary traction
for different values of the *solid elastic modulus*

Boundary traction (vs. solid elastic modulus)



[a2_case_001]

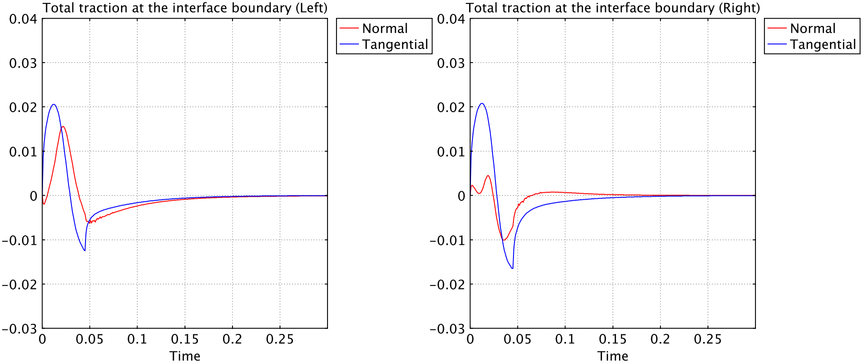


$$c_{01} := 1.0 \quad \text{Pa}$$

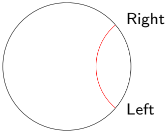
$$m_{01} := 1.0 \quad \text{Pa}$$

$$\mu_s := 0.5 \quad \text{Pa s}$$

Boundary traction (vs. solid elastic modulus)

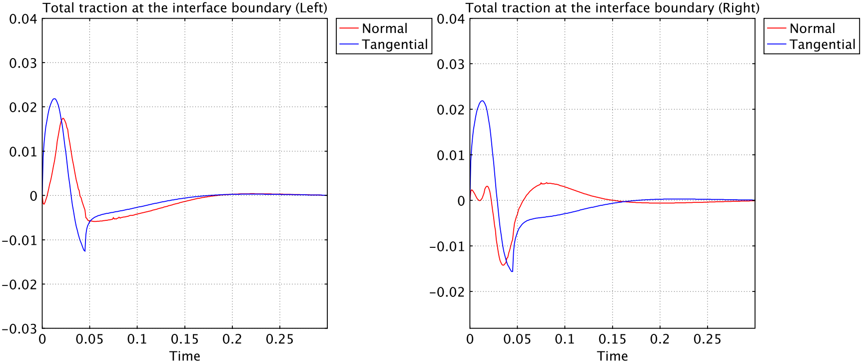


[a2_case_005]

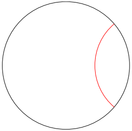


$$c_{01} := 0.5 \quad \text{Pa}$$
$$m_{01} := 10 \quad \text{Pa}$$
$$\mu_s := 0.5 \quad \text{Pa s}$$

Boundary traction (vs. solid elastic modulus)

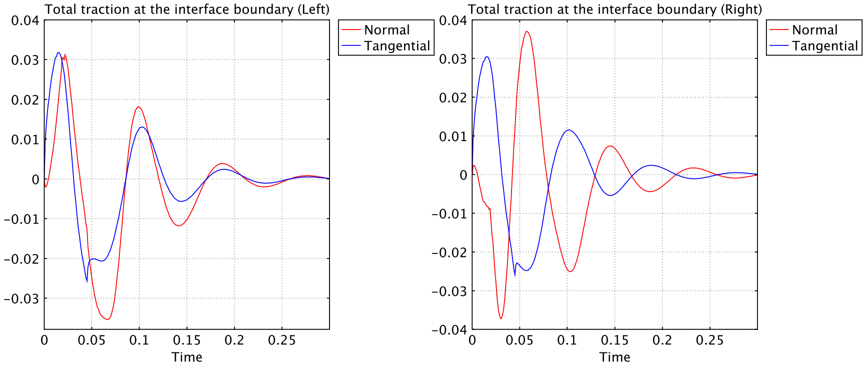


[a2_case_008]

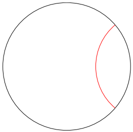


$$c_{01} := \color{red}{2.5} \quad \text{Pa}$$
$$m_{01} := 10 \quad \text{Pa}$$
$$\mu_s := 0.5 \quad \text{Pa s}$$

Boundary traction (vs. solid elastic modulus)

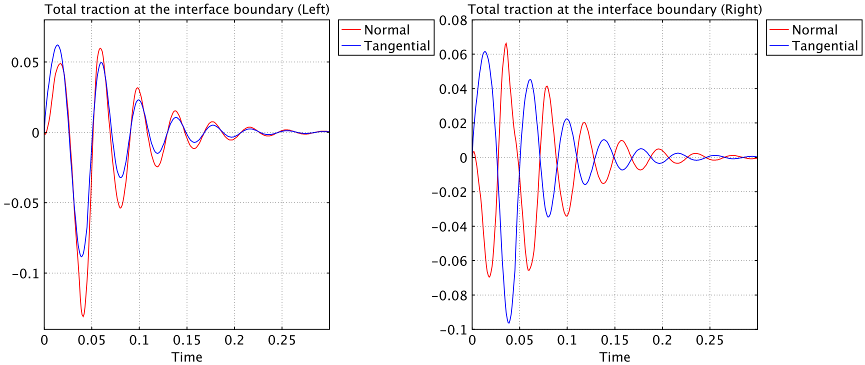


[a2_case_012]

