neo-Hookean strain energy

```
mF[λ_] := {{λ, 0, 0}, {0, 1/Sqrt[λ], 0}, {0, 0, 1/Sqrt[λ]}}
mC[λ_] := Transpose[mF[λ]].mF[λ]
I1[λ_] := Tr[mC[λ]]
I2[λ_] := 1/2 (I1[λ]^2 - Tr[mC[λ]^2])
I1[λ[t]]
```

$$\frac{2}{\lambda[t]}+\lambda[t]^2$$

```
I2[λ[t]]
```

$$\frac{1}{2}\left(-\frac{2}{\lambda[t]^2}-\lambda[t]^4+\left(\frac{2}{\lambda[t]}+\lambda[t]^2\right)^2\right)$$

```
φ[λ_] := c (I1[λ] - 3)
φ[λ[t]] // FullSimplify
```

$$c\left(-3+\frac{2}{\lambda[t]}+\lambda[t]^2\right)$$

```
D[φ[λ[t]], t] // FullSimplify
```

$$\frac{2\,c\left(-1+\lambda[t]^3\right)\lambda'[t]}{\lambda[t]^2}$$

```
D[φ[λ[t]], t] λ[t]/λ'[t] // FullSimplify
```

$$\frac{2\,c\left(-1+\lambda[t]^3\right)}{\lambda[t]}$$

```
σ0[λ1_] = % /. λ[t] → λ1 // Simplify
```

$$\frac{2\,c\left(-1+\lambda 1^3\right)}{\lambda 1}$$

```
σ0[λ[t]]
```

$$\frac{2\,c\left(-1+\lambda[t]^3\right)}{\lambda[t]}$$

Uniaxial traction for a viscoelastic material

```
viscoEq = {σ0[λ[t]] + 3 μ λ'[t]/λ[t] == p0}
```

$$\left\{\frac{2\,c\left(-1+\lambda[t]^3\right)}{\lambda[t]}+\frac{3\,\mu\,\lambda'[t]}{\lambda[t]}=p0\right\}$$

```
viscoEq /. {λ → (λ0 + β dϵ[#] &)} // FullSimplify
```

$$\left\{\frac{2\,c\left(-1+(\lambda 0+\beta\,d\epsilon[t])^3\right)+3\,\beta\,\mu\,d\epsilon'[t]}{\lambda 0+\beta\,d\epsilon[t]}=p0\right\}$$

```
viscoEqβ = Series[Evaluate[viscoEq /. {λ → (λ0 + β dϵ[#] &)}], {β, 0, 1}] // FullSimplify // Normal
```

$$\left\{\frac{2\,c\,\left(-1+\lambda0^3\right)}{\lambda0}+\frac{\beta\,\left(2\,\left(c+2\,c\,\lambda0^3\right)\,d\epsilon[t]+3\,\lambda0\,\mu\,d\epsilon'[t]\right)}{\lambda0^2}==p0\right\}$$

```
viscoEqβ0 = viscoEqβ[[1]] /. β → 0
```

$$\frac{2\,c\,\left(-1+\lambda0^3\right)}{\lambda0}==p0$$

```
p0StaSol = Solve[viscoEqβ0, p0][[1]]
```

$$\left\{p0\to\frac{2\,c\,\left(-1+\lambda0^3\right)}{\lambda0}\right\}$$

```
viscoEqLin = {a dϵ[t] + dϵ'[t] == 0, dϵ[0] == dϵ0}
```

$$\left\{a\,d\epsilon[t]+d\epsilon'[t]==0,\ d\epsilon[0]==d\epsilon0\right\}$$

```
viscoEqLin = viscoEqβ /. p0StaSol /. β → 1 // FullSimplify
```

$$\left\{\frac{2\,\left(c+2\,c\,\lambda0^3\right)\,d\epsilon[t]}{\lambda0}+3\,\mu\,d\epsilon'[t]==0\right\}$$

```
dϵSol = DSolve[Join[viscoEqLin, {dϵ[0] == dϵ0}], dϵ, t][[1]]
```

$$\left\{d\epsilon\to\text{Function}\left[\{t\},\ d\epsilon0\,e^{-\frac{2\,t\,\left(c+2\,c\,\lambda0^3\right)}{3\,\lambda0\,\mu}}\right]\right\}$$

```
λ0Sol = Assuming[λ0 > 0 && c > 0, Solve[viscoEqβ0, λ0] // FullSimplify]
```

$$\left\{\left\{\lambda0\to\frac{6^{1/3}\,c\,p0+\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{2/3}}{6^{2/3}\,c\,\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{1/3}}\right\},\right.$$

$$\left\{\lambda0\to\frac{-(-6)^{1/3}\,c\,p0+(-1)^{2/3}\,\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{2/3}}{6^{2/3}\,c\,\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{1/3}}\right\},$$

$$\left.\left\{\lambda0\to\frac{(-6)^{2/3}\,c\,p0-(-6)^{1/3}\,\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{2/3}}{6\,c\,\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{1/3}}\right\}\right\}$$

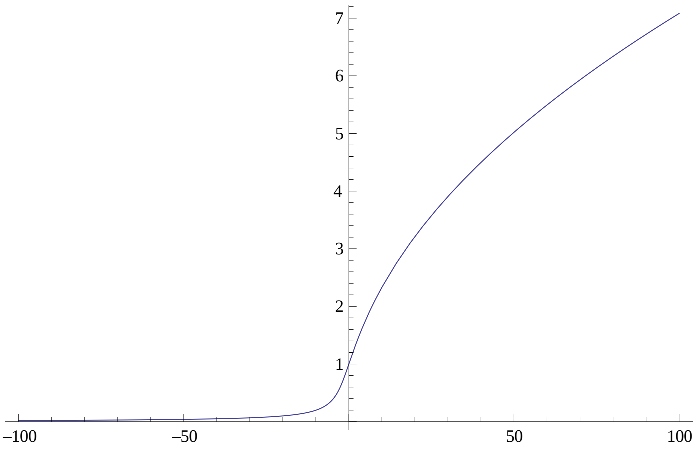
```
λ0Sol1 = λ0Sol[[1]]
```

$$\left\{\lambda0\to\frac{6^{1/3}\,c\,p0+\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{2/3}}{6^{2/3}\,c\,\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{1/3}}\right\}$$

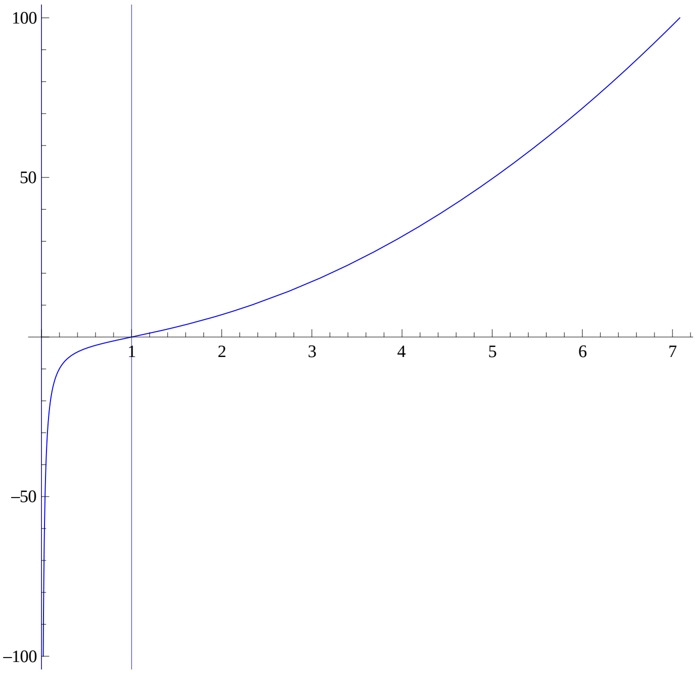
```
λ0f[p0_] = λ0 /. λ0Sol1 // FullSimplify
```

$$\frac{6^{1/3}\,c\,p0+\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{2/3}}{6^{2/3}\,c\,\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{1/3}}$$

```
Block[{c = 1}, Plot[λ0f[p0], {p0, -100, 100}]]
```




```
Block[{c = 1}, ParametricPlot[{λ0f[p0], p0}, {p0, -100, 100}, PlotRange → Automatic,
  AspectRatio → 1, GridLines → {{{0, {Blue}}, {1, {Blue}}}, None}, PlotStyle → {Blue}]]
```




```
viscoEqLin
```

$$\left\{\frac{2\left(c+2\,c\,\lambda 0^3\right) d\epsilon[t]}{\lambda 0}+3\,\mu\, d\epsilon'[t]==0\right\}$$

```
viscoEqLin1 = viscoEqLin[[1]]
```

$$\frac{2\left(c+2\,c\,\lambda 0^3\right) d\epsilon[t]}{\lambda 0}+3\,\mu\, d\epsilon'[t]==0$$

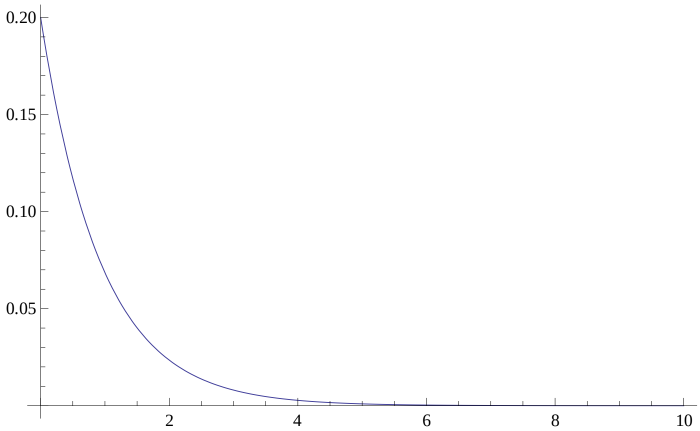
```
Block[{pc = 50}, viscoEqLin1]
```

$$\frac{2\left(c+2\,c\,\lambda 0^3\right) d\epsilon[t]}{\lambda 0}+3\,\mu\, d\epsilon'[t]==0$$

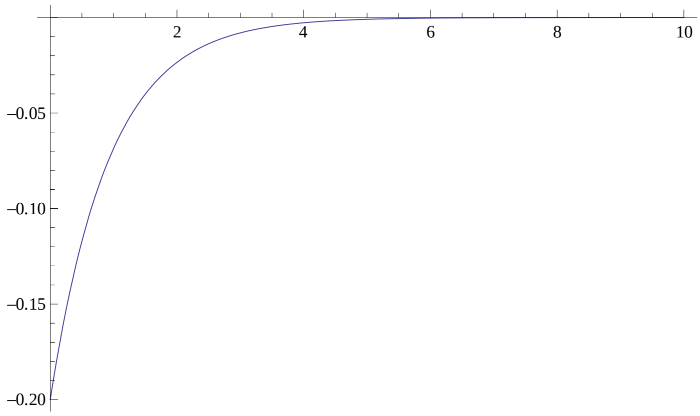
```
dϵs = dϵ /. dϵSol
```

$$\text{Function}\left[\{t\},\ d\epsilon 0\, e^{-\frac{2\,t\,\left(c+2\,c\,\lambda 0^3\right)}{3\,\lambda 0\,\mu}}\right]$$

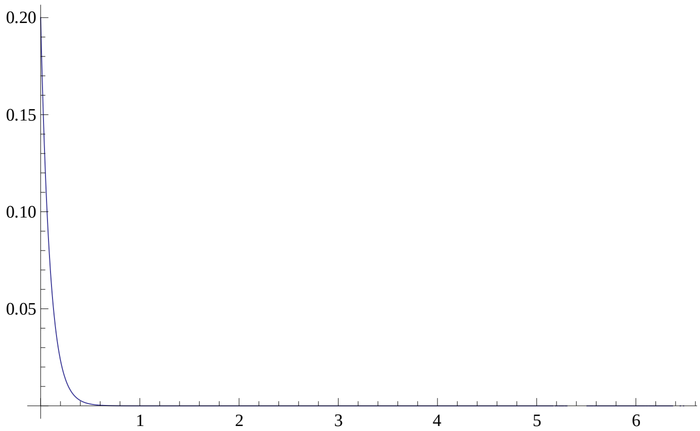
```
Block[{p0 = 15, c = 1, μ = 10, λ0 = λ0f[p0], tlim = 10, λN, dϵ0 = 0.2},
 Plot[dϵs[t], {t, 0, tlim}, PlotRange → All]]
```



```
Block[{p0 = 15, c = 1, μ = 10, λ0 = λ0f[p0], tlim = 10, λN, dϵ0 = -0.2},
 Plot[dϵs[t], {t, 0, tlim}, PlotRange → All]]
```



```
Block[{p0 = 15, c = 1, μ = 1, λ0 = λ0f[p0], tlim = 10, λN, dϵ0 = 0.2},
 Plot[dϵs[t], {t, 0, tlim}, PlotRange → All]]
```



```
viscoEqLin
```

$$\left\{\frac{2\left(c+2\,c\,\lambda 0^3\right)\,d\epsilon[t]}{\lambda 0}+3\,\mu\,d\epsilon'[t]==0\right\}$$

```
Block[{p0 = 50, c = 1, μ = 1, λ0 = λ0f[p0], tlim = 10, dϵ0 = 0.2}, λ0f[p0] // FullSimplify // N]
```

5.01988

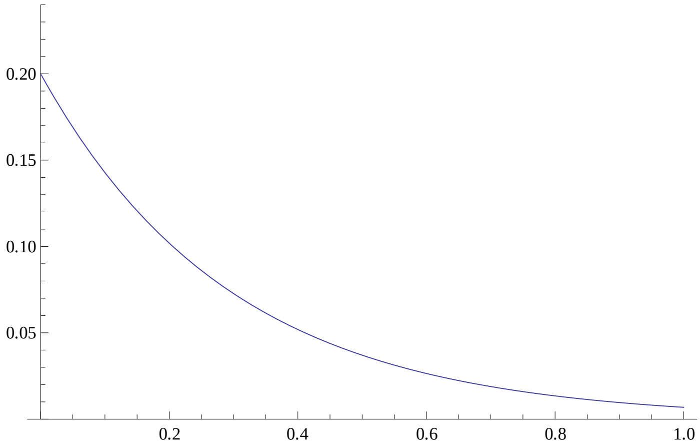
```
dϵs[t]
```

$d\epsilon 0\, e^{-\frac{2 t (c+2 c \lambda 0^3)}{3 \lambda 0 \mu}}$

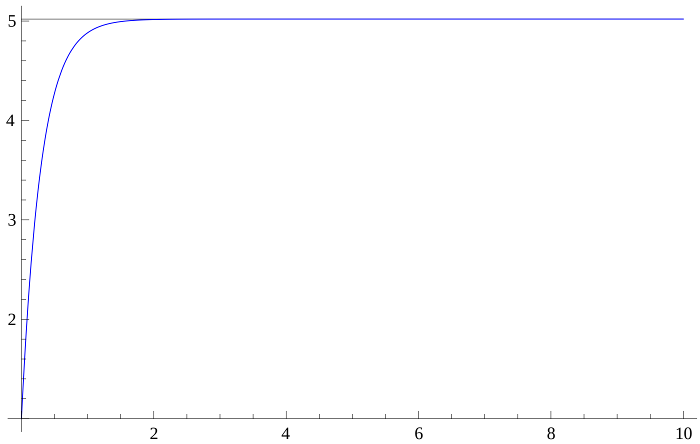
```
Block[{p0 = 50, c = 1, μ = 1, λ0 = λ0f[p0], tlim = 10}, dϵs[t] // Simplify // N // Chop]
```

$2.71828^{-33.7317 t}\, d\epsilon 0$

```
Block[{p0 = 50, c = 1, λ0 = λ0f[p0], dϵ0 = 0.2, μ = 10},
 Plot[dϵs[t], {t, 0, 1}, PlotRange → {0, 1.2 dϵ0}]]
```



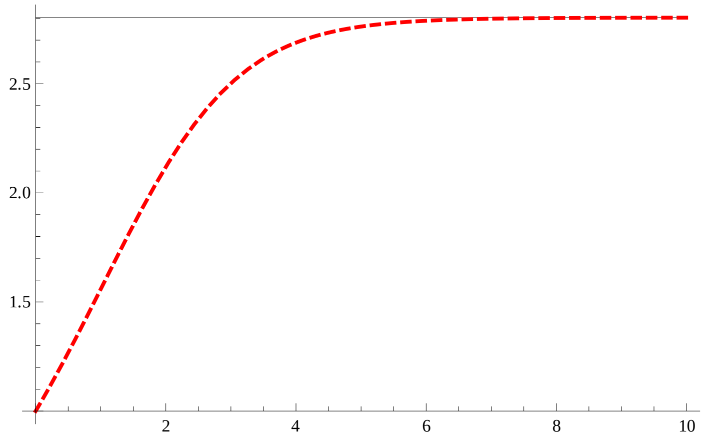

```
Block[{p0 = 50, c = 1, λ0 = λ0f[p0], dϵ0 = -(λ0 - 1), μ = 10, tlim = 10},
 Plot[{λ0, λ0 + dϵs[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]
```



