neo-Hookean strain energy

```
mF[λ_] := {{λ, 0, 0}, {0, 1/Sqrt[λ], 0}, {0, 0, 1/Sqrt[λ]}}
mC[λ_] := Transpose[mF[λ]].mF[λ]
I1[λ_] := Tr[mC[λ]]
I2[λ_] := 1/2 (I1[λ]^2 - Tr[mC[λ]^2])
I1[λ[t]]
```

$$\frac{2}{\lambda[t]} + \lambda[t]^2$$

```
I2[λ[t]]
```

$$\frac{1}{2}\left(-\frac{2}{\lambda[t]^2} - \lambda[t]^4 + \left(\frac{2}{\lambda[t]} + \lambda[t]^2\right)^2\right)$$

```
φ[λ_] := c (I1[λ] - 3)
φ[λ[t]] // FullSimplify
```

$$c\left(-3 + \frac{2}{\lambda[t]} + \lambda[t]^2\right)$$

```
D[φ[λ[t]], t] // FullSimplify
```

$$\frac{2\,c\left(-1 + \lambda[t]^3\right)\lambda'[t]}{\lambda[t]^2}$$

```
D[φ[λ[t]], t] λ[t]/λ'[t] // FullSimplify
```

$$\frac{2\,c\left(-1 + \lambda[t]^3\right)}{\lambda[t]}$$

```
σ0[λ1_] = % /. λ[t] → λ1 // Simplify
```

$$\frac{2\,c\left(-1 + \lambda 1^3\right)}{\lambda 1}$$

```
σ0[λ[t]]
```

$$\frac{2\,c\left(-1 + \lambda[t]^3\right)}{\lambda[t]}$$

Uniaxial traction for a viscoelastic material

```
viscoEq = {σ0[λ[t]] + 3 μ λ'[t]/λ[t] == p0}
```

$$\left\{\frac{2\,c\left(-1 + \lambda[t]^3\right)}{\lambda[t]} + \frac{3\,\mu\,\lambda'[t]}{\lambda[t]} == p0\right\}$$

```
viscoEq /. {λ → (λ0 + β dϵ[#] &)} // FullSimplify
```

$$\left\{\frac{2\,c\left(-1 + (\lambda 0 + \beta\, d\epsilon[t])^3\right) + 3\,\beta\,\mu\, d\epsilon'[t]}{\lambda 0 + \beta\, d\epsilon[t]} == p0\right\}$$

```
viscoEqβ = Series[Evaluate[viscoEq /. {λ → (λ0 + β dϵ[#] &)}], {β, 0, 1}] // FullSimplify // Normal
```

$$\left\{\frac{2\,c\,\left(-1+\lambda0^3\right)}{\lambda0}+\frac{\beta\,\left(2\,\left(c+2\,c\,\lambda0^3\right)\,d\epsilon[t]+3\,\lambda0\,\mu\,d\epsilon'[t]\right)}{\lambda0^2}=p0\right\}$$

```
viscoEqβ0 = viscoEqβ[[1]] /. β → 0
```

$$\frac{2\,c\,\left(-1+\lambda0^3\right)}{\lambda0}=p0$$

```
p0StaSol = Solve[viscoEqβ0, p0][[1]]
```

$$\left\{p0\to\frac{2\,c\,\left(-1+\lambda0^3\right)}{\lambda0}\right\}$$

```
viscoEqLin = {a dϵ[t] + dϵ'[t] == 0, dϵ[0] == dϵ0}
```

$$\left\{a\,d\epsilon[t]+d\epsilon'[t]=0,\ d\epsilon[0]=d\epsilon0\right\}$$

```
viscoEqLin = viscoEqβ /. p0StaSol /. β → 1 // FullSimplify
```

$$\left\{\frac{2\,\left(c+2\,c\,\lambda0^3\right)\,d\epsilon[t]}{\lambda0}+3\,\mu\,d\epsilon'[t]=0\right\}$$

```
dϵSol = DSolve[Join[viscoEqLin, {dϵ[0] == dϵ0}], dϵ, t][[1]]
```

$$\left\{d\epsilon\to\text{Function}\left[\{t\},\ d\epsilon0\,e^{-\frac{2\,t\,\left(c+2\,c\,\lambda0^3\right)}{3\,\lambda0\,\mu}}\right]\right\}$$

```
λ0Sol = Assuming[λ0 > 0 && c > 0, Solve[viscoEqβ0, λ0] // FullSimplify]
```

$$\left\{\left\{\lambda0\to\frac{6^{1/3}\,c\,p0+\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{2/3}}{6^{2/3}\,c\,\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{1/3}}\right\},\right.$$

$$\left\{\lambda0\to\frac{-(-6)^{1/3}\,c\,p0+(-1)^{2/3}\,\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{2/3}}{6^{2/3}\,c\,\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{1/3}}\right\},$$

$$\left.\left\{\lambda0\to\frac{(-6)^{2/3}\,c\,p0-(-6)^{1/3}\,\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{2/3}}{6\,c\,\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{1/3}}\right\}\right\}$$

