$\kappa : \mathcal{D} \to \bar{\mathcal{R}} \subset \mathcal{E}$ parameterization of $\bar{\mathcal{R}}$

$$\begin{tikzcd}
 & \bar{\mathcal{R}} \arrow[r, "\phi"] & \mathcal{R} \\
\mathcal{D} \arrow[ur, "\kappa"] & \mathcal{B} \arrow[u, "\bar{p}"'] &
\end{tikzcd}$$

$\phi_\kappa := \phi \circ \kappa : \mathcal{D} \to \mathcal{R}$ parameterization of $\mathcal{R}$

Description of ϕ through κ

$$\phi_\kappa(s_1, s_2, s_3) = \phi(\kappa(s_1, s_2, s_3))$$

$$\begin{aligned}
\phi_\kappa(s_1, s_2, s_3) = o &+ \phi_{\kappa 1}(s_1, s_2, s_3)\, e_1 \\
&+ \phi_{\kappa 2}(s_1, s_2, s_3)\, e_2 \\
&+ \phi_{\kappa 3}(s_1, s_2, s_3)\, e_3
\end{aligned}$$

Let us chose κ such that

$$\kappa(s_1, s_2, s_3) = o + s_1 e_1 + s_2 e_2 + s_3 e_3$$

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$$\bar{c}_1(h) = \bar{p}_0 + h e_1 = \kappa(s_1, s_2, s_3) + h e_1$$

$$= 0 + s_1 e_1 + s_2 e_2 + s_3 e_3 + h e_1$$

$$= \kappa(s_1 + h, s_2, s_3)$$

$$c_1(h) = \phi(\bar{c}_1(h)) = \phi(\kappa(s_1 + h, s_2, s_3))$$

$$= \phi_\kappa(s_1 + h, s_2, s_3) = 0 + \phi_{\kappa 1}(s_1 + h, s_2, s_3) e_1$$

$$+ \phi_{\kappa 2}(s_1 + h, s_2, s_3) e_2 + \phi_{\kappa 3}(s_1 + h, s_2, s_3) e_3$$

$$c_1'(0) = \lim_{h \to 0} (c_1(h) - c_1(0)) \frac{1}{h}$$

$$= \partial_1 \phi_{\kappa 1} e_1 + \partial_1 \phi_{\kappa 2} e_2 + \partial_1 \phi_{\kappa 3} e_3$$

$$c_2(h) = \phi_\kappa(s_1, s_2 + h, s_3) = \ldots$$

$$c_3(h) = \phi_\kappa(s_1, s_2, s_3 + h) = \ldots$$

$$\bar{c}(h) = \bar{p}_0 + h(\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3)$$

$$= \kappa(s_1 + h\alpha_1, s_2 + h\alpha_2, s_3 + h\alpha_3)$$

$$c(h) = \phi_\kappa(s_1 + h\alpha_1, s_2 + h\alpha_2, s_3 + h\alpha_3)$$

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$$\bar{c}'(0) = \lim_{h \to 0} \left(\bar{c}(h) - \bar{c}(0) \right) \frac{1}{h}$$

$$c'(0) = \lim_{h \to 0} \left(c(h) - c(0) \right) \frac{1}{h}$$

$$\bar{c}(h) = o + (s_1 + \alpha_1 h) e_1 + (s_2 + \alpha_2 h) e_2 + (s_3 + \alpha_3 h) e_3$$

$$\begin{aligned} c(h) = o &+ \phi_{\kappa 1}(s_1 + \alpha_1 h, s_2 + \alpha_2 h, s_3 + \alpha_3 h)\, e_1 \\ &+ \phi_{\kappa 2}(\qquad\qquad)\, e_2 \\ &+ \phi_{\kappa 3}(\qquad\qquad)\, e_3 \end{aligned}$$

$$\begin{aligned} c'(0) = {} & \partial_1 \phi_{\kappa 1} \alpha_1 e_1 + \partial_2 \phi_{\kappa 1} \alpha_2 e_1 + \partial_3 \phi_{\kappa 1} \alpha_3 e_1 \\ & + \partial_1 \phi_{\kappa 2} \alpha_1 e_2 + \partial_2 \phi_{\kappa 2} \alpha_2 e_2 + \partial_3 \phi_{\kappa 2} \alpha_3 e_2 \\ & + \partial_1 \phi_{\kappa 3} \alpha_1 e_3 + \partial_2 \phi_{\kappa 3} \alpha_2 e_3 + \partial_3 \phi_{\kappa 3} \alpha_3 e_3 \end{aligned}$$