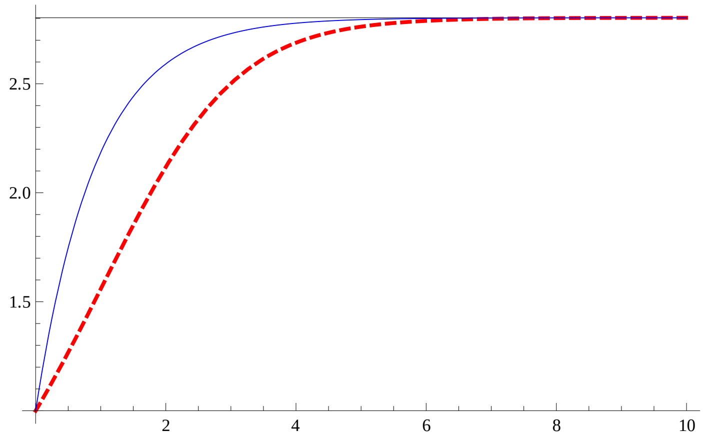

```
Block[{p0 = 5, c = 1}, λ0f[p0] // N // Chop]
```

1.75233

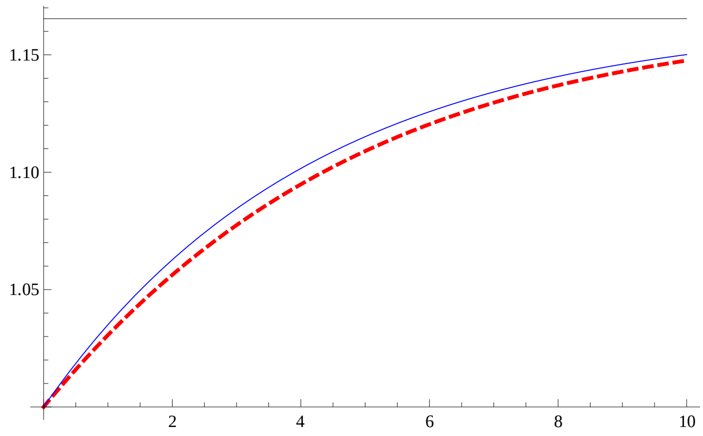
```
Block[{p0 = 15, c = 1, μ = 10, λ0 = λ0f[p0], tlim = 10, λN},
 λN = λ /. NDSolve[Join[viscoEq, {λ[0] == 1}], λ, {t, 0, tlim}][[1]];
Plot[{λ0, λN[t]}, {t, 0, tlim}, PlotRange → All,
  PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}]]
```



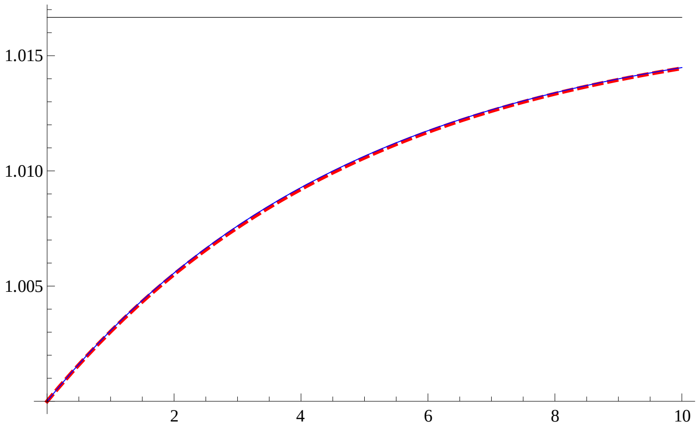

```
Block[{p0 = 15, c = 1, μ = 10, λ0 = λ0f[p0], dϵ0 = (1 - λ0), λN, tlim = 10},
 λN = λ /. NDSolve[Join[viscoEq, {λ[0] == 1}], λ, {t, 0, tlim}][[1]]; Show[Plot[{λ0, λN[t]},
   {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}],
  Plot[{λ0, λ0 + dϵs[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



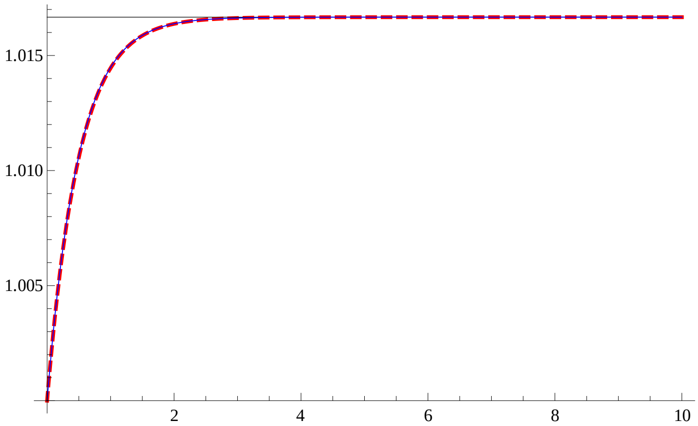

```
Block[{p0 = 1, c = 1, μ = 10, λ0 = λ0f[p0], dϵ0 = (1 - λ0), λN, tlim = 10},
 λN = λ /. NDSolve[Join[viscoEq, {λ[0] == 1}], λ, {t, 0, tlim}][[1]]; Show[Plot[{λ0, λN[t]},
   {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}],
  Plot[{λ0, λ0 + dϵs[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



```
Block[{p0 = 0.1, c = 1, μ = 10, λ0 = λ0f[p0], dϵ0 = (1 - λ0), λN, tlim = 10},
 λN = λ /. NDSolve[Join[viscoEq, {λ[0] == 1}], λ, {t, 0, tlim}][[1]]; Show[Plot[{λ0, λN[t]},
   {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}],
  Plot[{λ0, λ0 + dϵs[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```




```
Block[{p0 = 0.1, c = 1, μ = 1, λ0 = λ0f[p0], dϵ0 = (1 - λ0), λN, tlim = 10},
 λN = λ /. NDSolve[Join[viscoEq, {λ[0] == 1}], λ, {t, 0, tlim}][[1]]; Show[Plot[{λ0, λN[t]},
   {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}],
  Plot[{λ0, λ0 + dϵs[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



Oscillations for a viscoelastic material

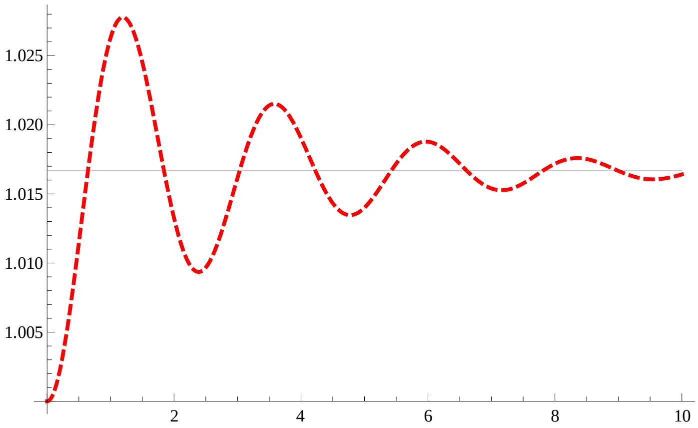
oscEq =

$$\left\{\sigma 0[\lambda[t]] + 3\mu \frac{\lambda'[t]}{\lambda[t]} == p0 - \frac{1}{2}\varrho\left(2\lambda[t] + \frac{\alpha^2}{\lambda[t]^2}\right)\lambda''[t] + \frac{3}{4}\varrho\frac{\alpha^2}{\lambda[t]^3}\lambda'[t]^2,\ \lambda[0] == \lambda 0 + d\epsilon 0,\ \lambda'[0] == 0\right\}$$

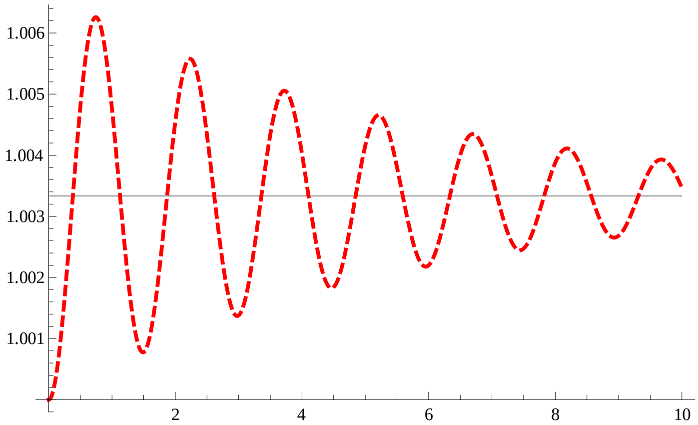
$$\left\{\frac{2\,c\,\left(-1 + \lambda[t]^3\right)}{\lambda[t]} + \frac{3\mu\lambda'[t]}{\lambda[t]} == p0 + \frac{3\alpha^2\varrho\lambda'[t]^2}{4\lambda[t]^3} - \frac{1}{2}\varrho\left(\frac{\alpha^2}{\lambda[t]^2} + 2\lambda[t]\right)\lambda''[t],\right.$$

$$\left.\lambda[0] == d\epsilon 0 + \lambda 0,\ \lambda'[0] == 0\right\}$$

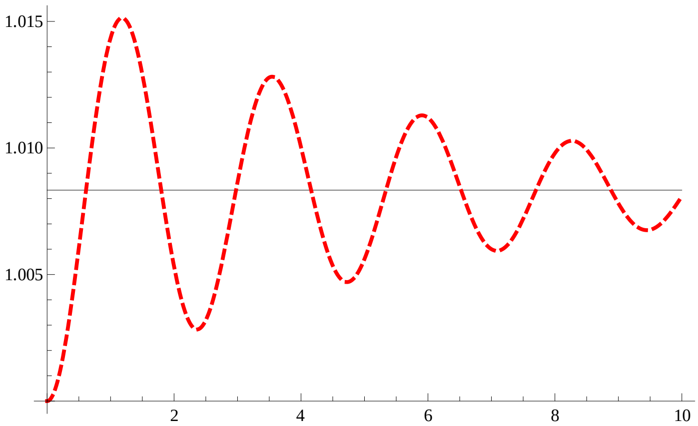
```
Block[{p0 = 0.1, c = 1, μ = 0.2, λ0 = λ0f[p0], dϵ0 = (1 - λ0), λN, tlim = 10, α = 0.1, ϱ = 10/12},
 λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]]; Show[Plot[{λ0, λN[t]}, {t, 0, tlim},
   PlotRange → All, PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}]]]
```



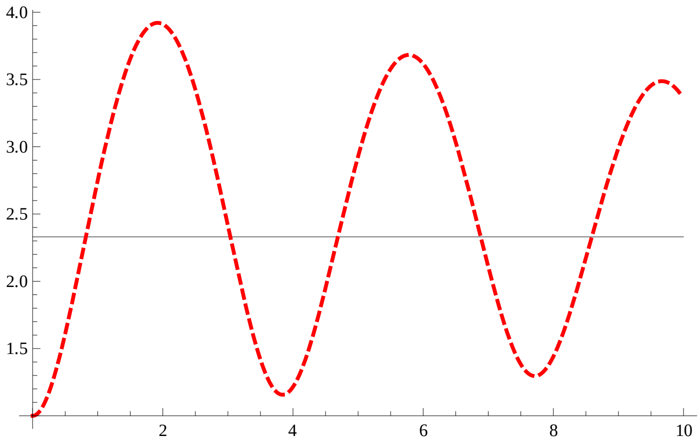

```
Block[{p0 = 0.1, c = 5, μ = 0.2, λ0 = λ0f[p0], dϵ0 = (1 - λ0), λN, tlim = 10, α = 0.1, ϱ = 20/12},
 λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]]; Show[Plot[{λ0, λN[t]}, {t, 0, tlim},
   PlotRange → All, PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}]]]
```



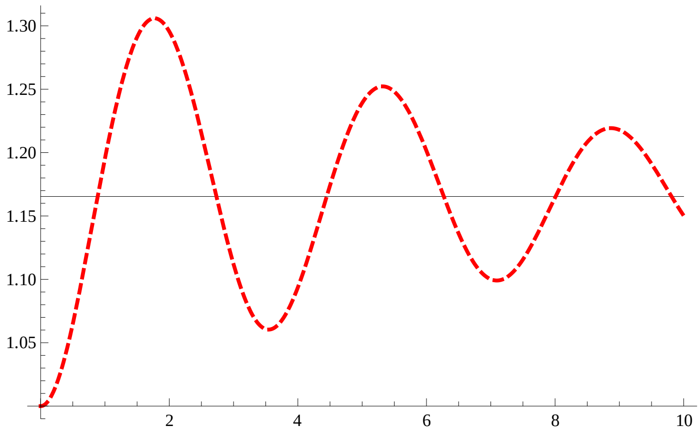

```
Block[{p0 = 0.1, c = 2, μ = 0.2, λ0 = λ0f[p0], dϵ0 = (1 - λ0), λN, tlim = 10, α = 0.1, ϱ = 20/12},
 λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]]; Show[Plot[{λ0, λN[t]}, {t, 0, tlim},
   PlotRange → All, PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}]]]
```



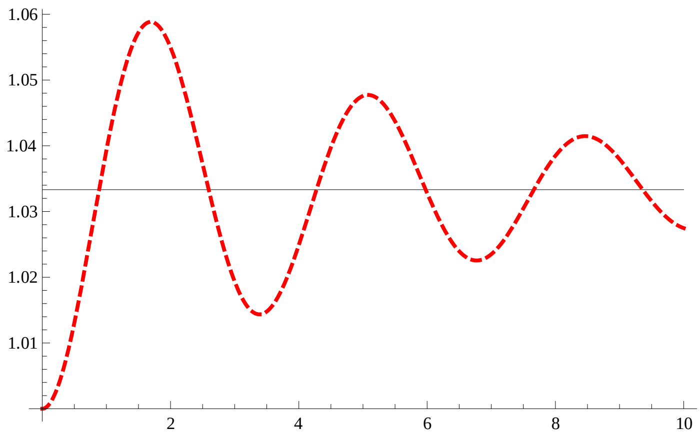
```
Block[{p0 = 10, c = 1, μ = 0.2, λ0 = λ0f[p0], dϵ0 = (1 - λ0), λN, tlim = 10, α = 0.1, ϱ = 20 / 12},
  λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]]; Show[Plot[{λ0, λN[t]}, {t, 0, tlim},
    PlotRange → All, PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}]]]
```




```
Block[{p0 = 1, c = 1, μ = 0.2, λ0 = λ0f[p0], dϵ0 = (1 - λ0), λN, tlim = 10, α = 0.1, ϱ = 20 / 12},
  λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]]; Show[Plot[{λ0, λN[t]}, {t, 0, tlim},
    PlotRange → All, PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}]]]
```




```
Block[{p0 = 0.2, c = 1, μ = 0.2, λ0 = λ0f[p0], dϵ0 = (1 - λ0), λN, tlim = 10, α = 0.1, ϱ = 20 / 12},
  λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]]; Show[Plot[{λ0, λN[t]}, {t, 0, tlim},
    PlotRange → All, PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}]]]
```



Small oscillations for a viscoelastic material

```
oscEq /. {λ → (λ0 + β dϵ[#] &)} // FullSimplify
```

$$\left\{\frac{2\,c\,\left(-1+(\lambda0+\beta\,d\epsilon[t])^3\right)+3\,\beta\,\mu\,d\epsilon'[t]}{\lambda0+\beta\,d\epsilon[t]} == p0+\frac{3\,\alpha^2\,\beta^2\,\varrho\,d\epsilon'[t]^2}{4\,(\lambda0+\beta\,d\epsilon[t])^3}-\frac{1}{2}\beta\,\varrho\left(2\,\lambda0+2\,\beta\,d\epsilon[t]+\frac{\alpha^2}{(\lambda0+\beta\,d\epsilon[t])^2}\right)d\epsilon''[t],\ d\epsilon0 == \beta\,d\epsilon[0],\ \beta\,d\epsilon'[0] == 0\right\}$$

```
oscEqβ = Series[Evaluate[oscEq /. {λ → (λ0 + β dϵ[#] &)}], {β, 0, 1}] // FullSimplify // Normal
```

$$\left\{-p0+\frac{2\,c\,\left(-1+\lambda0^3\right)}{\lambda0}+\frac{1}{2\,\lambda0^2}\beta\left(4\left(c+2\,c\,\lambda0^3\right)d\epsilon[t]+6\,\lambda0\,\mu\,d\epsilon'[t]+\left(\alpha^2+2\,\lambda0^3\right)\varrho\,d\epsilon''[t]\right) == 0,\ \lambda0+\beta\,d\epsilon[0] == d\epsilon0+\lambda0,\ \beta\,d\epsilon'[0] == 0\right\}$$

```
oscEqβ[[1]] /. β → 0
```

$$-p0+\frac{2\,c\,\left(-1+\lambda0^3\right)}{\lambda0} == 0$$

```
p0StaSol
```

$$\left\{p0\to\frac{2\,c\,\left(-1+\lambda0^3\right)}{\lambda0}\right\}$$

```
p0StaSol = Solve[oscEqβ[[1]] /. β → 0, p0][[1]]
```

$$\left\{p0\to\frac{2\,c\,\left(-1+\lambda0^3\right)}{\lambda0}\right\}$$

```
oscEqβ /. p0StaSol // Simplify
```

$$\left\{\frac{1}{\lambda0}\beta\left(4\left(c+2\,c\,\lambda0^3\right)d\epsilon[t]+6\,\lambda0\,\mu\,d\epsilon'[t]+\left(\alpha^2+2\,\lambda0^3\right)\varrho\,d\epsilon''[t]\right) == 0,\ d\epsilon0 == \beta\,d\epsilon[0],\ \beta\,d\epsilon'[0] == 0\right\}$$