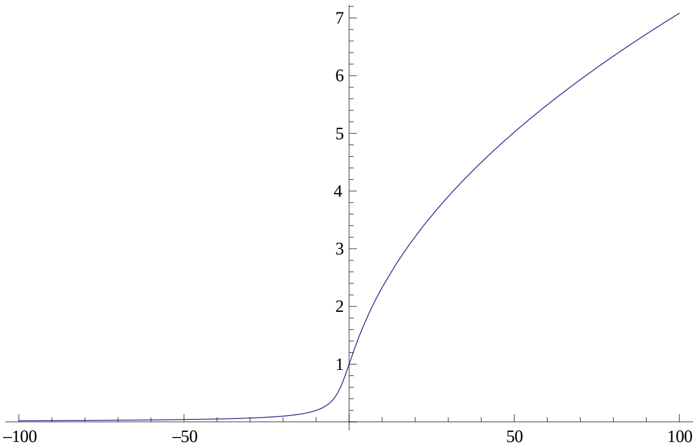
```
λ0Sol1 = λ0Sol[[1]]
```

$$\left\{\lambda0\to\frac{6^{1/3}\,c\,p0+\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{2/3}}{6^{2/3}\,c\,\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{1/3}}\right\}$$

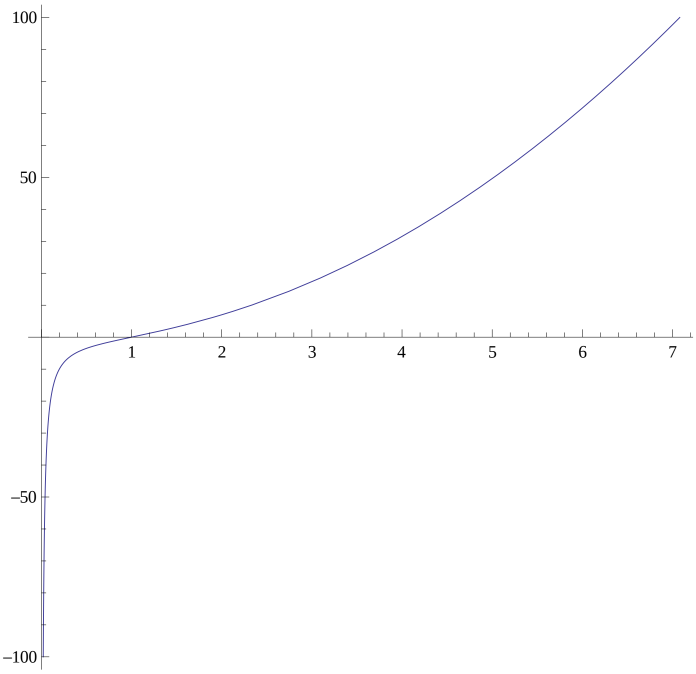
```
λ0f[p0_] = λ0 /. λ0Sol1 // FullSimplify
```

$$\frac{6^{1/3}\,c\,p0+\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{2/3}}{6^{2/3}\,c\,\left(18\,c^3+\sqrt{324\,c^6-6\,c^3\,p0^3}\right)^{1/3}}$$

```
Block[{c = 1}, Plot[λ0f[p0], {p0, -100, 100}]]
```




```
Block[{c = 1}, ParametricPlot[{λ0f[p0], p0}, {p0, -100, 100}, PlotRange → All, AspectRatio → 1]]
```




```
viscoEqLin
```

$$\left\{\frac{2\left(c+2\,c\,\lambda 0^3\right)\,d\epsilon[t]}{\lambda 0}+3\,\mu\,d\epsilon'[t]==0\right\}$$

```
viscoEqLin1 = viscoEqLin[[1]]
```

$$\frac{2\left(c+2\,c\,\lambda 0^3\right)\,d\epsilon[t]}{\lambda 0}+3\,\mu\,d\epsilon'[t]==0$$