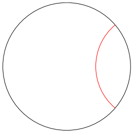


$$c_{01} := 20 \quad \text{Pa}$$
$$m_{01} := 20 \quad \text{Pa}$$
$$\mu_s := 0.5 \quad \text{Pa s}$$

Boundary traction (vs. solid elastic modulus)

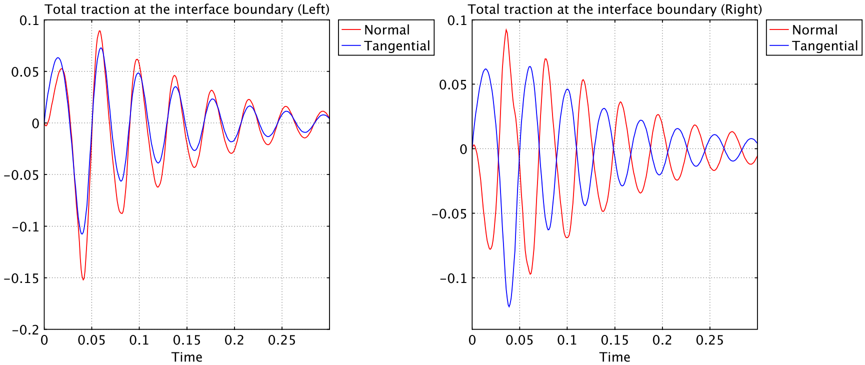


[a2_case_013]

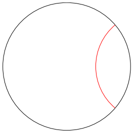


$$c_{01} := \color{red}{100} \quad \text{Pa}$$
$$m_{01} := 100 \quad \text{Pa}$$
$$\mu_s := 0.5 \quad \text{Pa s}$$

Boundary traction (vs. solid elastic modulus)



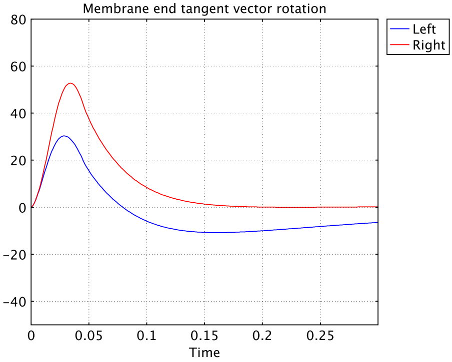
[a2_case_014]



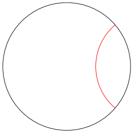
$$c_{01} := 100 \quad \text{Pa}$$
$$m_{01} := 100 \quad \text{Pa}$$
$$\mu_s := 0.3 \quad \text{Pa s}$$

Membrane oscillations
for different values of the *solid elastic modulus*

Membrane oscillations (vs. solid elastic modulus)

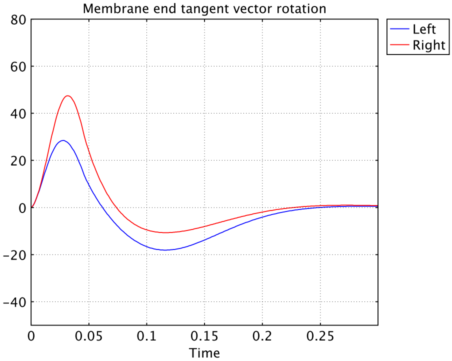


[a2_case_005]

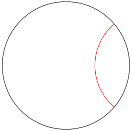


$$c_{01} := 0.5 \quad \text{Pa}$$
$$m_{01} := 10 \quad \text{Pa}$$
$$\mu_s := 0.5 \quad \text{Pa s}$$

Membrane oscillations (vs. solid elastic modulus)

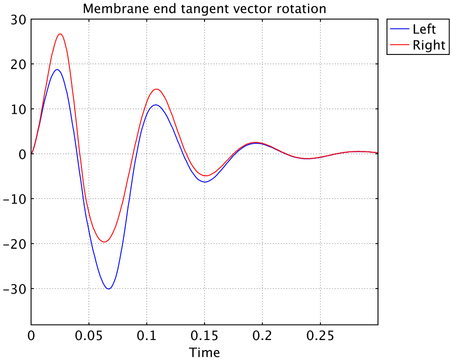


[a2_case_008]

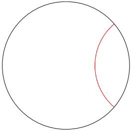


$$c_{01} := 2.5 \quad \text{Pa}$$
$$m_{01} := 10 \quad \text{Pa}$$
$$\mu_s := 0.5 \quad \text{Pa s}$$

Membrane oscillations (vs. solid elastic modulus)

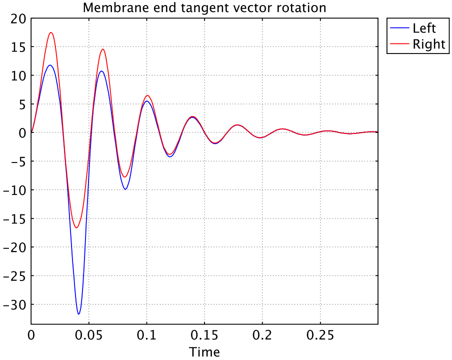


[a2_case_012]

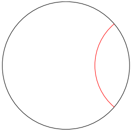


$$c_{01} := 20 \quad \text{Pa}$$
$$m_{01} := 20 \quad \text{Pa}$$
$$\mu_s := 0.5 \quad \text{Pa s}$$