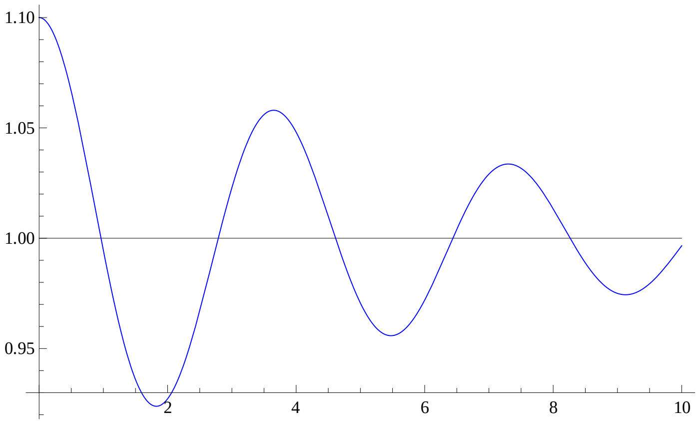
```
oscEqLin = oscEqβ /. p0StaSol /. β → 1 // FullSimplify
```

$$\left\{\frac{4\left(c+2\,c\,\lambda0^3\right)d\epsilon[t]+6\,\lambda0\,\mu\,d\epsilon'[t]+\left(\alpha^2+2\,\lambda0^3\right)\varrho\,d\epsilon''[t]}{\lambda0} == 0,\ d\epsilon0 == d\epsilon[0],\ d\epsilon'[0] == 0\right\}$$

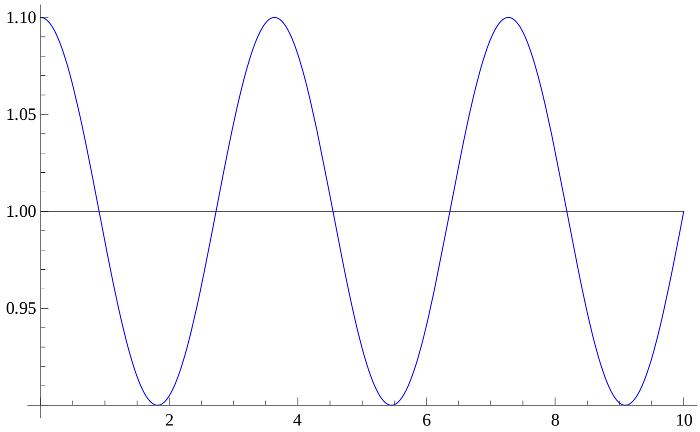
```
Block[{p0 = 0, c = 1, μ = 0.2, λ0 = λ0f[p0], dϵ0 = 1.6 (1 - λ0), dϵNlin, tlim = 10, α = 1, ϱ = 24}, dϵ0]
```

0.

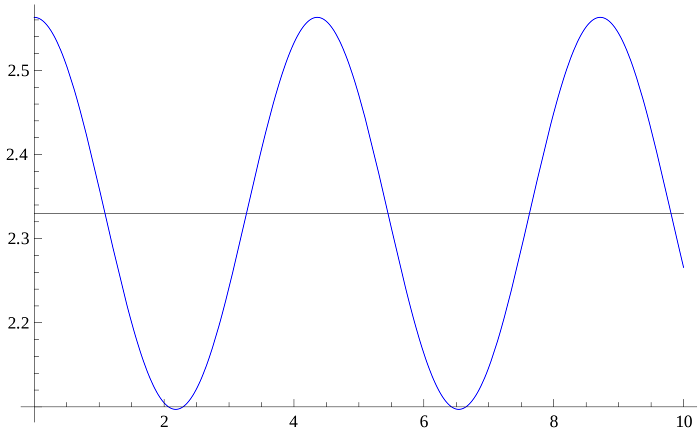
```
Block[{p0 = 0, c = 1, μ = 0.2, λ0 = λ0f[p0], dϵ0 = 0.1 λ0, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[
  Plot[{λ0, λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



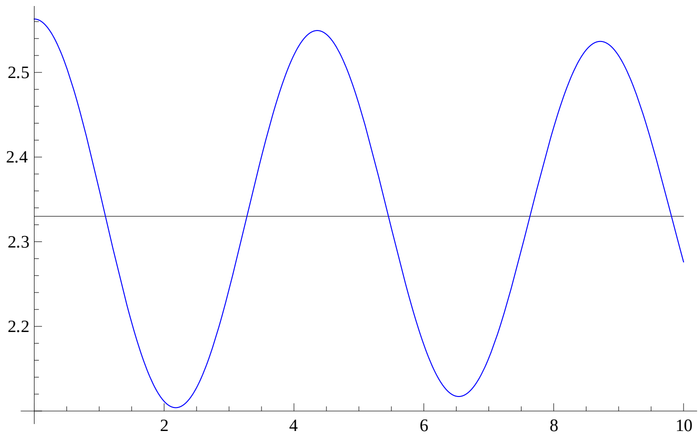
```
Block[{p0 = 0, c = 1, μ = 0, λ0 = λ0f[p0], dϵ0 = 0.1 λ0, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[
  Plot[{λ0, λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



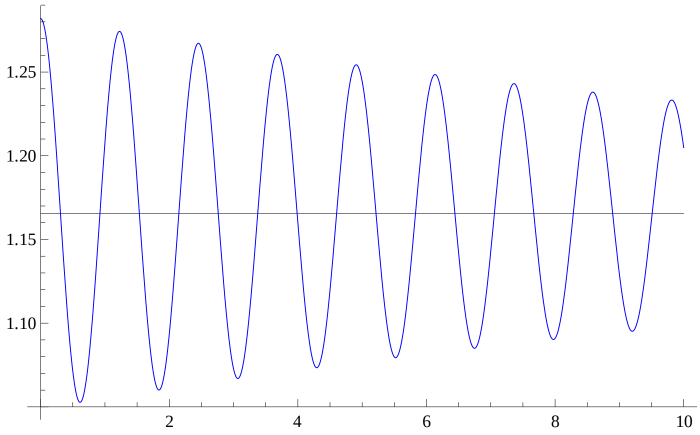

```
Block[{p0 = 10, c = 1, μ = 0, λ0 = λ0f[p0], dϵ0 = 0.1 λ0, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[
  Plot[{λ0, λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



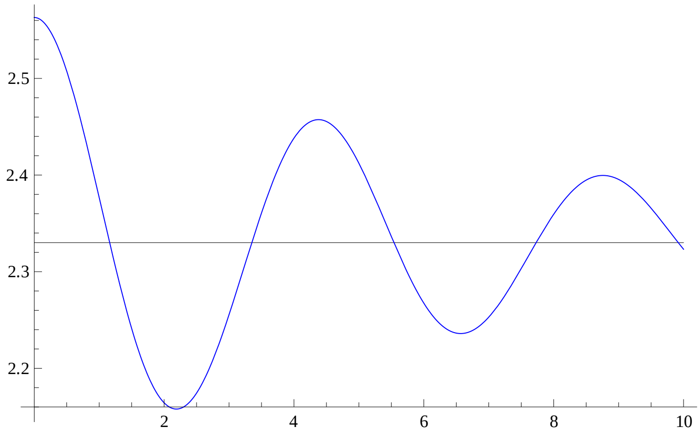

```
Block[{p0 = 10, c = 1, μ = 0.1, λ0 = λ0f[p0], dϵ0 = 0.1 λ0, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[
  Plot[{λ0, λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



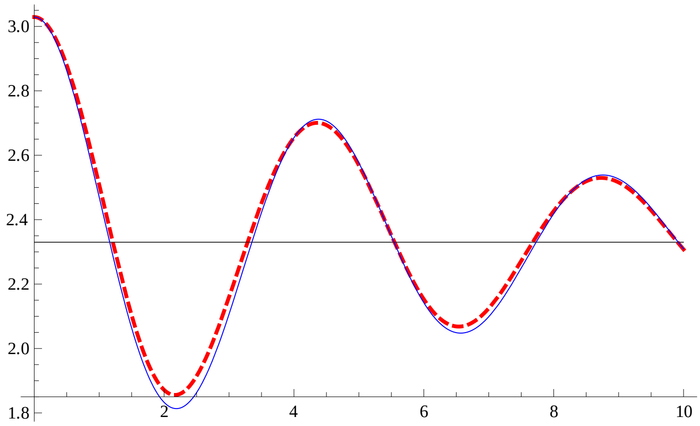
```
Block[{p0 = 10, c = 10, μ = 0.1, λ0 = λ0f[p0], dϵ0 = 0.1 λ0, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[
  Plot[{λ0, λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



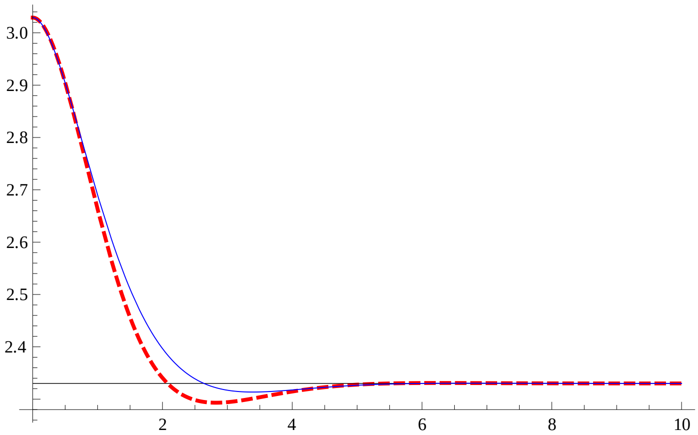

```
Block[{p0 = 10, c = 1, μ = 1, λ0 = λ0f[p0], dϵ0 = 0.1 λ0, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[
  Plot[{λ0, λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



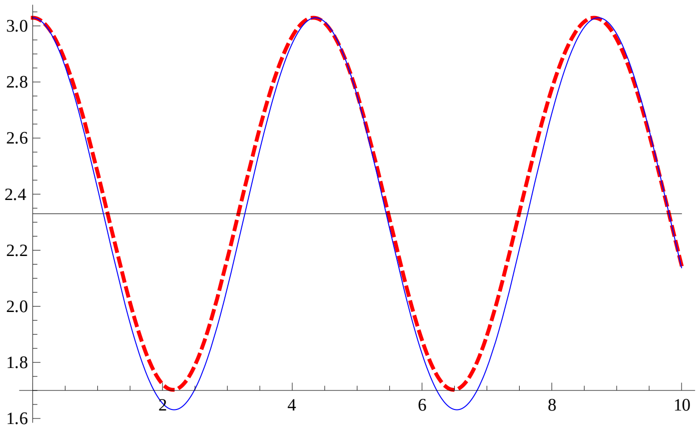

```
Block[{p0 = 10, c = 1, μ = 1, λ0 = λ0f[p0], dϵ0 = 0.3 λ0, λN, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]];
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[Plot[{λ0, λN[t]},
   {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}],
  Plot[{λ0, λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



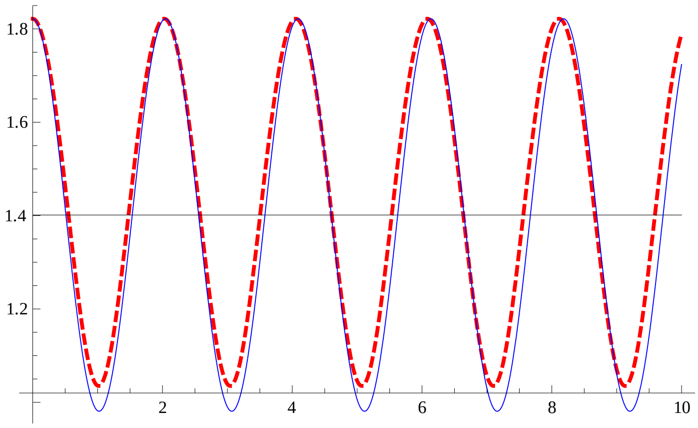
```
Block[{p0 = 10, c = 1, μ = 8, λ0 = λ0f[p0], dϵ0 = 0.3 λ0, λN, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]];
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[Plot[{λ0, λN[t]},
   {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}],
  Plot[{λ0, λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```




```
Block[{p0 = 10, c = 1, μ = 0, λ0 = λ0f[p0], λN, dϵ0 = 0.3 λ0, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]];
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[Plot[{λ0, λN[t]},
   {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}],
  Plot[{λ0, λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```




```
Block[{p0 = 10, c = 4, μ = 0, λ0 = λ0f[p0], λN, dϵ0 = 0.3 λ0, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]];
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[Plot[{λ0, λN[t]},
   {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}],
  Plot[{λ0, λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



Small oscillations for a viscoelastic material (critical viscosity value)

```
(oscEqLin[[1, 1]] - oscEqLin[[1, 2]]) λ0 // FullSimplify
```

$4\left(c+2\,c\,\lambda 0^3\right) d\epsilon[t]+6\,\lambda 0\,\mu\, d\epsilon'[t]+\left(\alpha^2+2\,\lambda 0^3\right)\varrho\, d\epsilon''[t]$

```
oscEqLin1 = (oscEqLin[[1, 1]] - oscEqLin[[1, 2]]) λ0/((α^2 + 2 λ0^3) ϱ) // FullSimplify
```

$$\frac{4\,c\,d\epsilon[t]+8\,c\,\lambda 0^3\,d\epsilon[t]+6\,\lambda 0\,\mu\,d\epsilon'[t]}{\alpha^2\varrho+2\,\lambda 0^3\varrho}+d\epsilon''[t]$$

```
oscEqLin1/(dϵ0 e^(t κ)) /. dϵ → (dϵ0 Exp[κ #] &) // FullSimplify
```

$$\kappa^2+\frac{4\,c+8\,c\,\lambda 0^3+6\,\kappa\,\lambda 0\,\mu}{\alpha^2\varrho+2\,\lambda 0^3\varrho}$$

```
κSol = Solve[% == 0, κ] // FullSimplify
```

$$\left\{\left\{\kappa\to-\frac{3\,\lambda 0\,\mu+\sqrt{9\,\lambda 0^2\mu^2-4\,c\left(1+2\,\lambda 0^3\right)\left(\alpha^2+2\,\lambda 0^3\right)\varrho}}{\left(\alpha^2+2\,\lambda 0^3\right)\varrho}\right\},\left\{\kappa\to\frac{-3\,\lambda 0\,\mu+\sqrt{9\,\lambda 0^2\mu^2-4\,c\left(1+2\,\lambda 0^3\right)\left(\alpha^2+2\,\lambda 0^3\right)\varrho}}{\left(\alpha^2+2\,\lambda 0^3\right)\varrho}\right\}\right\}$$

```
κ /. κSol[[1]] // FullSimplify
```

$$-\frac{3\,\lambda 0\,\mu+\sqrt{9\,\lambda 0^2\mu^2-4\,c\left(1+2\,\lambda 0^3\right)\left(\alpha^2+2\,\lambda 0^3\right)\varrho}}{\left(\alpha^2+2\,\lambda 0^3\right)\varrho}$$

```
κ /. κSol[[2]] // FullSimplify
```

$$\frac{-3\,\lambda 0\,\mu+\sqrt{9\,\lambda 0^2\mu^2-4\,c\left(1+2\,\lambda 0^3\right)\left(\alpha^2+2\,\lambda 0^3\right)\varrho}}{\left(\alpha^2+2\,\lambda 0^3\right)\varrho}$$

```
κdis = (κ /. κSol[[1]]) - (κ /. κSol[[2]]) // FullSimplify
```

$$-\frac{2\sqrt{9\,\lambda 0^2\mu^2-4\,c\left(1+2\,\lambda 0^3\right)\left(\alpha^2+2\,\lambda 0^3\right)\varrho}}{\left(\alpha^2+2\,\lambda 0^3\right)\varrho}$$

```
κdis2 = (Numerator[κdis]/2)^2 // FullSimplify
```

$9\,\lambda 0^2\mu^2-4\,c\left(1+2\,\lambda 0^3\right)\left(\alpha^2+2\,\lambda 0^3\right)\varrho$

```
κdis2 /. λ0 → 1 // Simplify
```