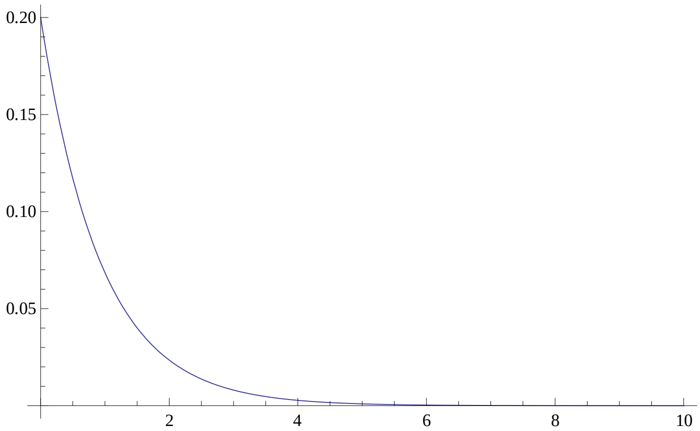
```
Block[{pc = 50}, viscoEqLin1]
```

$$\frac{2\left(c+2\,c\,\lambda 0^3\right)\,d\epsilon[t]}{\lambda 0}+3\,\mu\,d\epsilon'[t]==0$$

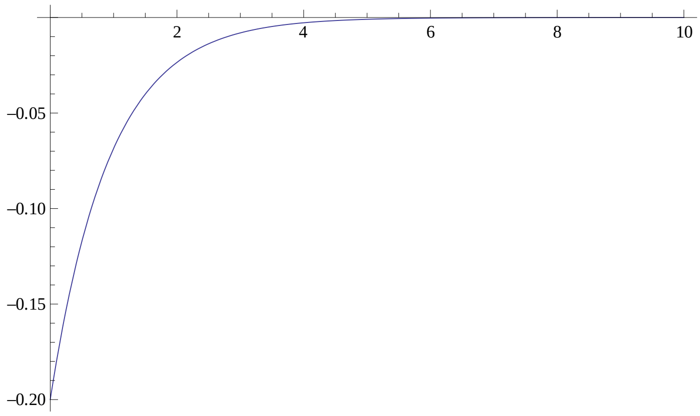
```
dϵS = dϵ /. dϵSol
```

$$\text{Function}\left[\{t\},\ d\epsilon 0\, e^{-\frac{2\,t\,\left(c+2\,c\,\lambda 0^3\right)}{3\,\lambda 0\,\mu}}\right]$$

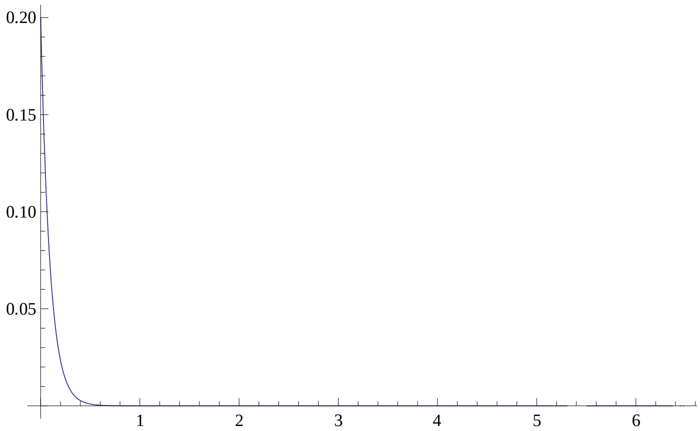
```
Block[{p0 = 15, c = 1, μ = 10, λ0 = λ0f[p0], tlim = 10, λN, dϵ0 = 0.2},
 Plot[dϵs[t], {t, 0, tlim}, PlotRange → All]]
```




```
Block[{p0 = 15, c = 1, μ = 10, λ0 = λ0f[p0], tlim = 10, λN, dϵ0 = -0.2},
 Plot[dϵs[t], {t, 0, tlim}, PlotRange → All]]
```




```
Block[{p0 = 15, c = 1, μ = 1, λ0 = λ0f[p0], tlim = 10, λN, dϵ0 = 0.2},
 Plot[dϵs[t], {t, 0, tlim}, PlotRange → All]]
```




```
viscoEqLin
```

$$\left\{\frac{2\left(c+2\,c\,\lambda 0^{3}\right)\,d\epsilon[t]}{\lambda 0}+3\,\mu\,d\epsilon'[t]==0\right\}$$

```
Block[{p0 = 50, c = 1, μ = 1, λ0 = λ0f[p0], tlim = 10, dϵ0 = 0.2}, λ0f[p0] // FullSimplify // N]
```