Membrane oscillations (vs. solid vitreous elastic modulus)



[a2_case_013]

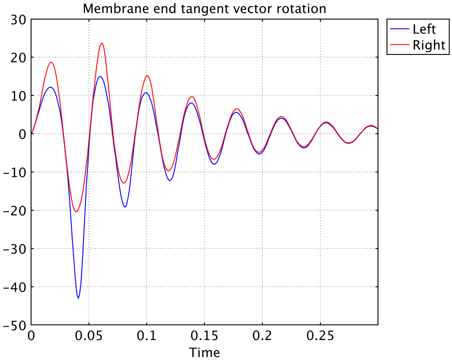


$c_{01} := 100$ Pa

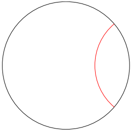
$m_{01} := 100$ Pa

$\mu_s := 0.5$ Pa s

Membrane oscillations (vs. solid vitreous elastic modulus)



[a2_case_014]



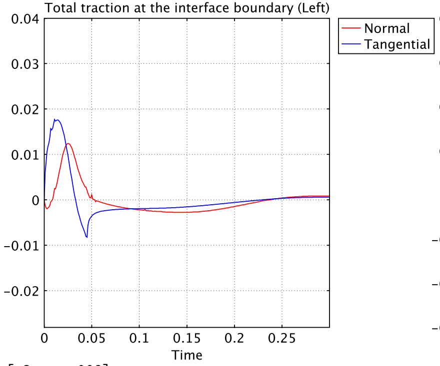
$c_{01} := 100$ Pa

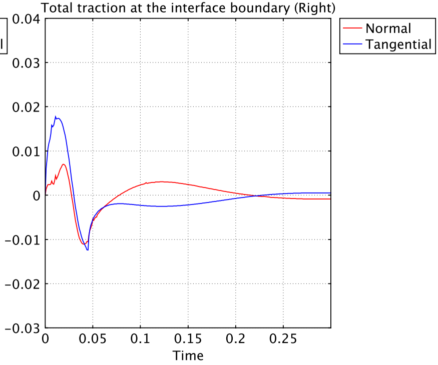
$m_{01} := 100$ Pa

$\mu_s := 0.3$ Pa s

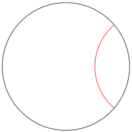
Boundary traction
for different values of the *solid viscosity*

Boundary traction (vs. solid viscosity)





[a2_case_009]

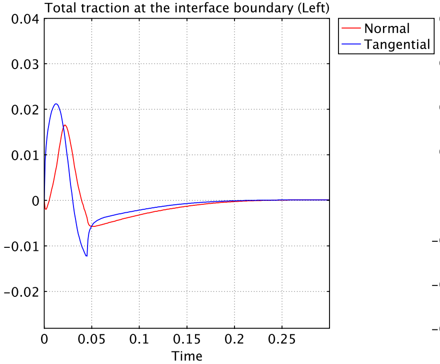


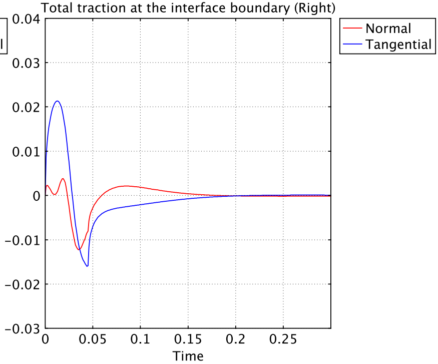
$$c_{01} := 1.5 \quad \text{Pa}$$

$$m_{01} := 10.0 \quad \text{Pa}$$

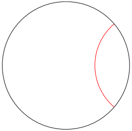
$$\mu_s := 0.25 \quad \text{Pa s}$$

Boundary traction (vs. solid viscosity)





[a2_case_006]

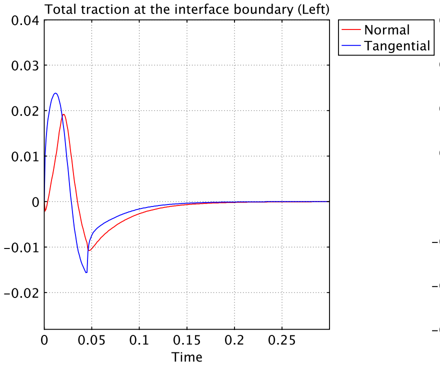


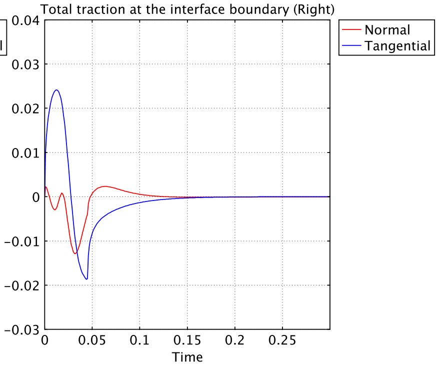
$c_{01} := 1.5$ Pa

$m_{01} := 10.0$ Pa

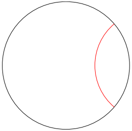
$\mu_s := 0.50$ Pa s

Boundary traction (vs. solid viscosity)





[a2_case_010]