$9\,\mu^2-12\,c\left(2+\alpha^2\right)\varrho$

```
Solve[κdis2 == 0 /. μ^2 → μ2, μ2][[1]]
```

$$\left\{\mu 2\to\frac{4\,c\left(1+2\,\lambda 0^3\right)\left(\alpha^2+2\,\lambda 0^3\right)\varrho}{9\,\lambda 0^2}\right\}$$

```
μ0 = Sqrt[μ2] /. Solve[κdis2 == 0 /. μ^2 → μ2, μ2][[1]] // FullSimplify // PowerExpand
```

$$\frac{2\sqrt{c}\sqrt{1+2\,\lambda 0^3}\sqrt{\alpha^2+2\,\lambda 0^3}\sqrt{\varrho}}{3\,\lambda 0}$$

```
dϵgSol = DSolve[oscEqLin1 == 0, dϵ, t] // FullSimplify
```

$$\left\{\left\{d\epsilon \to \text{Function}\left[\{t\},\ e^{\frac{t\left(-6\,\lambda 0\,\mu-\sqrt{36\,\lambda 0^2\,\mu^2-4\,(4\,c+8\,c\,\lambda 0^3)\,(\alpha^2\,\varrho+2\,\lambda 0^3\,\varrho)}\right)}{2\,(\alpha^2\,\varrho+2\,\lambda 0^3\,\varrho)}}\,C[1]+e^{\frac{t\left(-6\,\lambda 0\,\mu+\sqrt{36\,\lambda 0^2\,\mu^2-4\,(4\,c+8\,c\,\lambda 0^3)\,(\alpha^2\,\varrho+2\,\lambda 0^3\,\varrho)}\right)}{2\,(\alpha^2\,\varrho+2\,\lambda 0^3\,\varrho)}}\,C[2]\right]\right\}\right\}$$

```
κSol
```

$$\left\{\left\{\kappa \to -\frac{3\,\lambda 0\,\mu+\sqrt{9\,\lambda 0^2\,\mu^2-4\,c\,(1+2\,\lambda 0^3)\,(\alpha^2+2\,\lambda 0^3)\,\varrho}}{(\alpha^2+2\,\lambda 0^3)\,\varrho}\right\},\ \left\{\kappa \to \frac{-3\,\lambda 0\,\mu+\sqrt{9\,\lambda 0^2\,\mu^2-4\,c\,(1+2\,\lambda 0^3)\,(\alpha^2+2\,\lambda 0^3)\,\varrho}}{(\alpha^2+2\,\lambda 0^3)\,\varrho}\right\}\right\}$$

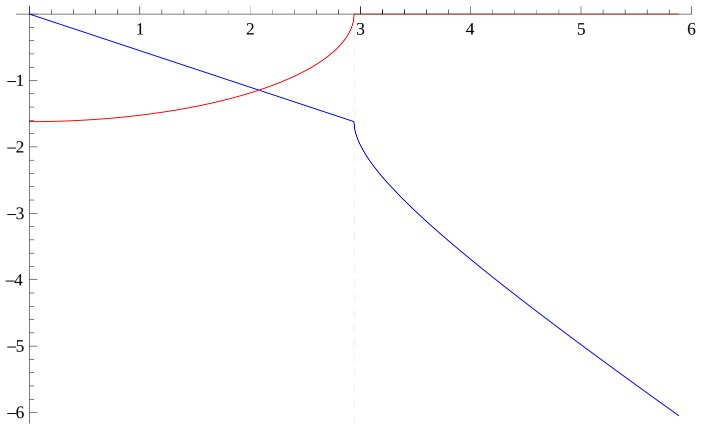
```
κ^2 /. κSol /. μ → ν μ0 // FullSimplify // PowerExpand // FullSimplify
```

$$\left\{\frac{4\,c\,(1+2\,\lambda 0^3)\left(\nu+\sqrt{-1+\nu^2}\right)^2}{(\alpha^2+2\,\lambda 0^3)\,\varrho},\ \frac{4\,c\,(1+2\,\lambda 0^3)\left(\nu-\sqrt{-1+\nu^2}\right)^2}{(\alpha^2+2\,\lambda 0^3)\,\varrho}\right\}$$

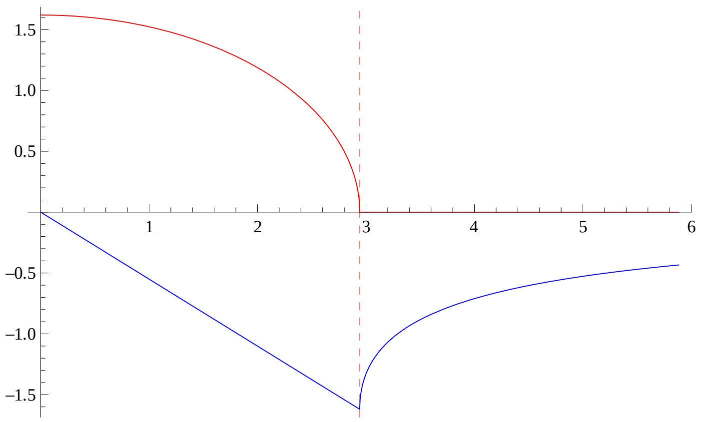
```
oscEqLin1 /. μ → ν μ0 // FullSimplify // PowerExpand // FullSimplify
```

$$\frac{1}{(\alpha^2+2\,\lambda 0^3)\,\varrho}4\left((c+2\,c\,\lambda 0^3)\,d\epsilon[t]+\sqrt{c}\,\sqrt{1+2\,\lambda 0^3}\,\sqrt{\alpha^2+2\,\lambda 0^3}\,\nu\,\sqrt{\varrho}\,d\epsilon'[t]\right)+d\epsilon''[t]$$

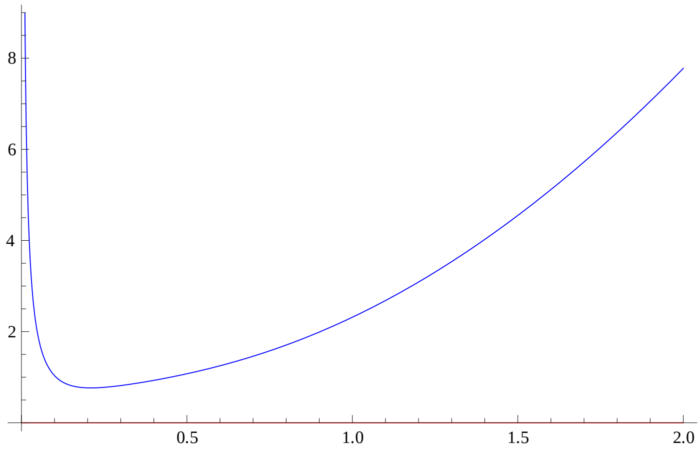
```
Block[{p0 = 1, c = 1, λ0 = λ0f[p0], dϵ0 = 0.3 λ0, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 Plot[{Re[κ /. κSol[[1]]], Im[κ /. κSol[[1]]]}, {μ, 0, 2 μ0},
  PlotStyle → {Blue, Red}, GridLines → {{{μ0, {Red, Dashed}}}, None}]]
```




```
Block[{p0 = 1, c = 1, λ0 = λ0f[p0], dϵ0 = 0.3 λ0, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 Plot[{Re[κ /. κSol[[2]]], Im[κ /. κSol[[2]]]}, {μ, 0, 2 μ0},
  PlotStyle → {Blue, Red}, GridLines → {{{μ0, {Red, Dashed}}}, None}]]
```



Block[{c = 1, α = 0.1, ϱ = 2}, Plot[{Re[μ0], Im[μ0]}, {λ0, 0, 2}, PlotStyle → {Blue, Red}]]



D[μ0, λ0] // FullSimplify

$$\frac{2\sqrt{c}\left(\lambda 0^3+8\,\lambda 0^6+\alpha^2\left(-1+\lambda 0^3\right)\right)\sqrt{\varrho}}{3\,\lambda 0^2\sqrt{1+2\,\lambda 0^3}\,\sqrt{\alpha^2+2\,\lambda 0^3}}$$

Solve[D[μ0, λ0] == 0, λ0] // FullSimplify

$$\left\{\left\{\lambda 0\to-\frac{1}{2}\left(-\frac{1}{2}\right)^{1/3}\left(-1-\alpha^2-\sqrt{1+34\,\alpha^2+\alpha^4}\right)^{1/3}\right\},\left\{\lambda 0\to\frac{\left(-1-\alpha^2-\sqrt{1+34\,\alpha^2+\alpha^4}\right)^{1/3}}{2\times 2^{1/3}}\right\},\right.$$
$$\left\{\lambda 0\to\left(-1-\alpha^2-\sqrt{1+34\,\alpha^2+\alpha^4}\right)^{1/3}\text{Root}\left[-1+16\,\#1^3\,\&,\,3\right]\right\},$$
$$\left\{\lambda 0\to-\frac{1}{2}\left(-\frac{1}{2}\right)^{1/3}\left(-1-\alpha^2+\sqrt{1+34\,\alpha^2+\alpha^4}\right)^{1/3}\right\},\left\{\lambda 0\to\frac{\left(-1-\alpha^2+\sqrt{1+34\,\alpha^2+\alpha^4}\right)^{1/3}}{2\times 2^{1/3}}\right\},$$
$$\left.\left\{\lambda 0\to\left(-1-\alpha^2+\sqrt{1+34\,\alpha^2+\alpha^4}\right)^{1/3}\text{Root}\left[-1+16\,\#1^3\,\&,\,3\right]\right\}\right\}$$

λ0μ0min = λ0 /. %[[5]]

$$\frac{\left(-1-\alpha^2+\sqrt{1+34\,\alpha^2+\alpha^4}\right)^{1/3}}{2\times 2^{1/3}}$$

% /. α → 0.9 // N

0.607845

Limit[λ0μ0min, α → 0]

0

Limit[λ0μ0min, α → ∞]

1

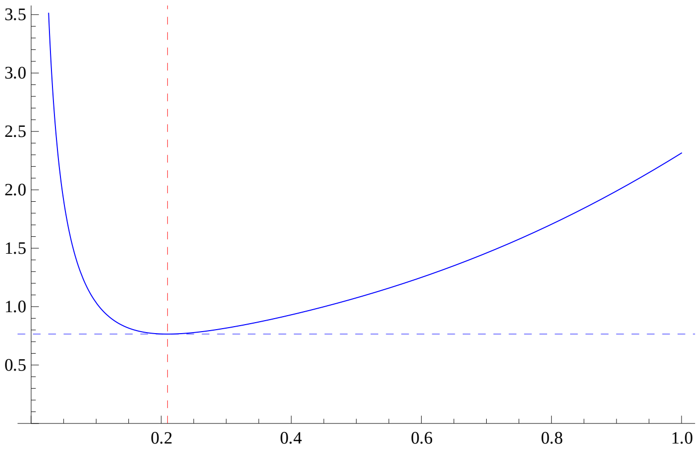
Assuming[α > 0, 0 < λ0μ0min < 1 // Refine]

True

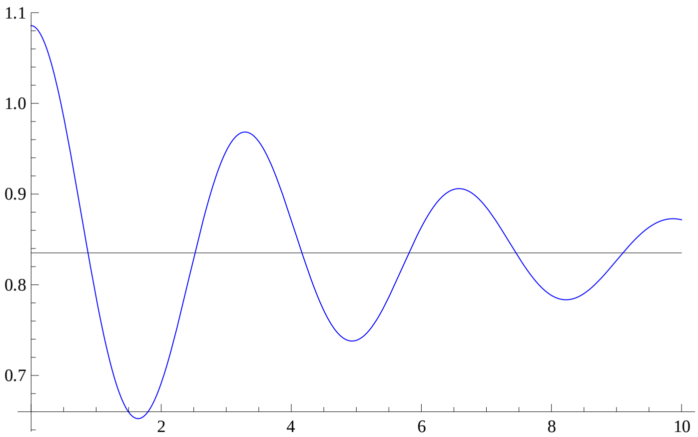
```
μ0min = μ0 /. λ0 → λ0μ0min // FullSimplify
```

$$\left(\sqrt{c}\,\sqrt{7-\alpha^2+\sqrt{1+34\,\alpha^2+\alpha^4}}\,\sqrt{-1+7\,\alpha^2+\sqrt{1+34\,\alpha^2+\alpha^4}}\,\sqrt{\varrho}\right)\Big/\left(3\times 2^{2/3}\left(-1-\alpha^2+\sqrt{1+34\,\alpha^2+\alpha^4}\right)^{1/3}\right)$$

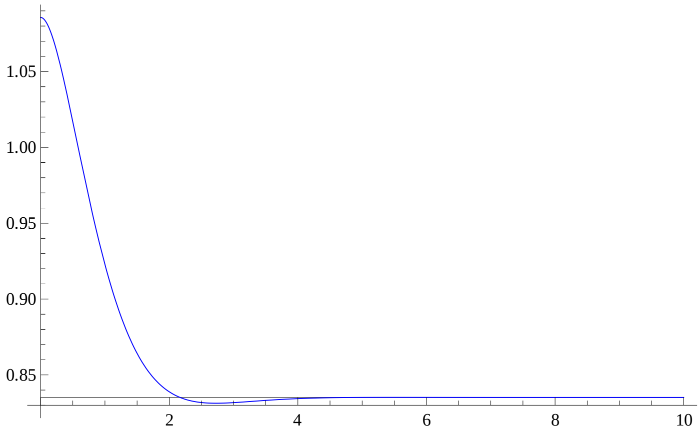
```
Block[{c = 1, α = 0.1, ϱ = 2}, Plot[μ0, {λ0, 0, 1}, PlotStyle → {Blue}, PlotRange → {0, Automatic},
  GridLines → {{{λ0μ0min, {Red, Dashed}}}, {{μ0min, {Blue, Dashed}}}}]]
```



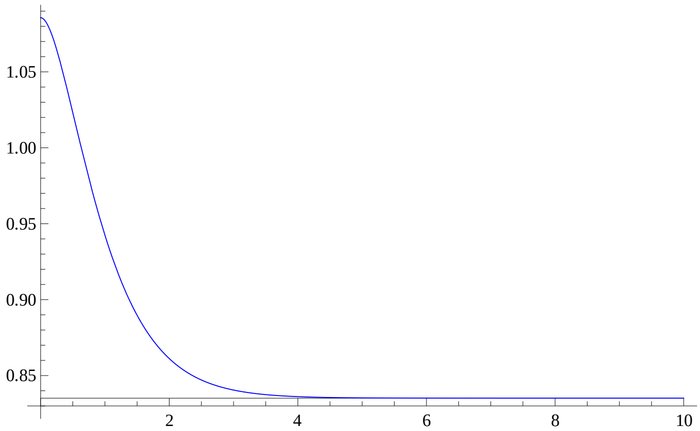

```
Block[{p0 = -1, c = 1, μ = 0.1 μ0, λ0 = λ0f[p0], dϵ0 = 0.3 λ0, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[
  Plot[{λ0, λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



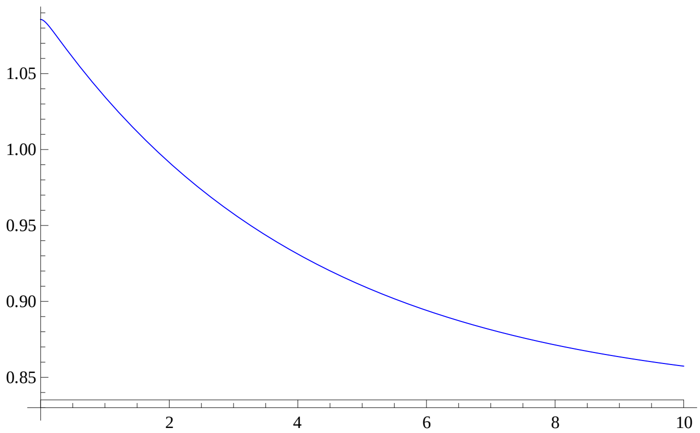
```
Block[{p0 = -1, c = 1, μ = 0.8 μ0, λ0 = λ0f[p0], dϵ0 = 0.3 λ0, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[
  Plot[{λ0, λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