5.01988

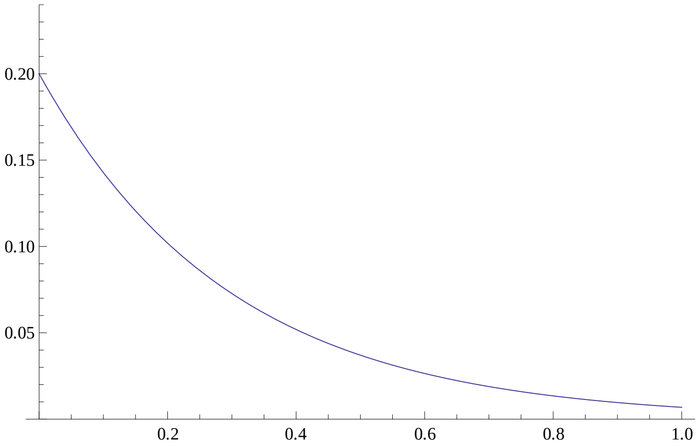
```
dϵs[t]
```

$d\epsilon 0\, e^{-\frac{2 t (c+2 c \lambda 0^3)}{3 \lambda 0 \mu}}$

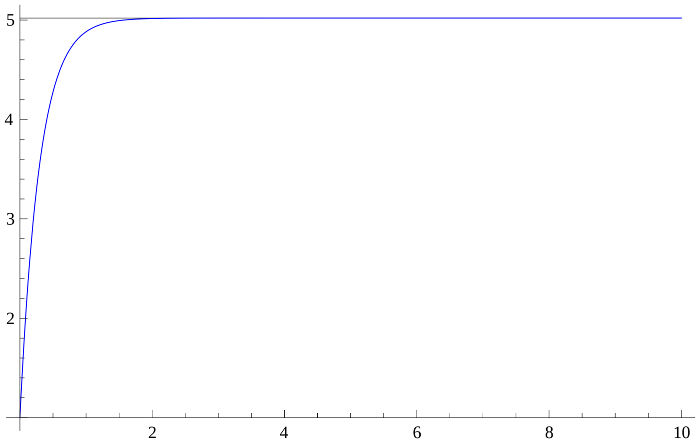
```
Block[{p0 = 50, c = 1, μ = 1, λ0 = λ0f[p0], tlim = 10}, dϵs[t] // Simplify // N // Chop]
```

$2.71828^{-33.7317\, t}\, d\epsilon 0$

```
Block[{p0 = 50, c = 1, λ0 = λ0f[p0], dϵ0 = 0.2, μ = 10},
 Plot[dϵs[t], {t, 0, 1}, PlotRange → {0, 1.2 dϵ0}]]
```



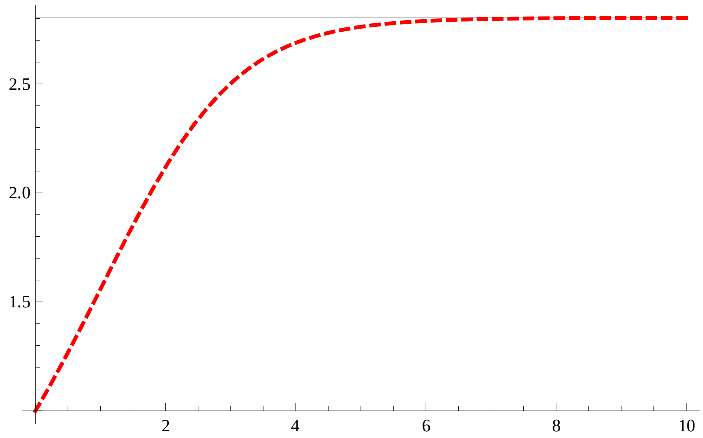

```
Block[{p0 = 50, c = 1, λ0 = λ0f[p0], dϵ0 = -(λ0 - 1), μ = 10, tlim = 10},
 Plot[{λ0, λ0 + dϵs[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]
```



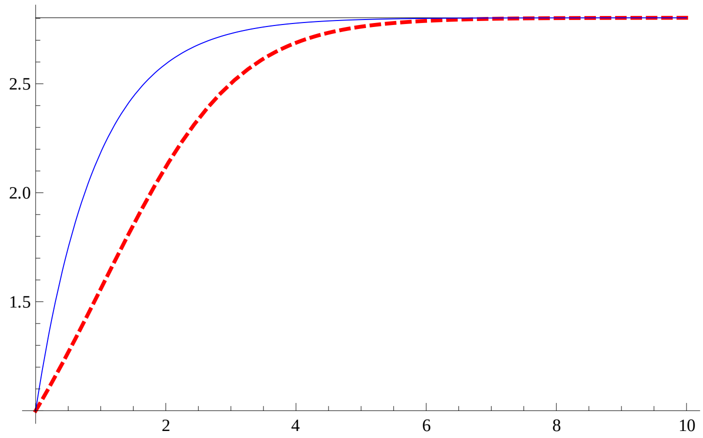

```
Block[{p0 = 5, c = 1}, λ0f[p0] // N // Chop]
```

