MECHANICS OF SOLIDS AND MATERIALS 2013-2014

2° Term. Class started on [2014-02-24], room A1.3

$(1-2)_1$ 16:00-18:00

Very small body: body point and its positions

$$p : \{A\} \times \mathbb{R} \to \mathcal{E}$$

Collection of body points

$$\mathcal{B} = \{A, B, C\}$$

placement $\quad p : \{A, B, C\} \to \mathcal{E}$

motion $\quad p : \{A, B, C\} \times \mathbb{R} \to \mathcal{E}$

one-parameter family of placements

trajectory, trajectories

shapes $\quad \mathrm{im}\, p \subset \mathcal{E}$

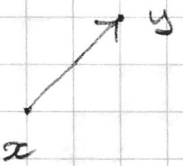
Operations on positions

$$x + u = y$$

$$\{\mathcal{E}, \mathcal{V}\}$$

translation space

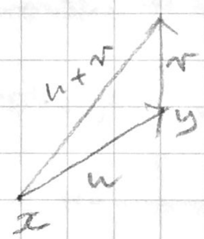
Euclidean space



$$y - x = u$$

for any two positions (x, y) there is a vector u translating x to y; this vector is unique

$$(x+u)+v = x+(u+v) \Rightarrow$$

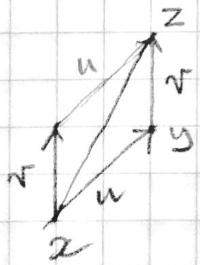


$$x + 0 = x$$

$$x+u=y \qquad \Rightarrow \quad y-x=u$$

$$(x+u)+(-u) = x+0 = x$$

$$y + (-u) = x \quad \Rightarrow x - y = -u$$



$$(x+u)+v = x+(u+v)$$

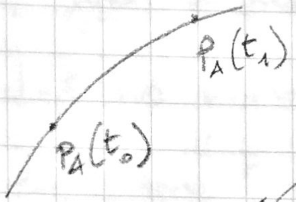
$$= x+(v+u) = (x+v)+u$$

Tuesday [2014-02-25]

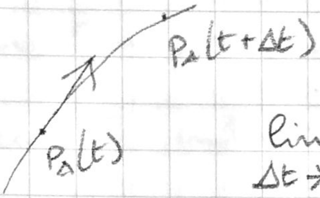
9:00 - 11:00

$(3\text{-}4)_1$

motion $p: \mathcal{B}\times\mathbb{R}\to\mathcal{E}$



trajectory parameterized by t

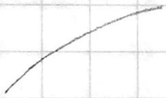


velocity vector

$$\lim_{\Delta t\to 0}\frac{1}{\Delta t}\left(p_A(t+\Delta t)-p_A(t)\right)$$

Curve-like shape parameterized by a scalar s

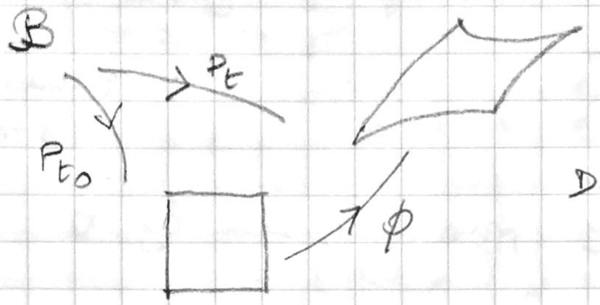
Square-like shape parameterized by two scalars s_1, s_2



$$\hat{x}: [0,\ell]\to p_t(\mathcal{B})$$

$$\hat{x}: [0,\ell_1]\times[0,\ell_2]\to p_t(\mathcal{B})$$

Dimension of the shape = Dimension of the body



Current shape

Deformation

Reference shape

motion description

$$\phi : \underbrace{p_{t_0}(\mathcal{B})}_{\bar{\mathcal{R}}} \times \mathbb{R} \to \mathcal{E}$$

$$\phi(\bar{\mathcal{R}}, t) = \underbrace{\mathcal{R}_t}_{\text{"region" of } \mathcal{E}}$$

We will discover properties of a deformation by comparing shapes

Wednesday [2014-02-26]

(5-6)$_1$ 11:00 – 13:00

Rigid deformation (it's not an OXYMORON)

let us introduce first the "distance" between two positions.

We need to introduce a scalar product in $\mathcal{V}$ and the induced norm.

The distance between any two position is left unchanged by a rigid deformation.

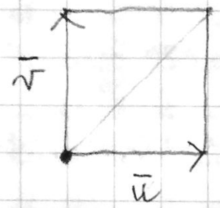
$$\|p_A - p_B\| = \|\bar{p}_A - \bar{p}_B\|$$

$$\|u + v\|^2 = (u+v)\cdot(u+v) = u\cdot u + 2u\cdot v + v\cdot v$$
$$= \|u\|^2 + 2u\cdot v + \|v\|^2$$

$$\|\bar{u} + \bar{v}\|^2 = \|\bar{u}\|^2 + 2\,\bar{u}\cdot\bar{v} + \|\bar{v}\|^2$$

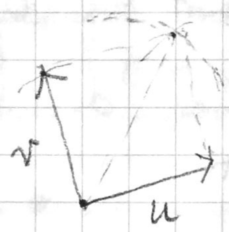
Subtracting we get $\quad 0 = u\cdot v - \bar{u}\cdot\bar{v}$

He the scalar product of any two vectors is left unchanged



$$\bar{u} \mapsto u \quad , \quad \bar{v} \mapsto v$$

$$\bar{u} \cdot \bar{v} = 0 \quad \Rightarrow \quad u \cdot v = 0$$



$$\bar{u} + \bar{v} \mapsto u + v \qquad \text{linearity}$$

$$u = R\bar{u} \quad , \quad v = R\bar{v}$$

$$u \cdot v = \bar{u} \cdot \bar{v} \quad \Rightarrow \quad R\bar{u} \cdot R\bar{v} = \bar{u} \cdot \bar{v}$$

$$R^T R\, \bar{u} \cdot \bar{v} = \bar{u} \cdot \bar{v}$$

$$\left(R^T R - I\right) \bar{u} \cdot \bar{v} = 0$$

Rigid deformation representation

$$\phi(\bar{p}_A) = \phi(\bar{p}_O) + R\left(\bar{p}_A - \bar{p}_O\right)$$

Rigid motion

$$\phi(\bar{p}_A, t) = \phi(\bar{p}_O, t) + R(t)\left(\bar{p}_A - \bar{p}_O\right)$$