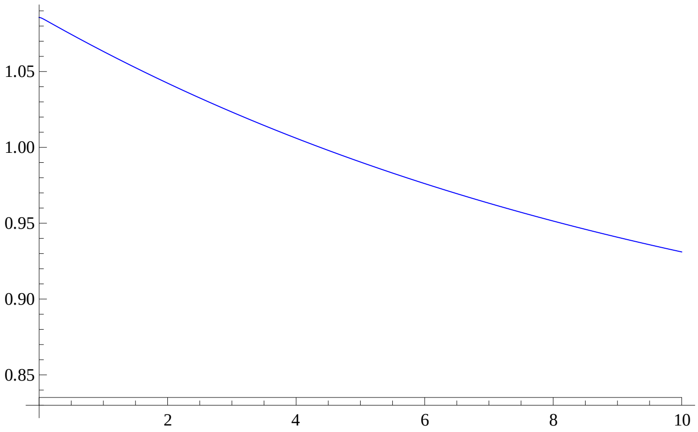

```
Block[{p0 = -1, c = 1, μ = μ0, λ0 = λ0f[p0], dϵ0 = 0.3 λ0, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[
  Plot[{λ0, λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



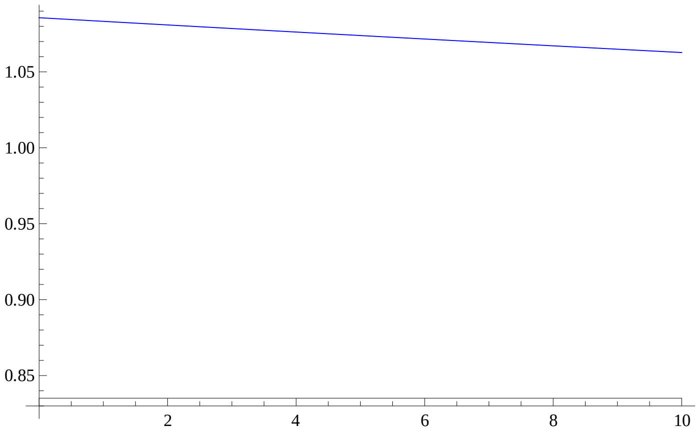

```
Block[{p0 = -1, c = 1, μ = 4 μ0, λ0 = λ0f[p0], dϵ0 = 0.3 λ0, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[
  Plot[{λ0, λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



```
Block[{p0 = -1, c = 1, μ = 10 μ0, λ0 = λ0f[p0], dϵ0 = 0.3 λ0, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[
  Plot[{λ0, λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```




```
Block[{p0 = -1, c = 1, μ = 100 μ0, λ0 = λ0f[p0], dϵ0 = 0.3 λ0, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[
  Plot[{λ0, λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



Small oscillations for an elastic material (μ=0)

```
κSol /. λ0 → 1 /. μ → 0 // Simplify
```

$$\left\{\left\{\kappa \to \frac{2\sqrt{3}\,c}{\sqrt{-c\,(2+\alpha^2)\,\varrho}}\right\}, \left\{\kappa \to -\frac{2\sqrt{3}\,c}{\sqrt{-c\,(2+\alpha^2)\,\varrho}}\right\}\right\}$$

```
κSol /. μ → 0 // Simplify
```

$$\left\{\left\{\kappa \to \frac{2\,c\,(1+2\,\lambda 0^3)}{\sqrt{-c\,(1+2\,\lambda 0^3)\,(\alpha^2+2\,\lambda 0^3)\,\varrho}}\right\}, \left\{\kappa \to -\frac{2\,c\,(1+2\,\lambda 0^3)}{\sqrt{-c\,(1+2\,\lambda 0^3)\,(\alpha^2+2\,\lambda 0^3)\,\varrho}}\right\}\right\}$$

```
DSolve[oscEqLin1 == 0 /. μ → 0, dϵ, t]
```

$$\left\{\left\{d\epsilon \to \text{Function}\left[\{t\},\ e^{\frac{2\,t\,\sqrt{-c-2\,c\,\lambda 0^3}}{\sqrt{\alpha^2\,\varrho+2\,\lambda 0^3\,\varrho}}}\,C[1] + e^{-\frac{2\,t\,\sqrt{-c-2\,c\,\lambda 0^3}}{\sqrt{\alpha^2\,\varrho+2\,\lambda 0^3\,\varrho}}}\,C[2]\right]\right\}\right\}$$

```
dϵ[t] /. %
```

$$\left\{e^{\frac{2\,t\,\sqrt{-c-2\,c\,\lambda 0^3}}{\sqrt{\alpha^2\,\varrho+2\,\lambda 0^3\,\varrho}}}\,C[1] + e^{-\frac{2\,t\,\sqrt{-c-2\,c\,\lambda 0^3}}{\sqrt{\alpha^2\,\varrho+2\,\lambda 0^3\,\varrho}}}\,C[2]\right\}$$

oscEqLin1 /. μ → ν μ0 // Simplify // PowerExpand // Simplify

$$\frac{1}{\left(\alpha^2 + 2\,\lambda 0^3\right)\varrho}4\,\left(\left(c + 2\,c\,\lambda 0^3\right) d\epsilon[t] + \sqrt{c}\,\sqrt{1 + 2\,\lambda 0^3}\,\sqrt{\alpha^2 + 2\,\lambda 0^3}\,\nu\,\sqrt{\varrho}\,d\epsilon'[t]\right) + d\epsilon''[t]$$

κSol

$$\left\{\left\{\kappa \to -\frac{3\,\lambda 0\,\mu + \sqrt{9\,\lambda 0^2\,\mu^2 - 4\,c\,\left(1 + 2\,\lambda 0^3\right)\,\left(\alpha^2 + 2\,\lambda 0^3\right)\,\varrho}}{\left(\alpha^2 + 2\,\lambda 0^3\right)\,\varrho}\right\},\right.$$
$$\left.\left\{\kappa \to \frac{-3\,\lambda 0\,\mu + \sqrt{9\,\lambda 0^2\,\mu^2 - 4\,c\,\left(1 + 2\,\lambda 0^3\right)\,\left(\alpha^2 + 2\,\lambda 0^3\right)\,\varrho}}{\left(\alpha^2 + 2\,\lambda 0^3\right)\,\varrho}\right\}\right\}$$

κSol /. μ → ν μ0 // Simplify // PowerExpand // Simplify

$$\left\{\left\{\kappa \to -\frac{2\,\sqrt{c}\,\sqrt{1 + 2\,\lambda 0^3}\,\left(\nu + \sqrt{-1 + \nu^2}\right)}{\sqrt{\alpha^2 + 2\,\lambda 0^3}\,\sqrt{\varrho}}\right\},\ \left\{\kappa \to \frac{2\,\sqrt{c}\,\sqrt{1 + 2\,\lambda 0^3}\,\left(-\nu + \sqrt{-1 + \nu^2}\right)}{\sqrt{\alpha^2 + 2\,\lambda 0^3}\,\sqrt{\varrho}}\right\}\right\}$$

κSol0 = κSol /. μ → 0 // Simplify // PowerExpand // Simplify

$$\left\{\left\{\kappa \to -\frac{2\,i\,\sqrt{c}\,\sqrt{1 + 2\,\lambda 0^3}}{\sqrt{\alpha^2 + 2\,\lambda 0^3}\,\sqrt{\varrho}}\right\},\ \left\{\kappa \to \frac{2\,i\,\sqrt{c}\,\sqrt{1 + 2\,\lambda 0^3}}{\sqrt{\alpha^2 + 2\,\lambda 0^3}\,\sqrt{\varrho}}\right\}\right\}$$

κ /. κSol0⟦2⟧ // Simplify

$$\frac{2\,i\,\sqrt{c}\,\sqrt{1 + 2\,\lambda 0^3}}{\sqrt{\alpha^2 + 2\,\lambda 0^3}\,\sqrt{\varrho}}$$

oscEqLin1

$$\frac{4\,c\,d\epsilon[t] + 8\,c\,\lambda 0^3\,d\epsilon[t] + 6\,\lambda 0\,\mu\,d\epsilon'[t]}{\alpha^2\,\varrho + 2\,\lambda 0^3\,\varrho} + d\epsilon''[t]$$

$\frac{\text{oscEqLin1}}{e^{t\,I\,\kappa}}$ /. μ → 0 /. {dϵ → (Exp[I κ #] &)} // FullSimplify

$$-\kappa^2 + \frac{4\,c + 8\,c\,\lambda 0^3}{\alpha^2\,\varrho + 2\,\lambda 0^3\,\varrho}$$

Assuming[c > 0 && λ0 > 0 && ϱ > 0, Solve[% == 0, κ] // Refine]

$$\left\{\left\{\kappa \to -\frac{2\,\sqrt{c + 2\,c\,\lambda 0^3}}{\sqrt{\alpha^2\,\varrho + 2\,\lambda 0^3\,\varrho}}\right\},\ \left\{\kappa \to \frac{2\,\sqrt{c + 2\,c\,\lambda 0^3}}{\sqrt{\alpha^2\,\varrho + 2\,\lambda 0^3\,\varrho}}\right\}\right\}$$

κSol0

$$\left\{\left\{\kappa \to -\frac{2\,i\,\sqrt{c}\,\sqrt{1 + 2\,\lambda 0^3}}{\sqrt{\alpha^2 + 2\,\lambda 0^3}\,\sqrt{\varrho}}\right\},\ \left\{\kappa \to \frac{2\,i\,\sqrt{c}\,\sqrt{1 + 2\,\lambda 0^3}}{\sqrt{\alpha^2 + 2\,\lambda 0^3}\,\sqrt{\varrho}}\right\}\right\}$$

Assuming[c > 0 && λ0 > 0 && ϱ > 0, % /. λ0 → 1 // FullSimplify]

$$\left\{\left\{\kappa \to -\frac{2\,i\,\sqrt{3}\,c}{\sqrt{c\,\left(2 + \alpha^2\right)\,\varrho}}\right\},\ \left\{\kappa \to \frac{2\,i\,\sqrt{3}\,c}{\sqrt{c\,\left(2 + \alpha^2\right)\,\varrho}}\right\}\right\}$$

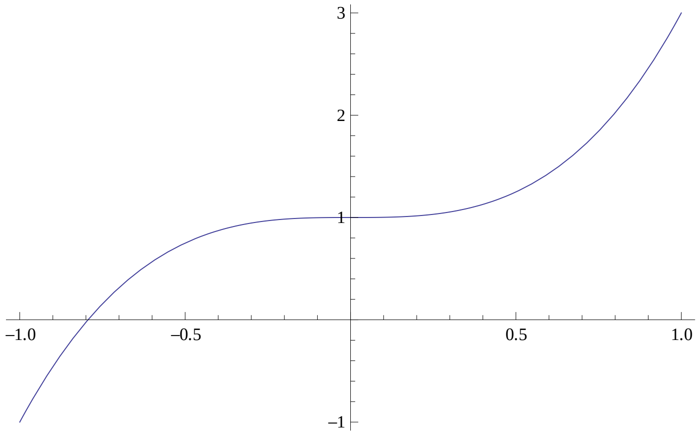
μ0

$$\frac{2\,\sqrt{c}\,\sqrt{1 + 2\,\lambda 0^3}\,\sqrt{\alpha^2 + 2\,\lambda 0^3}\,\sqrt{\varrho}}{3\,\lambda 0}$$

κSol /. μ → 0 // Simplify // PowerExpand // Simplify

$$\left\{\left\{\kappa \to -\frac{2\, i\, \sqrt{c}\, \sqrt{1+2\,\lambda 0^3}}{\sqrt{\alpha^2+2\,\lambda 0^3}\, \sqrt{\varrho}}\right\}, \left\{\kappa \to \frac{2\, i\, \sqrt{c}\, \sqrt{1+2\,\lambda 0^3}}{\sqrt{\alpha^2+2\,\lambda 0^3}\, \sqrt{\varrho}}\right\}\right\}$$

Plot[(1 + 2 λ0³), {λ0, -1, 1}]



Stability and small oscillations

Clear[p0f]; p0f = (p0f /. DSolve[viscoEqβ0 /. p0 → p0f[λ0], p0f, λ0]〚1〛)

$$\text{Function}\left[\{\lambda 0\}, \frac{2\, c\, (-1+\lambda 0^3)}{\lambda 0}\right]$$

p0f'[1]

$6\, c$

p0f[λ0]

$$\frac{2\, c\, (-1+\lambda 0^3)}{\lambda 0}$$

p0f''[λ0]

$$\frac{4\, c\, (-1+\lambda 0^3)}{\lambda 0^3}$$

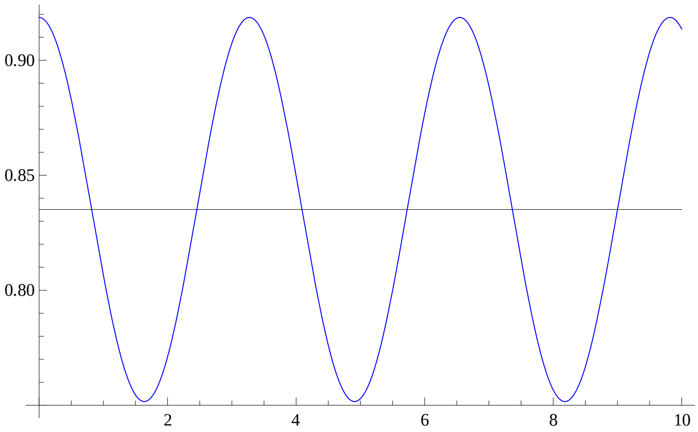
p0f''[1]

0

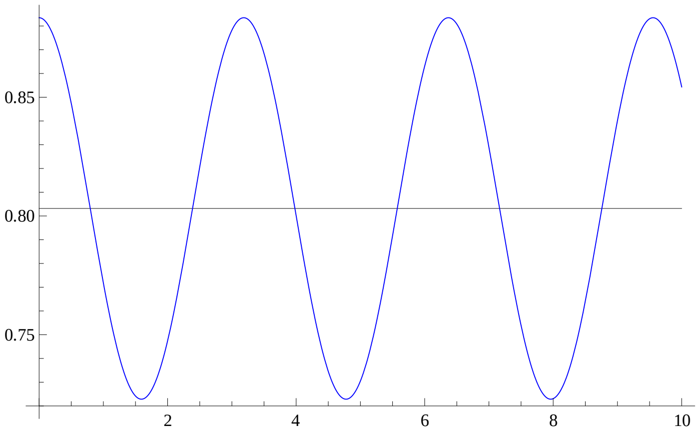
Solve[μ0 == 0, λ0]

$$\left\{\left\{\lambda 0 \to \left(-\frac{1}{2}\right)^{1/3}\right\}, \left\{\lambda 0 \to -\frac{1}{2^{1/3}}\right\}, \left\{\lambda 0 \to -\frac{(-1)^{2/3}}{2^{1/3}}\right\},\right.$$
$$\left.\left\{\lambda 0 \to \left(-\frac{1}{2}\right)^{1/3} \alpha^{2/3}\right\}, \left\{\lambda 0 \to -\frac{\alpha^{2/3}}{2^{1/3}}\right\}, \left\{\lambda 0 \to -\frac{(-1)^{2/3}\, \alpha^{2/3}}{2^{1/3}}\right\}\right\}$$

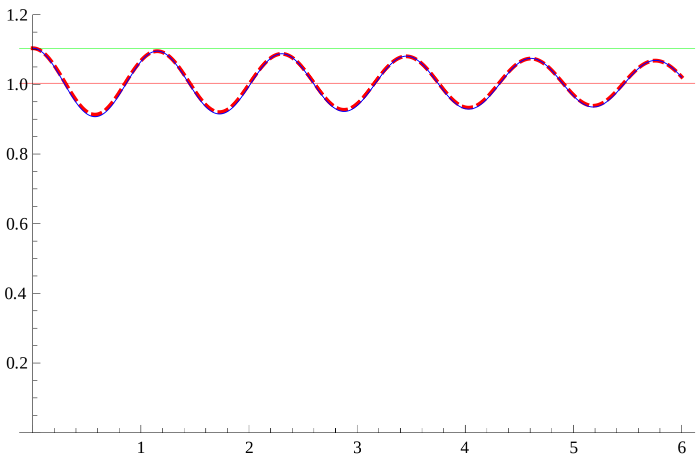
```
Block[{p0 = -1, c = 1, μ = 0, λ0 = λ0f[p0], dϵ0 = 0.1 λ0, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[
  Plot[{λ0, λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



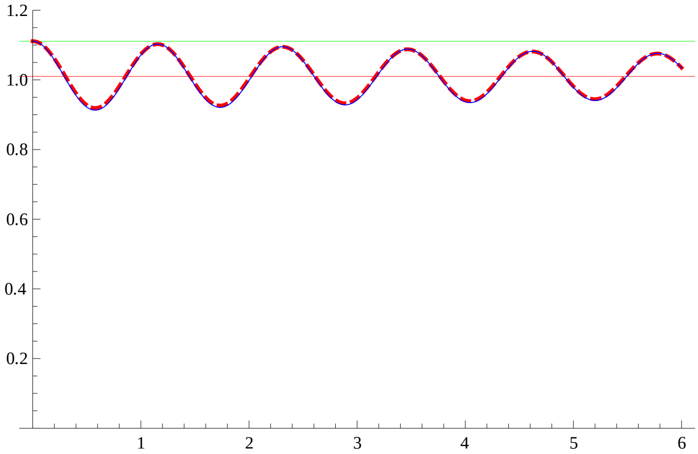

```
Block[{p0 = -1.2, c = 1, μ = 0, λ0 = λ0f[p0], dϵ0 = 0.1 λ0, dϵNlin, tlim = 10, α = 0.1, ϱ = 2},
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[
  Plot[{λ0, λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