1.75233

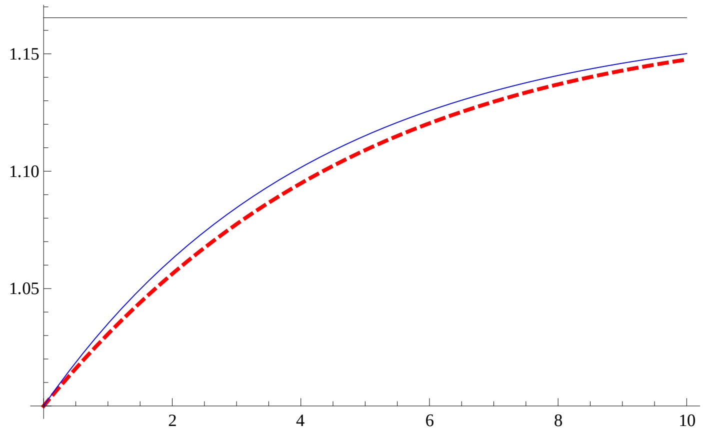
```
Block[{p0 = 15, c = 1, μ = 10, λ0 = λ0f[p0], tlim = 10, λN},
 λN = λ /. NDSolve[Join[viscoEq, {λ[0] == 1}], λ, {t, 0, tlim}][[1]];
Plot[{λ0, λN[t]}, {t, 0, tlim}, PlotRange → All,
  PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}]]
```



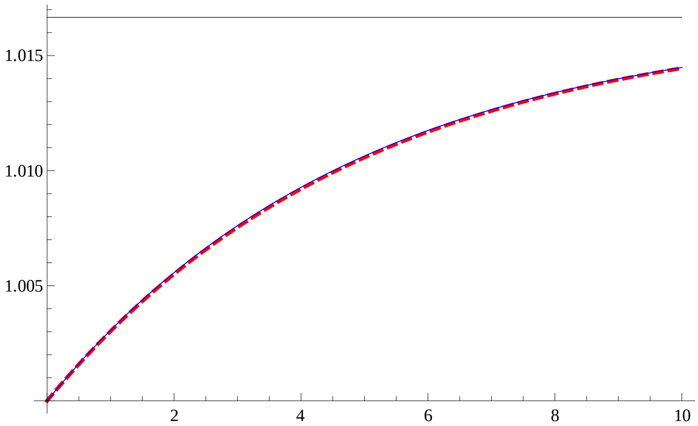
```
Block[{p0 = 15, c = 1, μ = 10, λ0 = λ0f[p0], dϵ0 = (1 - λ0), λN, tlim = 10},
 λN = λ /. NDSolve[Join[viscoEq, {λ[0] == 1}], λ, {t, 0, tlim}][[1]]; Show[Plot[{λ0, λN[t]},
   {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}],
  Plot[{λ0, λ0 + dϵs[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



```
Block[{p0 = 1, c = 1, μ = 10, λ0 = λ0f[p0], dϵ0 = (1 - λ0), λN, tlim = 10},
 λN = λ /. NDSolve[Join[viscoEq, {λ[0] == 1}], λ, {t, 0, tlim}][[1]]; Show[Plot[{λ0, λN[t]},
   {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}],
  Plot[{λ0, λ0 + dϵs[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```



```
Block[{p0 = 0.1, c = 1, μ = 10, λ0 = λ0f[p0], dϵ0 = (1 - λ0), λN, tlim = 10},
 λN = λ /. NDSolve[Join[viscoEq, {λ[0] ⩵ 1}], λ, {t, 0, tlim}]⟦1⟧; Show[Plot[{λ0, λN[t]},
   {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}],
  Plot[{λ0, λ0 + dϵs[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```




```
Block[{p0 = 0.1, c = 1, μ = 1, λ0 = λ0f[p0], dϵ0 = (1 - λ0), λN, tlim = 10},
 λN = λ /. NDSolve[Join[viscoEq, {λ[0] ⩵ 1}], λ, {t, 0, tlim}]⟦1⟧; Show[Plot[{λ0, λN[t]},
   {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Red, Dashed, Thick}}],
  Plot[{λ0, λ0 + dϵs[t]}, {t, 0, tlim}, PlotRange → All, PlotStyle → {{Black, Thin}, {Blue}}]]]
```