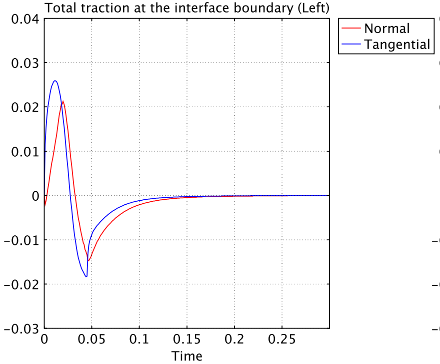


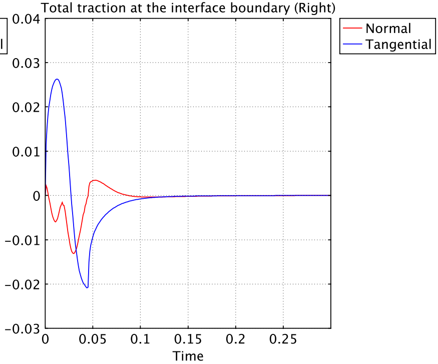
$$c_{01} := 1.5 \quad \text{Pa}$$

$$m_{01} := 10.0 \quad \text{Pa}$$

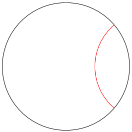
$$\mu_s := 0.75 \quad \text{Pa s}$$

Boundary traction (vs. solid viscosity)





[a2_case_011]



$$c_{01} := 1.5 \quad \text{Pa}$$

$$m_{01} := 10.0 \quad \text{Pa}$$

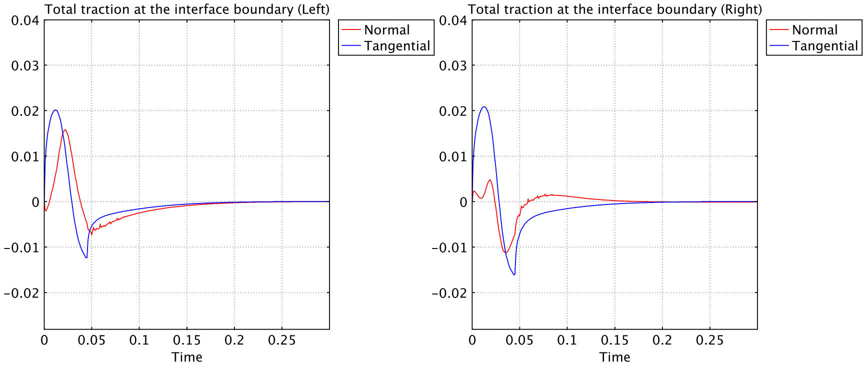
$$\mu_s := 1.00 \quad \text{Pa s}$$

Damping factors and natural frequencies

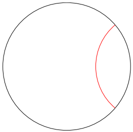
Case	c_{01} (Pa)	m_{01} (Pa)	μ (Pa s)	λ_1	λ_2	λ_3
008	2.5	10	0.5	$-32.90 \pm 15.13i$	$-34.31 \pm 16.04i$	$-16.93 \pm 19.18i$
009	1.5	10	0.25	$-8.64 \pm 17.95i$	$-16.88 \pm 23.15i$	$-17.14 \pm 23.20i$
012	20	20	0.5	$-17.49 \pm 71.11i$	$-33.87 \pm 97.69i$	$-33.60 \pm 98.14i$
013	100	100	0.5	$-17.77 \pm 163.42i$	$-34.08 \pm 229.22i$	$-33.66 \pm 229.60i$
014	100	100	0.3	$-11.03 \pm 164.02i$	$-20.75 \pm 230.81i$	$-20.28 \pm 231.17i$

Boundary traction
for different values of the *membrane elastic modulus*

Boundary traction (vs. membrane elastic modulus)



[a2_case_001]

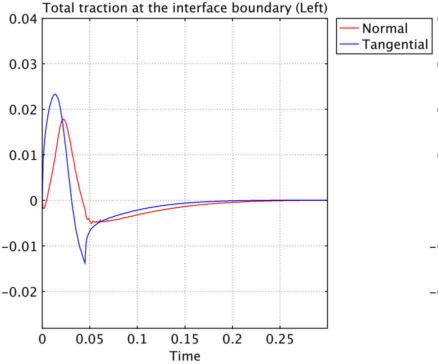


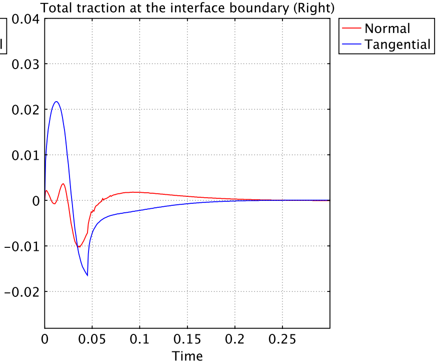
$c_{01} := 1.0$ Pa

$m_{01} := 1.0$ Pa

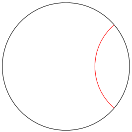
$\mu_s := 0.50$ Pa s

Boundary traction (vs. membrane elastic modulus)





[a2_case_004]