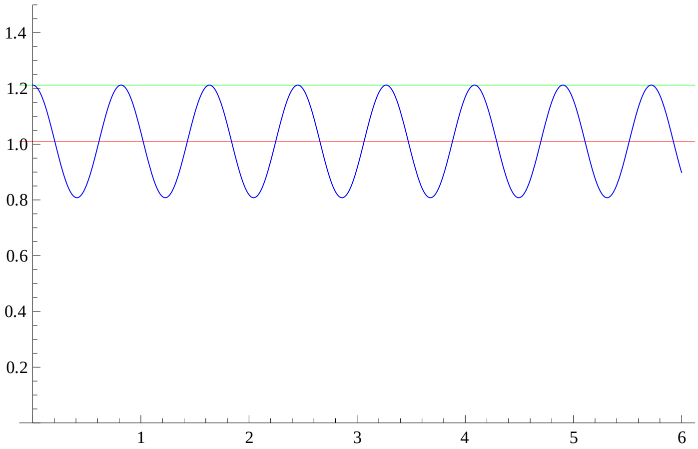

```
Block[{p0 = 0.2, c = 10, μ = 0.1, λ0 = λ0f[p0], λN, dϵ0 = 0.1 λ0, dϵNlin, tlim = 6, α = 0.1, ϱ = 2},
 λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]];
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
 Show[Plot[{λN[t]}, {t, 0, tlim}, PlotRange → {0, 1.2},
   PlotStyle → {{Red, Dashed, Thick}}, GridLines → {None, {{λ0, Red}, {λ0 + dϵ0, Green}}}],
  Plot[{λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → {0, 1.2}, PlotStyle → {{Blue}},
   GridLines → {None, {{λ0, Red}, {λ0 + dϵ0, Green}}}]]]
```



```
Block[{p0 = 0.6, c = 10, μ = 0.1, λ0 = λ0f[p0], λN, dϵ0 = 0.1 λ0, dϵNlin, tlim = 6, α = 0.1, ϱ = 2},
 λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]];
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
 Show[Plot[{λN[t]}, {t, 0, tlim}, PlotRange → {0, 1.2},
   PlotStyle → {{Red, Dashed, Thick}}, GridLines → {None, {{λ0, Red}, {λ0 + dϵ0, Green}}}],
  Plot[{λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → {0, 1.2}, PlotStyle → {{Blue}},
   GridLines → {None, {{λ0, Red}, {λ0 + dϵ0, Green}}}]]]
```



```
Block[{p0 = 0.6, c = 10, μ = 0, λ0 = λ0f[p0], λN, dϵ0 = 0.2 λ0, dϵNlin, tlim = 6, α = 0.1, ϱ = 1},
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
 Show[Plot[{λ0 + dϵNlin[t]}, {t, 0, tlim}, PlotRange → {0, 1.5},
   PlotStyle → {{Blue}}, GridLines → {None, {{λ0, Red}, {λ0 + dϵ0, Green}}}]]]
```



```
κSol // FullSimplify
```

$$\left\{\left\{\kappa \to -\frac{3\,\lambda 0\,\mu + \sqrt{9\,\lambda 0^2\,\mu^2 - 4\,c\,(1 + 2\,\lambda 0^3)\,(\alpha^2 + 2\,\lambda 0^3)\,\varrho}}{(\alpha^2 + 2\,\lambda 0^3)\,\varrho}\right\},\ \left\{\kappa \to \frac{-3\,\lambda 0\,\mu + \sqrt{9\,\lambda 0^2\,\mu^2 - 4\,c\,(1 + 2\,\lambda 0^3)\,(\alpha^2 + 2\,\lambda 0^3)\,\varrho}}{(\alpha^2 + 2\,\lambda 0^3)\,\varrho}\right\}\right\}$$

```
μ0^2
```

$$\frac{4\,c\,(1 + 2\,\lambda 0^3)\,(\alpha^2 + 2\,\lambda 0^3)\,\varrho}{9\,\lambda 0^2}$$

```
κSol /. μ → ν μ0 // FullSimplify // PowerExpand // FullSimplify
```

$$\left\{\left\{\kappa \to -\frac{2\sqrt{c}\,\sqrt{1+2\,\lambda 0^3}\,\left(\nu+\sqrt{-1+\nu^2}\right)}{\sqrt{\alpha^2+2\,\lambda 0^3}\,\sqrt{\varrho}}\right\},\ \left\{\kappa \to \frac{2\sqrt{c}\,\sqrt{1+2\,\lambda 0^3}\,\left(-\nu+\sqrt{-1+\nu^2}\right)}{\sqrt{\alpha^2+2\,\lambda 0^3}\,\sqrt{\varrho}}\right\}\right\}$$

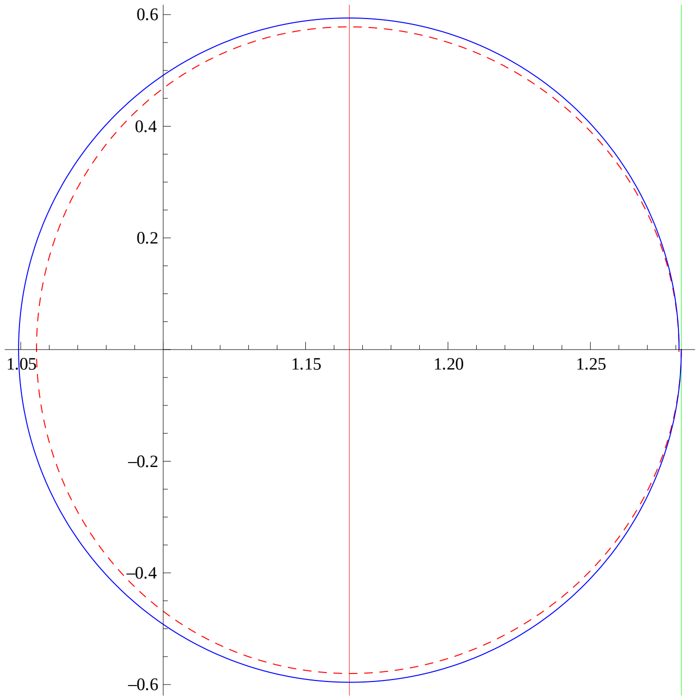
```
% /. ν → 0
```

$$\left\{\left\{\kappa \to -\frac{2\,i\,\sqrt{c}\,\sqrt{1+2\,\lambda 0^3}}{\sqrt{\alpha^2+2\,\lambda 0^3}\,\sqrt{\varrho}}\right\},\ \left\{\kappa \to \frac{2\,i\,\sqrt{c}\,\sqrt{1+2\,\lambda 0^3}}{\sqrt{\alpha^2+2\,\lambda 0^3}\,\sqrt{\varrho}}\right\}\right\}$$

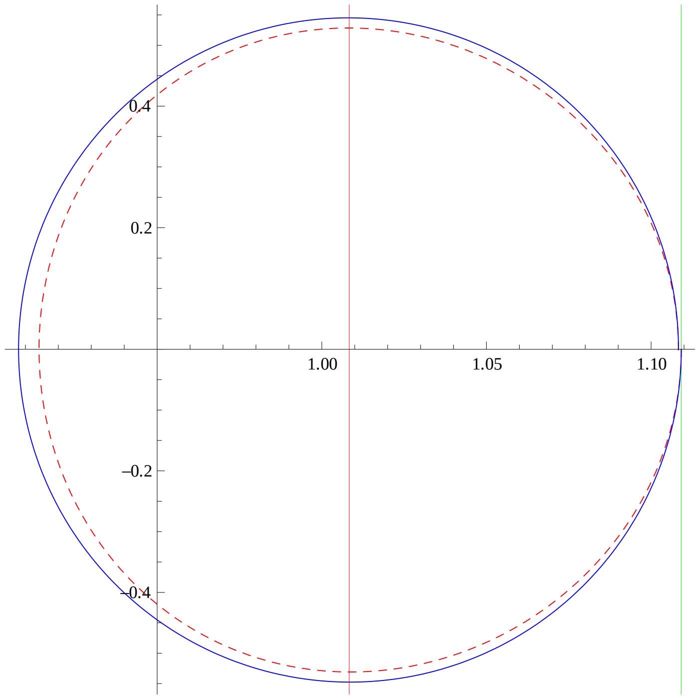
```
κSol0 // FullSimplify // PowerExpand
```

$$\left\{\left\{\kappa \to -\frac{2\,i\,\sqrt{c}\,\sqrt{1+2\,\lambda 0^3}}{\sqrt{\alpha^2+2\,\lambda 0^3}\,\sqrt{\varrho}}\right\},\ \left\{\kappa \to \frac{2\,i\,\sqrt{c}\,\sqrt{1+2\,\lambda 0^3}}{\sqrt{\alpha^2+2\,\lambda 0^3}\,\sqrt{\varrho}}\right\}\right\}$$

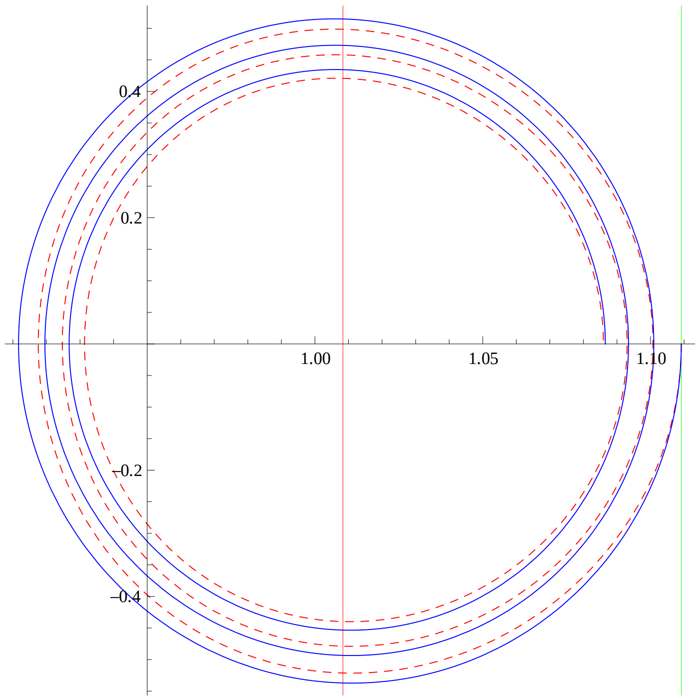
```
Block[{p0 = 10, c = 10, μ = 0.01, λ0 = λ0f[p0], λN, dϵ0 = 0.1 λ0, dϵNlin, tlim = tper,
   tper = Max[2 π / Im[κ] /. κSol], α = 0.1, ϱ = 2}, λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]];
  dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
  Show[ParametricPlot[{λN[t], λN'[t]}, {t, 0, tlim}, AspectRatio → 1, PlotRange → All,
    PlotStyle → {{Red, Dashed}}, GridLines → {{{λ0, Red}, {λ0 + dϵ0, Green}}, None}],
   ParametricPlot[{λ0 + dϵNlin[t], dϵNlin'[t]}, {t, 0, tlim}, AspectRatio → 1,
    PlotRange → All, PlotStyle → {{Blue}}, GridLines → {{{λ0, Red}, {λ0 + dϵ0, Green}}, None}]]]
```



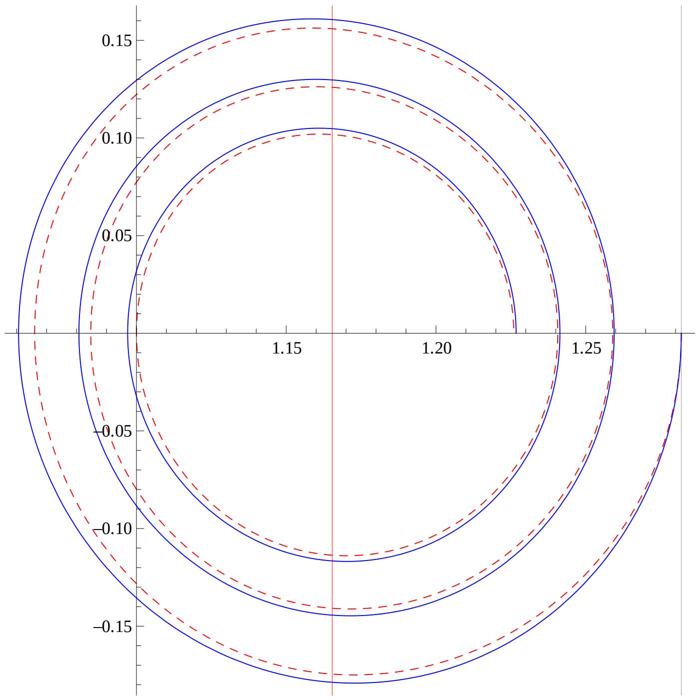
```
Block[{p0 = 0.5, c = 10, μ = 0.01, λ0 = λ0f[p0], λN, dϵ0 = 0.1 λ0, dϵNlin, tlim = tper,
   tper = Max[2 π / Im[κ] /. κSol], α = 0.1, ϱ = 2}, λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]];
  dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
  Show[ParametricPlot[{λN[t], λN'[t]}, {t, 0, tlim}, AspectRatio → 1, PlotRange → All,
    PlotStyle → {{Red, Dashed}}, GridLines → {{{λ0, Red}, {λ0 + dϵ0, Green}}, None}],
   ParametricPlot[{λ0 + dϵNlin[t], dϵNlin'[t]}, {t, 0, tlim}, AspectRatio → 1,
    PlotRange → All, PlotStyle → {{Blue}}, GridLines → {{{λ0, Red}, {λ0 + dϵ0, Green}}, None}]]]
```



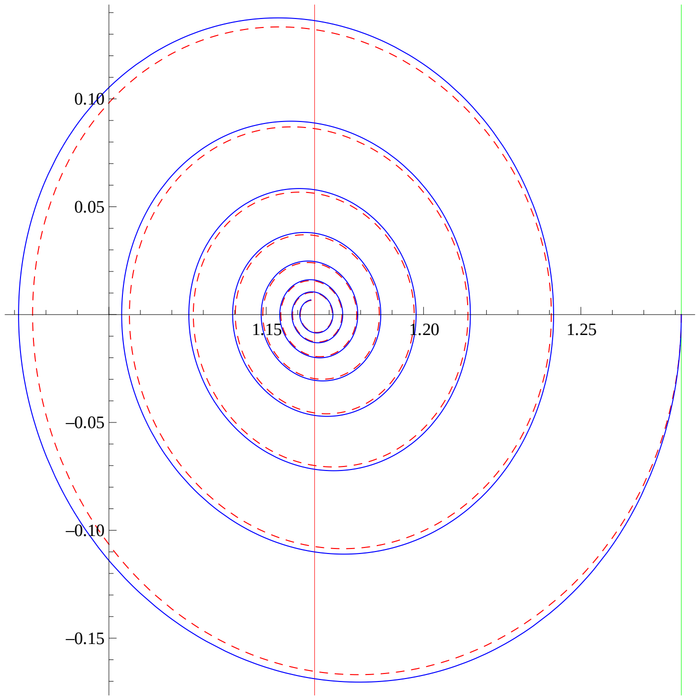
```
Block[{p0 = 0.5, c = 10, μ = 0.1, λ0 = λ0f[p0], λN, dϵ0 = 0.1 λ0, dϵNlin, tlim = 3 tper,
  tper = Max[2 π/Im[κ] /. κSol], α = 0.1, ϱ = 2}, λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]];
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
 Show[ParametricPlot[{λN[t], λN'[t]}, {t, 0, tlim}, AspectRatio → 1, PlotRange → All,
   PlotStyle → {{Red, Dashed}}, GridLines → {{{λ0, Red}, {λ0 + dϵ0, Green}}, None}],
  ParametricPlot[{λ0 + dϵNlin[t], dϵNlin'[t]}, {t, 0, tlim}, AspectRatio → 1,
   PlotRange → All, PlotStyle → {{Blue}}, GridLines → {{{λ0, Red}, {λ0 + dϵ0, Green}}, None}]]]
```