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$$c'_1 = (\partial_1 \phi_{k1}) e_1 + (\partial_1 \phi_{k2}) e_2 + (\partial_1 \phi_{k3}) e_3$$

$$c'_2 = (\partial_2 \phi_{k1}) e_1 + (\partial_2 \phi_{k2}) e_2 + (\partial_1 \phi_{k3} e_3$$

$$c'_3 = (\partial_3 \phi_{k1}) e_1 + (\partial_3 \phi_{k2}) e_2 + (\partial_3 \phi_{k3}) e_3$$

$$c'_1 = \phi_{k1,1}\, e_1 + \phi_{k2,1}\, e_2 + \phi_{k3,1}\, e_3$$

$$c'_2 = \phi_{k1,2}\, e_1 + \phi_{k2,2}\, e_2 + \phi_{k3,2}\, e_3$$

$$c'_3 = \phi_{k1,3}\, e_1 + \phi_{k2,3}\, e_2 + \phi_{k3,3}\, e_3$$

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$$\bar{c}(h) = \bar{p}_0 + h\,\bar{u} = \overset{\kappa(s_1, s_2, s_3)}{\bar{p}_0} + h\,\underbrace{(\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3)}_{\bar{u}}$$

$$c(h) = \phi(\bar{c}(h)) = \phi(\kappa(s_1+\alpha_1 h, s_2+\alpha_2 h, s_3+\alpha_3 h))$$
$$= \phi_\kappa(s_1+\alpha_1 h, s_2+\alpha_2 h, s_3+\alpha_3 h)$$

$$\lim_{h\to 0} \frac{\bar{c}(h)-\bar{c}(0)}{h} = \bar{u}$$

$$\begin{aligned}
\frac{c(h)-c(0)}{h} = {} & \frac{1}{h}\Big(\phi_{\kappa 1}(s_1+\alpha_1 h, s_2+\alpha_2 h, s_3+\alpha_3 h) \\
& \qquad - \phi_{\kappa 1}(s_1, s_2, s_3)\Big)\, e_1 \\
& + \frac{1}{h}\Big(\phi_{\kappa 2}(\quad , \quad , \quad) \\
& \qquad - \phi_{\kappa 2}(s_1, s_2, s_3)\Big)\, e_2 \\
& + \frac{1}{h}\Big(\phi_{\kappa 3}(\quad , \quad , \quad) \\
& \qquad - \phi_{\kappa 3}(s_1, s_2, s_3)\Big)\, e_3
\end{aligned}$$

$$\begin{aligned}
c'(0) = \lim_{h\to 0} \frac{c(h)-c(0)}{h} = {} & (\phi_{\kappa 1,1}\,\alpha_1 + \phi_{\kappa 1,2}\,\alpha_2 + \phi_{\kappa 1,3}\,\alpha_3)\, e_1 \\
& + (\phi_{\kappa 2,1}\,\alpha_1 + \phi_{\kappa 2,2}\,\alpha_2 + \phi_{\kappa 2,3}\,\alpha_3)\, e_2 \\
& + (\phi_{\kappa 3,1}\,\alpha_1 + \phi_{\kappa 3,2}\,\alpha_2 + \phi_{\kappa 3,3}\,\alpha_3)\, e_3
\end{aligned}$$

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$$c' = \alpha_1 c'_1 + \alpha_2 c'_2 + \alpha_3 c'_3$$

$$\bar{c}'_1 \mapsto c'_1 \qquad \bar{u}_1 \mapsto u_1$$

$$\bar{c}'_2 \mapsto c'_2 \qquad \bar{u}_2 \mapsto u_2$$

$$\bar{c}'_3 \mapsto c'_3 \qquad \bar{u}_3 \mapsto u_3$$

$$\alpha_1 \bar{c}'_1 + \alpha_2 \bar{c}'_2 + \bar{\alpha}'_3 \mapsto \alpha_1 c'_1 + \alpha_2 c'_2 + \alpha_3 c'_3$$

$\Rightarrow$ There exists a Tensor $\quad F(\bar{p}_0)\,\bar{c}'_1 = c'_1$

[...]

Matrix of $F(\bar{p}_0)$ $\qquad \bar{p}_0 = \bar{c}_i(0)$

$$c'_1 = F(\bar{p}_0)\,e_1 \qquad c'_2 = F(\bar{p}_0)\,e_2 \qquad c'_3 = F(\bar{p}_0)\,e_3$$

$$\begin{pmatrix} \phi_{k1,1} & \phi_{k1,2} & \phi_{k1,3} \\ \phi_{k2,1} & \phi_{k2,2} & \phi_{k2,3} \\ \phi_{k3,1} & \phi_{k3,2} & \phi_{k3,3} \end{pmatrix}$$

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$$c(h) = \phi(\bar{c}(h))$$

$$c(h) = c(0) + F(\bar{c}(0))(\bar{c}(h) - \bar{c}(0)) + o(h)$$

$$\lim_{h\to 0} c(h) = c(0) \quad \Rightarrow \quad \lim_{h\to 0} o(h) = 0$$

$$\lim_{h\to 0} \frac{c(h)-c(0)}{h} = F(\bar{c}(0)) \lim \frac{\bar{c}(h)-\bar{c}(0)}{h} + \lim_{h\to 0} \frac{o(h)}{h}$$

$$c'(0) = F(\bar{c}(0))\,\bar{c}'(0) + \lim_{h\to 0} \frac{o(h)}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{o(h)}{h} = 0$$