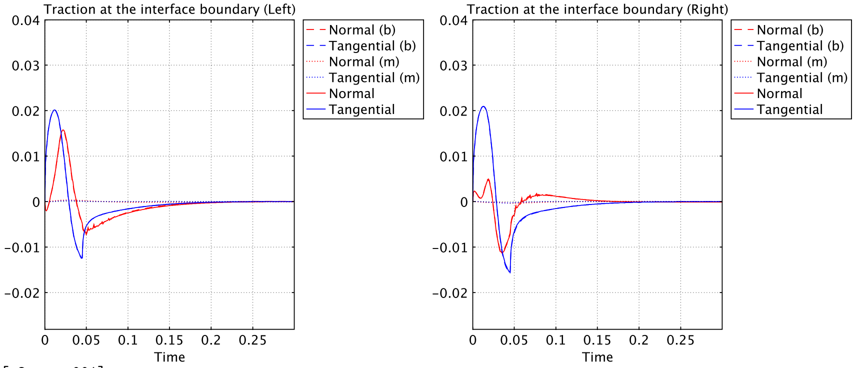


$$c_{01} := 1.0 \quad \text{Pa}$$

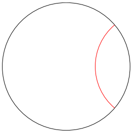
$$m_{01} := 50.0 \quad \text{Pa}$$

$$\mu_s := 0.50 \quad \text{Pa s}$$

Boundary traction (vs. membrane elastic modulus)



[a2_case_001]

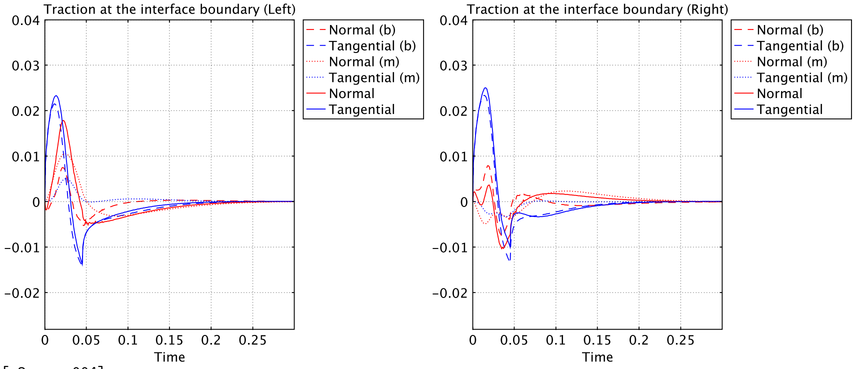


$$c_{01} := 1.0 \quad \text{Pa}$$

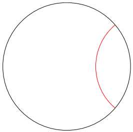
$$m_{01} := {\color{red}1.0} \quad \text{Pa}$$

$$\mu_s := 0.50 \quad \text{Pa s}$$

Boundary traction (vs. membrane elastic modulus)



[a2_case_004]

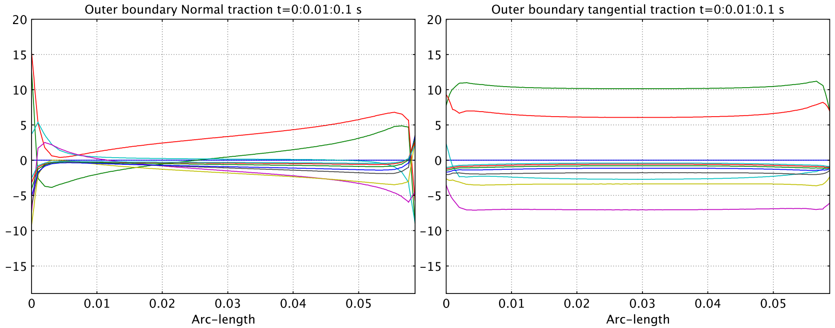


$$c_{01} := 1.0 \quad \text{Pa}$$

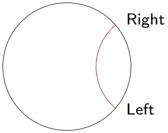
$$m_{01} := 50.0 \quad \text{Pa}$$

$$\mu_s := 0.50 \quad \text{Pa s}$$

Boundary traction (vs. solid elastic modulus)



[a2_case_001]

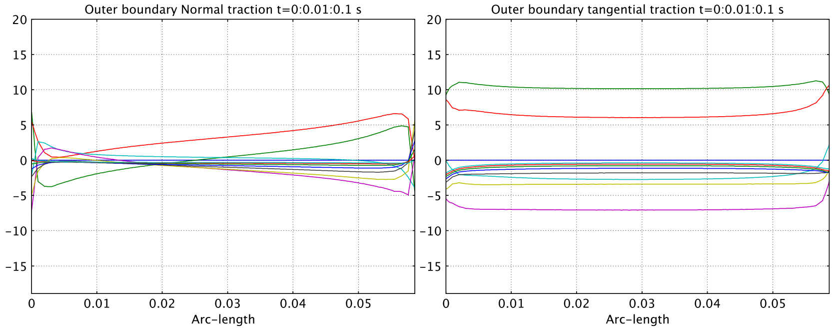


$$c_{01} := 1.0 \quad \text{Pa}$$

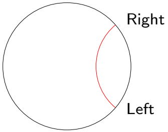
$$m_{01} := {\color{red}1.0} \quad \text{Pa}$$

$$\mu_s := 0.5 \quad \text{Pa s}$$

Boundary traction (vs. solid elastic modulus)



[a2_case_004]