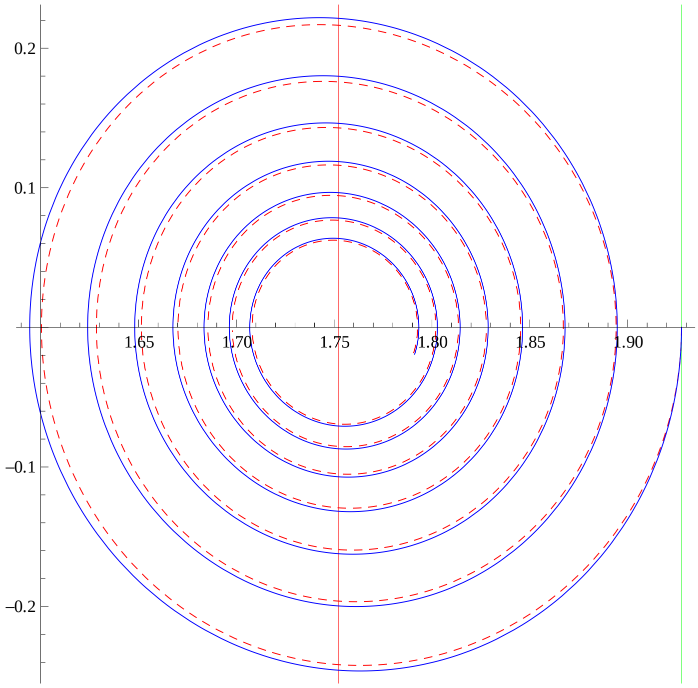
```
Block[{p0 = 1, c = 1, μ = 0.1, λ0 = λ0f[p0], λN, dϵ0 = 0.1 λ0, dϵNlin, tlim = 3 tper,
   tper = Max[2 π / Im[κ] /. κSol], α = 0.1, ϱ = 2}, λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]];
  dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
  Show[ParametricPlot[{λN[t], λN'[t]}, {t, 0, tlim}, AspectRatio → 1, PlotRange → All,
    PlotStyle → {{Red, Dashed}}, GridLines → {{{λ0, Red}, {λ0 + dϵ0, Green}}, None}],
   ParametricPlot[{λ0 + dϵNlin[t], dϵNlin'[t]}, {t, 0, tlim}, AspectRatio → 1,
    PlotRange → All, PlotStyle → {{Blue}}, GridLines → {{{λ0, Red}, {λ0 + dϵ0, Green}}, None}]]]
```



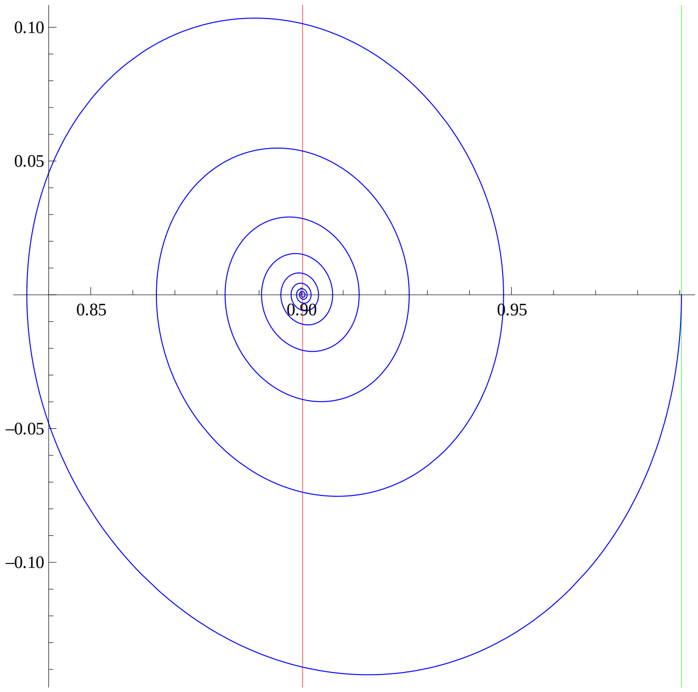
```
Block[{p0 = 1, c = 1, μ = 0.2, λ0 = λ0f[p0], λN, dϵ0 = 0.1 λ0, dϵNlin, tlim = 30, α = 0.1, ϱ = 2},
 λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]];
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
 Show[ParametricPlot[{λN[t], λN'[t]}, {t, 0, tlim}, AspectRatio → 1, PlotRange → All,
   PlotStyle → {{Red, Dashed}}, GridLines → {{{λ0, Red}, {λ0 + dϵ0, Green}}, None}],
  ParametricPlot[{λ0 + dϵNlin[t], dϵNlin'[t]}, {t, 0, tlim}, AspectRatio → 1,
   PlotRange → All, PlotStyle → {{Blue}}, GridLines → {{{λ0, Red}, {λ0 + dϵ0, Green}}, None}]]]
```



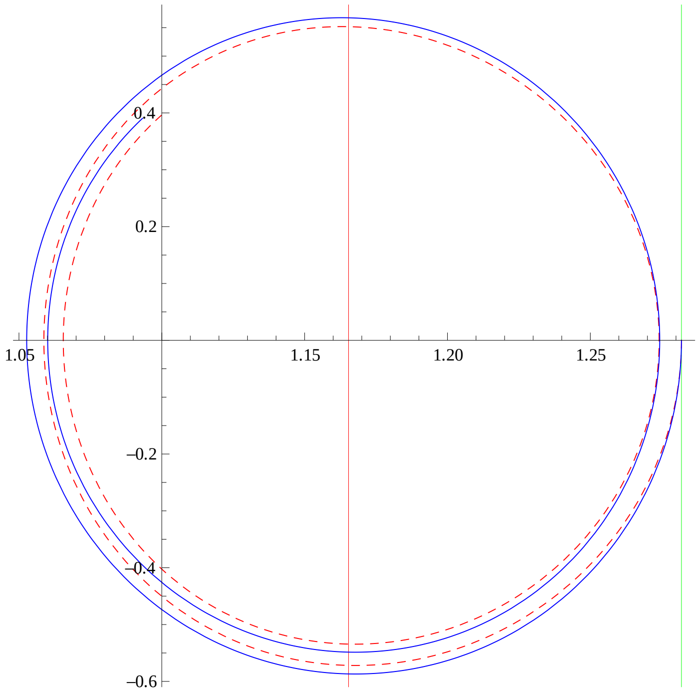
```
Block[{p0 = 5, c = 1, μ = 0.2, λ0 = λ0f[p0], λN, dϵ0 = 0.1 λ0, dϵNlin, tlim = 30, α = 0.1, ϱ = 2},
 λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]];
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
 Show[ParametricPlot[{λN[t], λN'[t]}, {t, 0, tlim}, AspectRatio → 1, PlotRange → All,
   PlotStyle → {{Red, Dashed}}, GridLines → {{{λ0, Red}, {λ0 + dϵ0, Green}}, None}],
  ParametricPlot[{λ0 + dϵNlin[t], dϵNlin'[t]}, {t, 0, tlim}, AspectRatio → 1,
   PlotRange → All, PlotStyle → {{Blue}}, GridLines → {{{λ0, Red}, {λ0 + dϵ0, Green}}, None}]]]
```



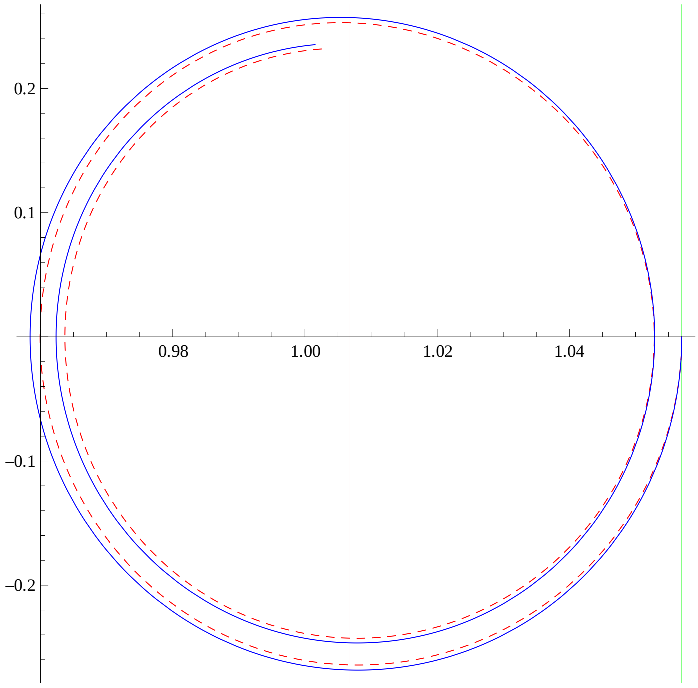

```
Block[{p0 = -0.6, c = 1, μ = 0.2, λ0 = λ0f[p0], λN, dϵ0 = 0.1 λ0, dϵNlin, tlim = 30, α = 0.1, ϱ = 2},
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
 Show[ParametricPlot[{λ0 + dϵNlin[t], dϵNlin'[t]}, {t, 0, tlim}, AspectRatio → 1,
   PlotRange → All, PlotStyle → {{Blue}}, GridLines → {{{λ0, Red}, {λ0 + dϵ0, Green}}, None}]]]
```



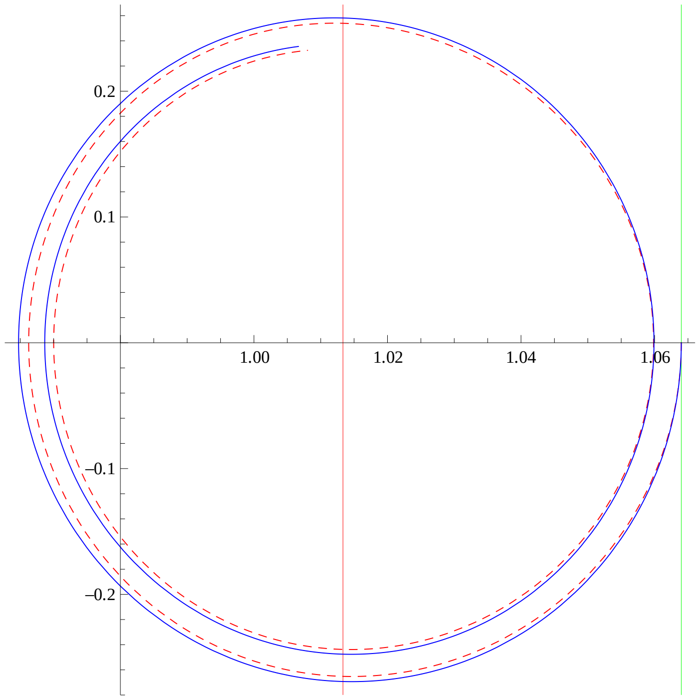
```
Block[{p0 = 10, c = 10, μ = 0.1, λ0 = λ0f[p0], λN, dϵ0 = 0.1 λ0, dϵNlin, tlim = 2, α = 0.1, ϱ = 2},
 λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]];
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
 Show[ParametricPlot[{λN[t], λN'[t]}, {t, 0, tlim}, AspectRatio → 1, PlotRange → All,
   PlotStyle → {{Red, Dashed}}, GridLines → {{{λ0, Red}, {λ0 + dϵ0, Green}}, None}],
  ParametricPlot[{λ0 + dϵNlin[t], dϵNlin'[t]}, {t, 0, tlim}, AspectRatio → 1,
   PlotRange → All, PlotStyle → {{Blue}}, GridLines → {{{λ0, Red}, {λ0 + dϵ0, Green}}, None}]]]
```



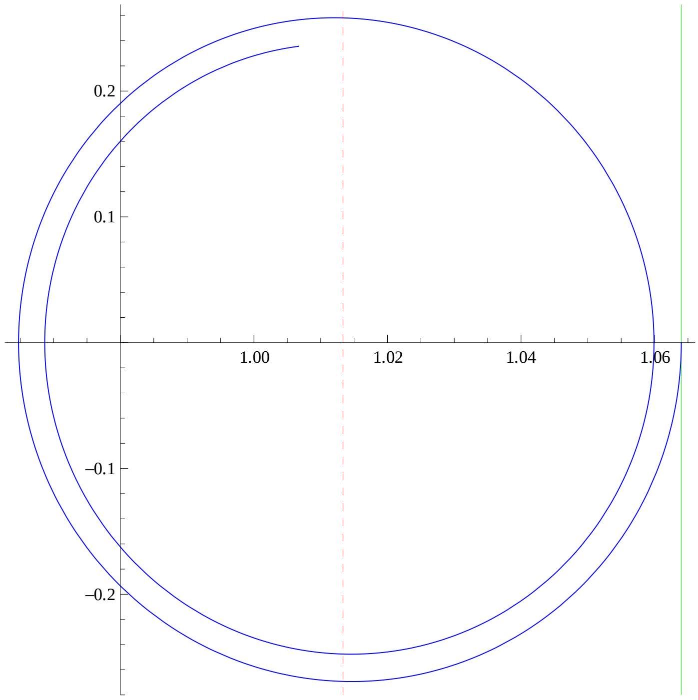
```
Block[{p0 = 0.4, c = 10, μ = 0.1, λ0 = λ0f[p0], λN, dϵ0 = 0.05 λ0, dϵNlin, tlim = 2, α = 0.1, ϱ = 2},
 λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]];
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
 Show[ParametricPlot[{λN[t], λN'[t]}, {t, 0, tlim}, AspectRatio → 1, PlotRange → All,
   PlotStyle → {{Red, Dashed}}, GridLines → {{{λ0, Red}, {λ0 + dϵ0, Green}}, None}],
  ParametricPlot[{λ0 + dϵNlin[t], dϵNlin'[t]}, {t, 0, tlim}, AspectRatio → 1,
   PlotRange → All, PlotStyle → {{Blue}}, GridLines → {{{λ0, Green}, {λ0 + dϵ0, Red}}, None}]]]
```



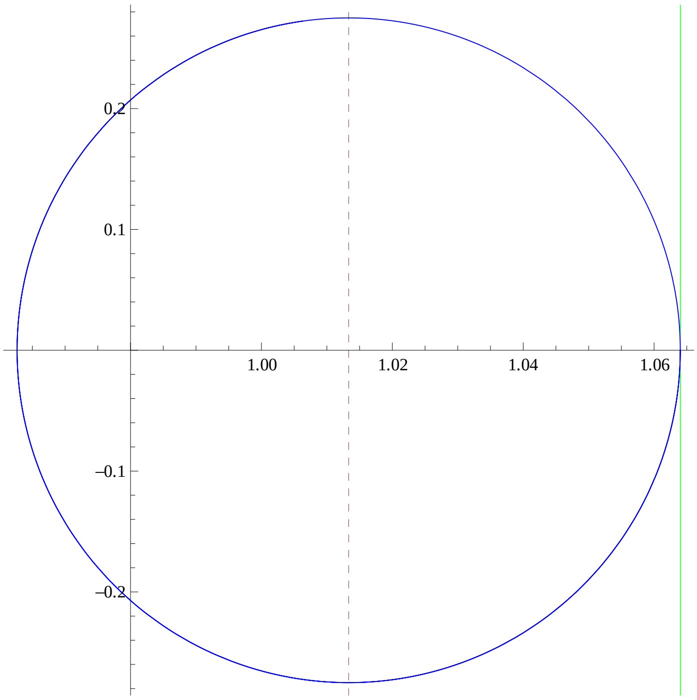
```
Block[{p0 = 0.8, c = 10, μ = 0.1, λ0 = λ0f[p0], λN, dϵ0 = 0.05 λ0, dϵNlin, tlim = 2, α = 0.1, ϱ = 2},
 λN = λ /. NDSolve[oscEq, λ, {t, 0, tlim}][[1]];
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
 Show[ParametricPlot[{λN[t], λN'[t]}, {t, 0, tlim}, AspectRatio → 1, PlotRange → All,
   PlotStyle → {{Red, Dashed}}, GridLines → {{{λ0, {Red}}, {λ0 + dϵ0, Green}}, None}],
  ParametricPlot[{λ0 + dϵNlin[t], dϵNlin'[t]}, {t, 0, tlim}, AspectRatio → 1, PlotRange → All,
   PlotStyle → {{Blue}}, GridLines → {{{λ0, {Red}}, {λ0 + dϵ0, Green}}, None}]]]
```



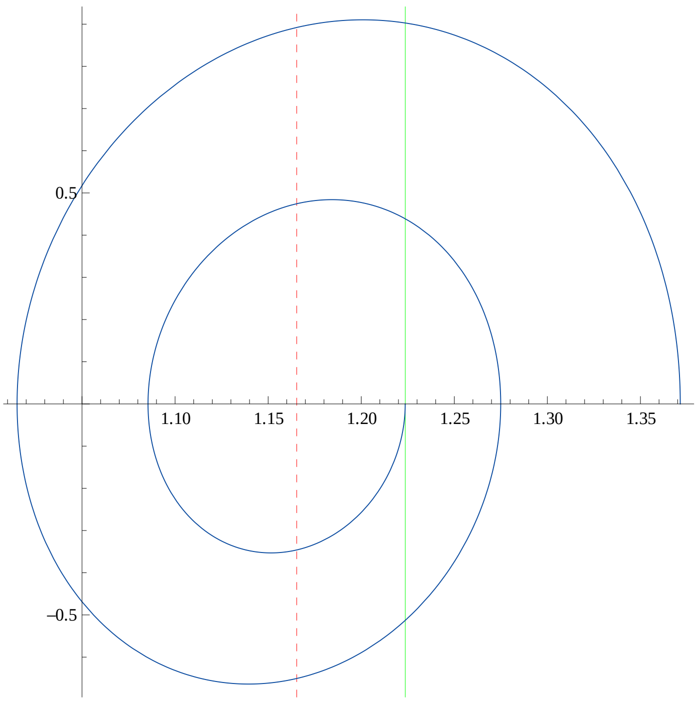

```
Block[{p0 = 0.8, c = 10, μ = 0.1, λ0 = λ0f[p0], λN, dϵ0 = 0.05 λ0, dϵNlin, tlim = 2, α = 0.1, ϱ = 2},
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[
  ParametricPlot[{λ0 + dϵNlin[t], dϵNlin'[t]}, {t, 0, tlim}, AspectRatio → 1, PlotRange → All,
   PlotStyle → {{Blue}}, GridLines → {{{λ0, {Red, Dashed}}, {λ0 + dϵ0, Green}}, None}]]]
```