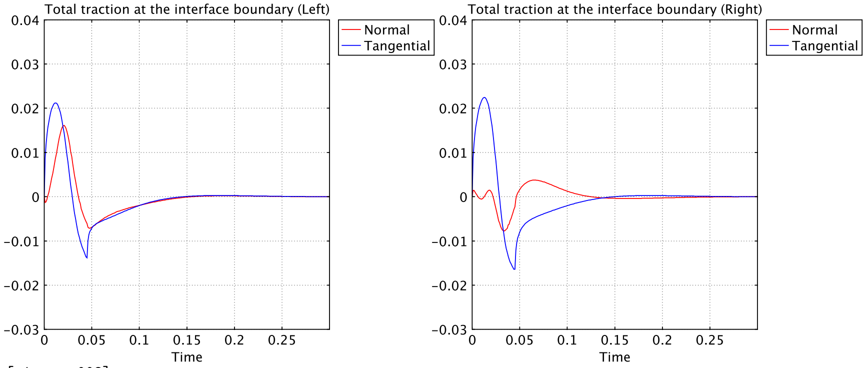
c_{01} := 1.0 Pa

m_{01} := 50 Pa

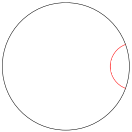
μ_s := 0.5 Pa s

Comparing different PVD conformations

Different PVD conformations



[a1_case_008]

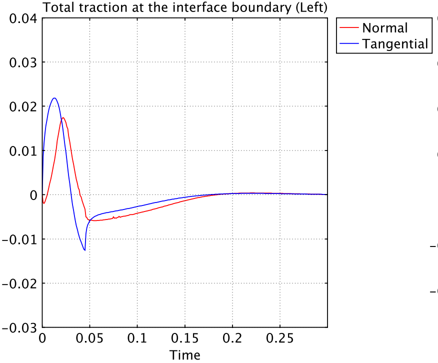


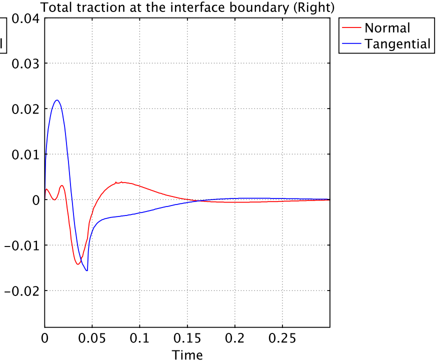
$$c_{01} := 2.5 \quad \text{Pa}$$

$$m_{01} := 10.0 \quad \text{Pa}$$

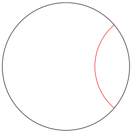
$$\mu_s := 0.50 \quad \text{Pa s}$$

Different PVD conformations





[a2_case_008]

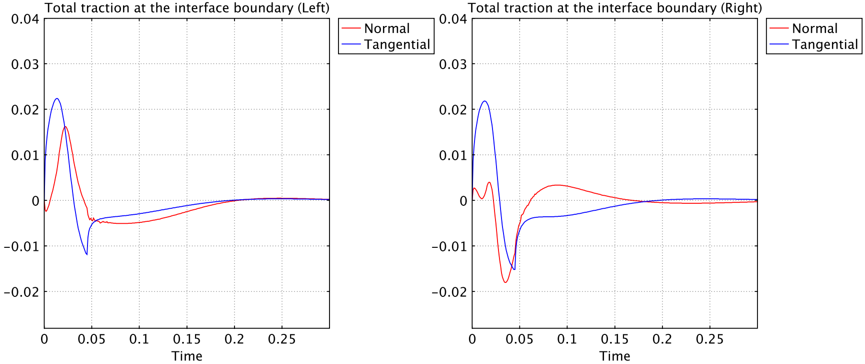


$$c_{01} := 2.5 \quad \text{Pa}$$

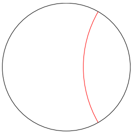
$$m_{01} := 10.0 \quad \text{Pa}$$

$$\mu_s := 0.50 \quad \text{Pa s}$$

Different PVD conformations

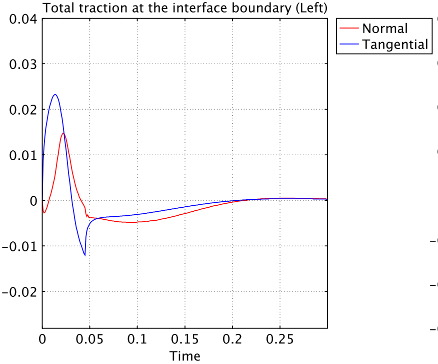


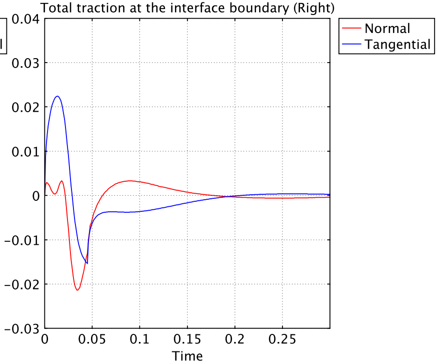
[a3_case_008]



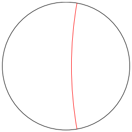
$$c_{01} := 2.5 \quad \text{Pa}$$
$$m_{01} := 10.0 \quad \text{Pa}$$
$$\mu_s := 0.50 \quad \text{Pa s}$$

Different PVD conformations





[a4_case_008]

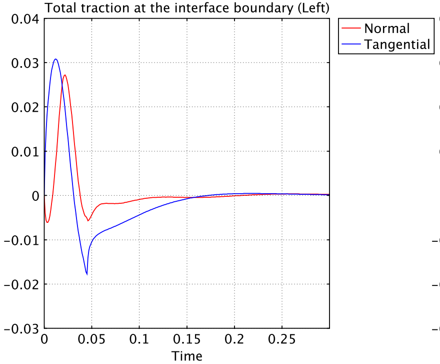


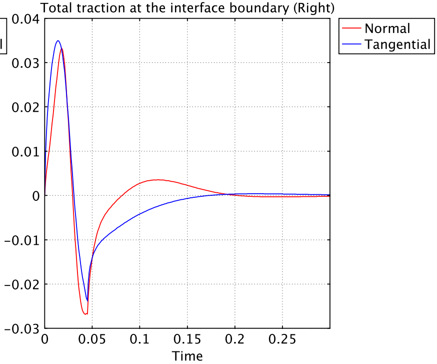
$$c_{01} := 2.5 \quad \text{Pa}$$

$$m_{01} := 10.0 \quad \text{Pa}$$

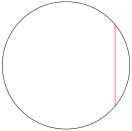
$$\mu_s := 0.50 \quad \text{Pa s}$$

Different PVD conformations





[b6_case_008]

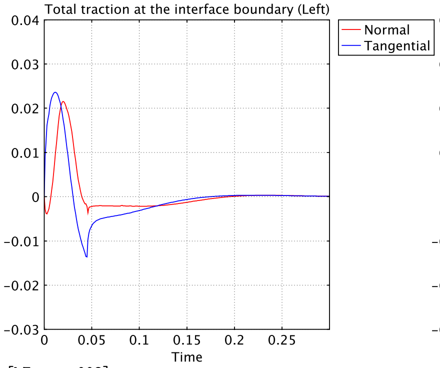


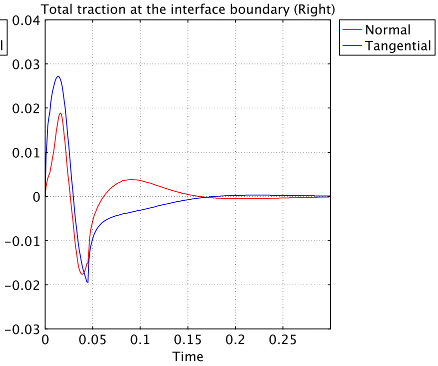
$$c_{01} := 2.5 \quad \text{Pa}$$

$$m_{01} := 10.0 \quad \text{Pa}$$

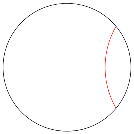
$$\mu_s := 0.50 \quad \text{Pa s}$$

Different PVD conformations





[b7_case_008]

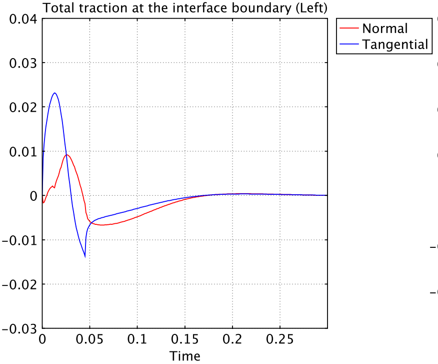


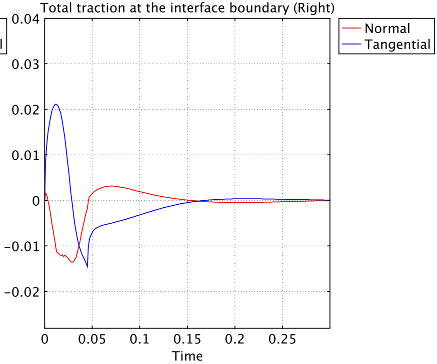
$$c_{01} := 2.5 \quad \text{Pa}$$

$$m_{01} := 10.0 \quad \text{Pa}$$

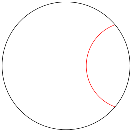
$$\mu_s := 0.50 \quad \text{Pa s}$$

Different PVD conformations





[b8_case_008]



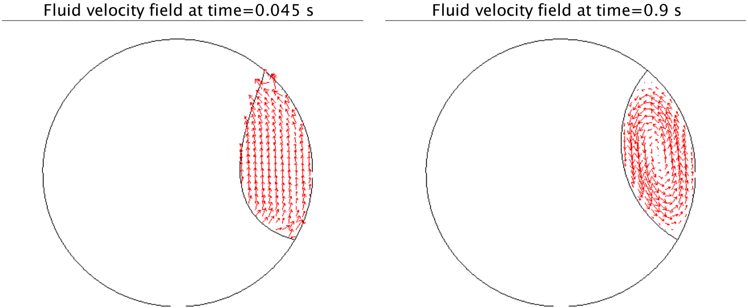
$$c_{01} := 2.5 \quad \text{Pa}$$

$$m_{01} := 10.0 \quad \text{Pa}$$

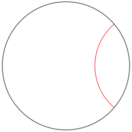
$$\mu_s := 0.50 \quad \text{Pa s}$$

Velocity field and pressure in the fluid part of the vitreous

Fluid vitreous velocity field



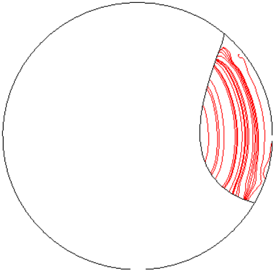
[a2_case_003]