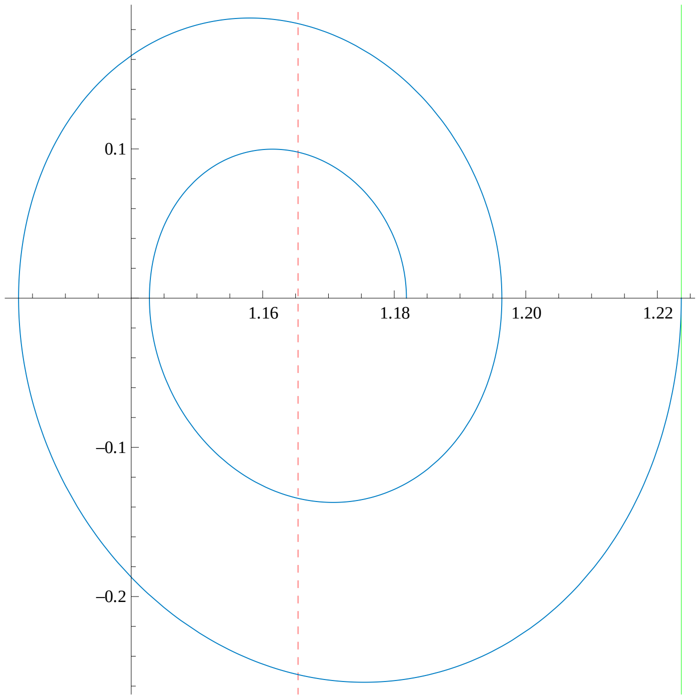
```
Block[{p0 = 0.8, c = 10, μ = 0, λ0 = λ0f[p0], λN, dϵ0 = 0.05 λ0, dϵNlin, tlim = 2, α = 0.1, ϱ = 2},
 dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]]; Show[
  ParametricPlot[{λ0 + dϵNlin[t], dϵNlin'[t]}, {t, 0, tlim}, AspectRatio → 1, PlotRange → All,
   PlotStyle → {{Blue}}, GridLines → {{{λ0, {Red, Dashed}}, {λ0 + dϵ0, Green}}, None}]]]
```



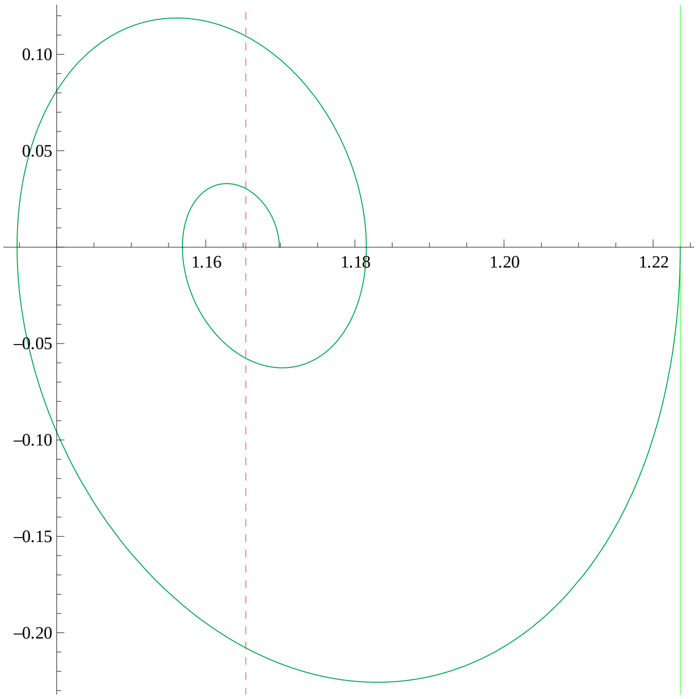

```
phpl0 = Block[{p0 = 10, c = 10, μ = -0.1 μ0, λ0 = λ0f[p0], λN,
   dϵ0 = 0.05 λ0, dϵNlin, tlim = 2 tper, tper = Max[2 π / Im[κ] /. κSol], α = 0.1, ϱ = 2},
  dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
  Show[ParametricPlot[{λ0 + dϵNlin[t], dϵNlin'[t]}, {t, 0, tlim},
    AspectRatio → 1, PlotRange → All, PlotStyle → {{CMYKColor[1, 0.8, 0]}},
    PlotRange → All, GridLines → {{{λ0, {Red, Dashed}}, {λ0 + dϵ0, Green}}, None}]]]
```



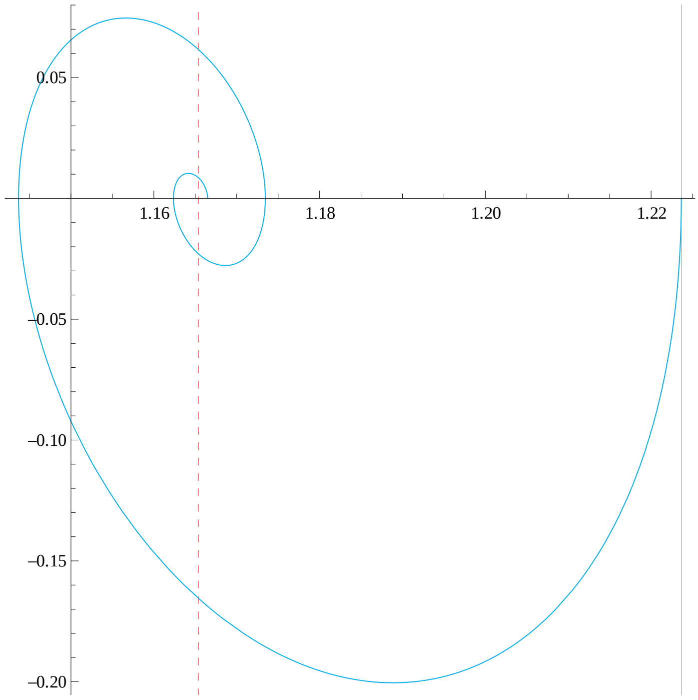
```
phpl1 = Block[{p0 = 10, c = 10, μ = 0.1 μ0, λ0 = λ0f[p0], λN,
    dϵ0 = 0.05 λ0, dϵNlin, tlim = 2 tper, tper = Max[2 π/Im[κ] /. κSol], α = 0.1, ϱ = 2},
   dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
   Show[ParametricPlot[{λ0 + dϵNlin[t], dϵNlin'[t]}, {t, 0, tlim},
     AspectRatio → 1, PlotRange → All, PlotStyle → {{CMYKColor[1, 0.4, 0]}},
     PlotRange → All, GridLines → {{{λ0, {Red, Dashed}}, {λ0 + dϵ0, Green}}, None}]]]
```



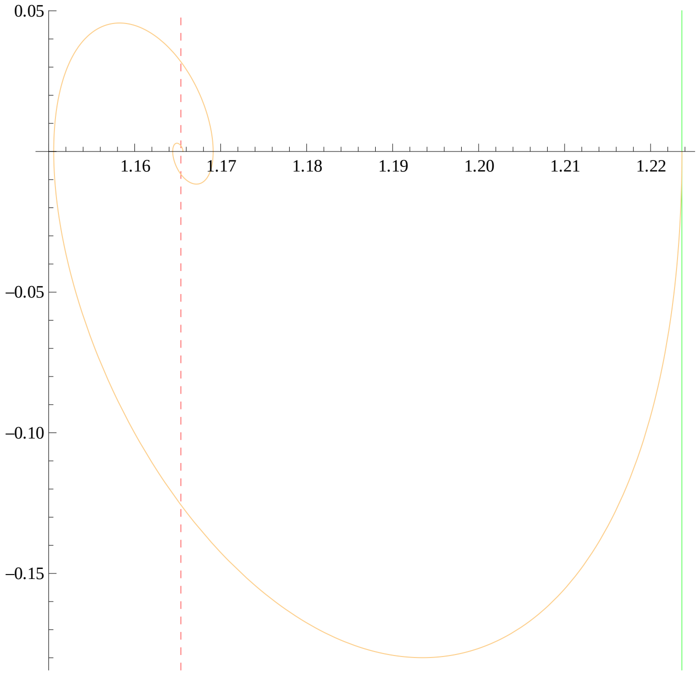
```
phpl2 = Block[{p0 = 10, c = 10, μ = 0.2 μ0, λ0 = λ0f[p0], λN,
    dϵ0 = 0.05 λ0, dϵNlin, tlim = 2 tper, tper = Max[2π/Im[κ] /. κSol], α = 0.1, ϱ = 2},
   dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
   Show[ParametricPlot[{λ0 + dϵNlin[t], dϵNlin'[t]}, {t, 0, tlim},
     AspectRatio → 1, PlotRange → All, PlotStyle → {{CMYKColor[1, 0, 1]}},
     PlotRange → All, GridLines → {{{λ0, {Red, Dashed}}, {λ0 + dϵ0, Green}}, None}]]]
```



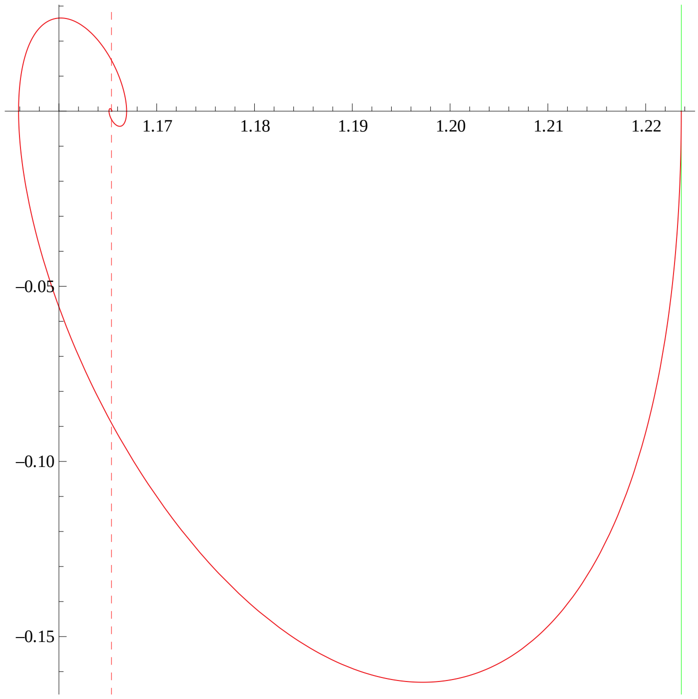
```
phpl3 = Block[{p0 = 10, c = 10, μ = 0.3 μ0, λ0 = λ0f[p0], λN,
    dϵ0 = 0.05 λ0, dϵNlin, tlim = 2 tper, tper = Max[2 π / Im[κ] /. κSol], α = 0.1, ϱ = 2},
   dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
   Show[ParametricPlot[{λ0 + dϵNlin[t], dϵNlin'[t]}, {t, 0, tlim},
     AspectRatio → 1, PlotRange → All, PlotStyle → {{CMYKColor[1, 0, 0]}},
     PlotRange → All, GridLines → {{{λ0, {Red, Dashed}}, {λ0 + dϵ0, Green}}, None}]]]
```



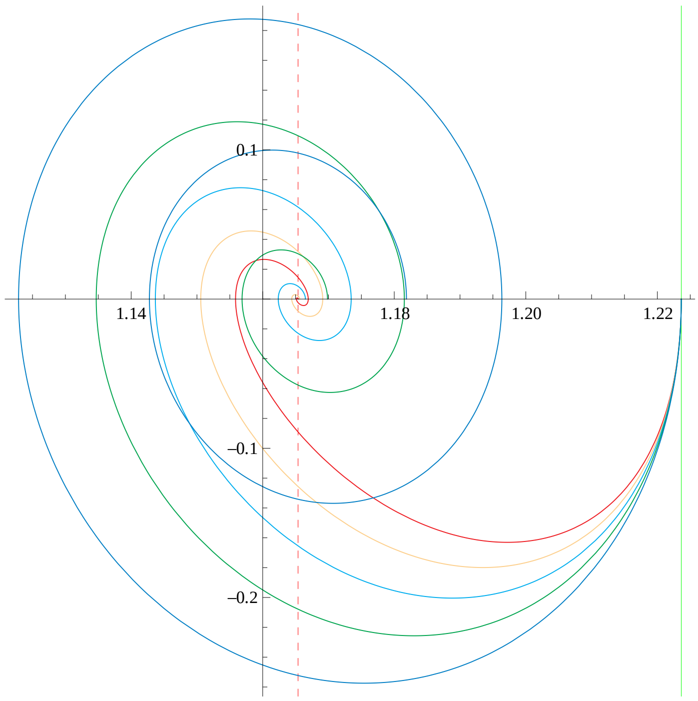
```
phpl4 = Block[{p0 = 10, c = 10, μ = 0.4 μ0, λ0 = λ0f[p0], λN,
    dϵ0 = 0.05 λ0, dϵNlin, tlim = 2 tper, tper = Max[2 π/Im[κ] /. κSol], α = 0.1, ϱ = 2},
  dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
  Show[ParametricPlot[{λ0 + dϵNlin[t], dϵNlin'[t]}, {t, 0, tlim},
     AspectRatio → 1, PlotRange → All, PlotStyle → {{CMYKColor[0, 0.2, 0.5]}},
     PlotRange → All, GridLines → {{{λ0, {Red, Dashed}}, {λ0 + dϵ0, Green}}, None}]]]
```



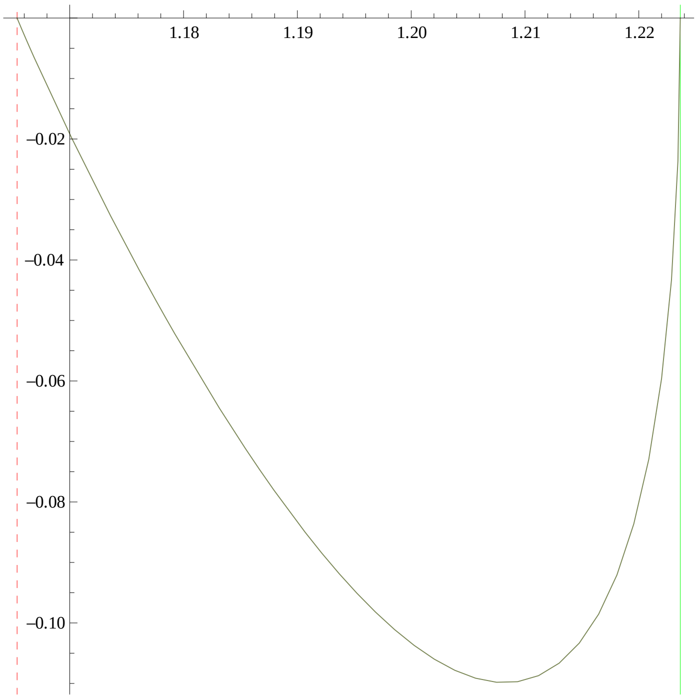
```
phpl5 = Block[{p0 = 10, c = 10, μ = 0.5 μ0, λ0 = λ0f[p0], λN,
    dϵ0 = 0.05 λ0, dϵNlin, tlim = 2 tper, tper = Max[2 π / Im[κ] /. κSol], α = 0.1, ϱ = 2},
   dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
   Show[ParametricPlot[{λ0 + dϵNlin[t], dϵNlin'[t]}, {t, 0, tlim},
     AspectRatio → 1, PlotRange → All, PlotStyle → {{CMYKColor[0, 1, 1]}},
     PlotRange → All, GridLines → {{{λ0, {Red, Dashed}}, {λ0 + dϵ0, Green}}, None}]]]
```




```
Show[phpl5, phpl4, phpl3, phpl2, phpl1, PlotRange → All]
```



```
phpl6 = Block[{p0 = 10, c = 10, μ = 0.999 μ0, λ0 = λ0f[p0], λN,
    dϵ0 = 0.05 λ0, dϵNlin, tlim = 2 tper, tper = Max[2 π / Im[κ] /. κSol], α = 0.1, ϱ = 2},
   dϵNlin = dϵ /. NDSolve[oscEqLin, dϵ, {t, 0, tlim}][[1]];
   Show[ParametricPlot[{λ0 + dϵNlin[t], dϵNlin'[t]}, {t, 0, tlim},
     AspectRatio → 1, PlotRange → All, PlotStyle → {{CMYKColor[0.6, 0.4, 0.8]}},
     PlotRange → All, GridLines → {{{λ0, {Red, Dashed}}, {λ0 + dϵ0, Green}}, None}]]]
```




```
Show[phpl6, phpl5, PlotRange → All]
```