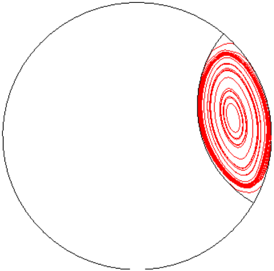
$$c_{01} := 0.5 \quad \text{Pa}$$
$$m_{01} := 10 \quad \text{Pa}$$
$$\mu_s := 0.5 \quad \text{Pa s}$$

Fluid vitreous velocity field

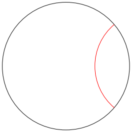
Streamlines of the fluid flow at time=0.045 s



Streamlines of the fluid flow at time=0.9 s



[a2_case_003]

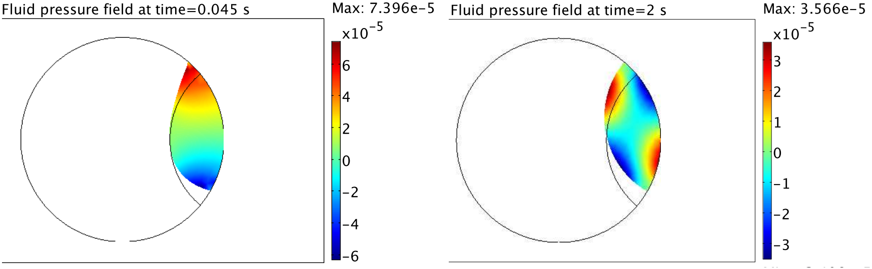


$$c_{01} := 0.5 \quad \text{Pa}$$

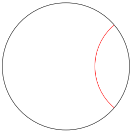
$$m_{01} := 10 \quad \text{Pa}$$

$$\mu_s := 0.5 \quad \text{Pa s}$$

Fluid vitreous pressure field

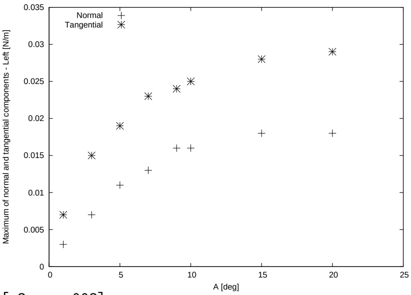


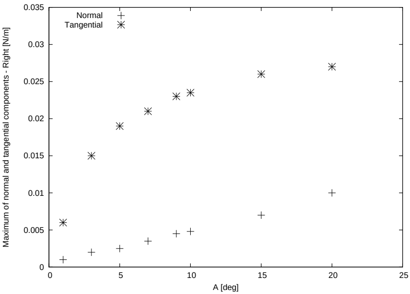
[a2_case_003]



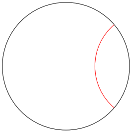
$$
\begin{aligned}
c_{01} &:= 0.5 && \text{Pa} \\
m_{01} &:= 10 && \text{Pa} \\
\mu_s &:= 0.5 && \text{Pa s}
\end{aligned}
$$

Increasing saccade amplitudes





[a2_case_008]



$$c_{01} := 2.5 \quad \text{Pa}$$

$$m_{01} := 10 \quad \text{Pa}$$

$$\mu_s := 0.5 \quad \text{Pa s}$$

Appendix: Piola stress and material properties

Frame indifference

$$\mathsf{S}\cdot\mathsf{W}\mathsf{F}=0\quad\forall\mathsf{W}\mid\operatorname{sym}\mathsf{W}=0\quad\Rightarrow\quad\operatorname{skw}\mathsf{S}\mathsf{F}^{\mathsf{T}}=0$$

Mooney-Rivlin strain energy (incompressible material):

$$\varphi(\mathsf{F}):=c_{10}(\imath_1(\mathsf{C})-3)+c_{01}(\imath_2(\mathsf{C})-3)$$

Stress response (energetic + reactive + dissipative):

$$\mathsf{S}\mathsf{F}^{\mathsf{T}}=\widehat{\mathsf{S}}(\mathsf{F})\mathsf{F}^{\mathsf{T}}-\pi\,\mathsf{I}+\mu\,\dot{\mathsf{F}}\mathsf{F}^{-1}$$

$$\widehat{\mathsf{S}}(\mathsf{F})\cdot\dot{\mathsf{F}}=\frac{d\varphi(\mathsf{F})}{dt}\quad\Rightarrow\quad\widehat{\mathsf{S}}(\mathsf{F})\,\mathsf{F}^{\mathsf{T}}=2(c_{10}\mathsf{F}\,\mathsf{F}^{\mathsf{T}}-c_{01}\mathsf{F}^{-\mathsf{T}}\mathsf{F}^{-1})$$

Dissipation principle:

$$\mathsf{S}\cdot\dot{\mathsf{F}}-d\varphi(\mathsf{F})/dt\geq 0\quad\Rightarrow\quad\mu\geq 0$$

Appendix: Pressure

Pressure π is the *reactive* part of $\mathbf{T}$. In an incompressible solid/fluid the velocity fields are said to be *isochoric*. The trace of the velocity gradient turns out to be zero.
A reactive stress, whose power is zero for any isochoric velocity field, has to be a spherical tensor $-\pi\,\mathbf{I}$:

$$\pi\,\mathbf{I}\cdot\mathbf{G} = \pi\,\mathrm{tr}\,\mathbf{G} = 0$$