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Any parameterization is "local" in general

$$z(\zeta_1, \zeta_2, \zeta_3) = p_0 + \zeta_1 e_1 + \zeta_2 e_2 + \zeta_3 e_3$$

$$c(h) = z(c_{z_1}(h), c_{z_2}(h), c_{z_3}(h))$$
$$= p_0 + c_{z_1}(h) e_1 + c_{z_2}(h) e_2 + c_{z_3}(h) e_3$$

$$c'(0) = \underbrace{c'_{z_1}(0)}_{\alpha_1} e_1 + \underbrace{c'_{z_2}(0)}_{\alpha_2} e_2 + \underbrace{c'_{z_3}(0)}_{\alpha_3} e_3$$

$$v(z(\zeta_1, \zeta_2, \zeta_3)) = v_z(\zeta_1, \zeta_2, \zeta_3)$$
$$= v_{z_1}(\quad) e_1 + v_{z_2}(\quad) e_2 + v_{z_3}(\quad) e_3$$

$$v(c(h)) = v_z(c_{z_1}(h), c_{z_2}(h), c_{z_3}(h))$$

$$\lim \left(v(c(h)) - v(c(0))\right)\frac{1}{h} = \partial_1 v_{z_1} \alpha_1 + \dots \qquad (\rightarrow)$$

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$$c(0) = p_0 + c_{z1}(0)\, e_1 + c_{z2}(0)\, e_2 + c_{z3}(0)\, e_3$$

$$c(h) = p_0 + c_{z1}(h)\, e_1 + c_{z2}(h)\, e_2 + c_{z3}(h)\, e_3$$

$$c'(0) = \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 \qquad \alpha_i := c'_{zi}(0)$$

$$\begin{aligned} v(c(h)) = {} & v_{z1}\big(c_{z1}(h), c_{z2}(h), c_{z3}(h)\big)\, e_1 \\ & + v_{z2}\big(c_{z1}(h), \quad , \quad\big)\, e_2 \\ & + v_{z3}\big(c_{z1}(h), \quad , \quad\big)\, e_3 \end{aligned}$$

$$\lim_{h\to 0} \frac{v(c(h)) - v(c(0))}{h} =$$

$$\begin{aligned} & (\partial_1 v_{z1}\, \alpha_1 + \partial_2 v_{z1}\, \alpha_2 + \partial_3 v_{z1}\, \alpha_3)\, e_1 \\ + & (\partial_1 v_{z2}\, \alpha_1 + \partial_2 v_{z2}\, \alpha_2 + \partial_3 v_{z2}\, \alpha_3)\, e_2 \\ + & (\partial_1 v_{z3}\, \alpha_1 + \partial_2 v_{z3}\, \alpha_2 + \partial_3 v_{z3}\, \alpha_3)\, e_3 \end{aligned}$$

$$\begin{aligned} = {} & \alpha_1 (\partial_1 v_{z1}\, e_1 + \partial_1 v_{z2}\, e_2 + \partial_1 v_{z3}\, e_3) \\ & + \alpha_2 (\partial_2 v_{z1}\, e_1 + \partial_2 v_{z2}\, e_2 + \partial_2 v_{z3}\, e_3) \\ & + \alpha_3 (\partial_3 v_{z1}\, e_1 + \partial_3 v_{z2}\, e_2 + \partial_3 v_{z3}\, e_3) \end{aligned}$$

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This result can be proved to be independent of the parameterization

$$\lim_{h\to 0} \frac{v(c(h))-v(c(0))}{h} = \nabla v(p_0)\, c'(0)$$

Now it's worth noting that the difference vector

$$o(h) := v(c(h)) - \Big(v(c(0)) + \nabla v(p_0)\big(c(h)-c(0)\big)\Big)$$

has the property

$$\lim_{h\to 0} \frac{o(h)}{h} = \lim_{h\to 0} \frac{v(c(h))-v(c(0))}{h} - \nabla v(p_0) \lim_{h\to 0} \frac{c(h)-c(0)}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{o(h)}{h} = 0$$

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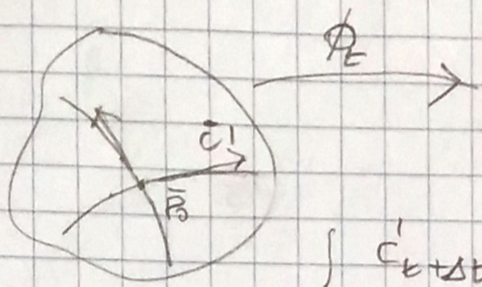
$$c'(0) = F(\bar{p}_0)\,\bar{c}'(0) \qquad \bar{c}(0) = \bar{p}_0\,,\ c(0) = p_0$$



$$\begin{cases} c'_{t+\Delta t}(0) = F_{t+\Delta t}(\bar{p}_0)\,\bar{c}'(0) \\ c'_t(0) = F_t(\bar{p}_0)\,\bar{c}'(0) \end{cases}$$

$$\dot{c}'_t(0) = \dot{F}_t(\bar{p}_0)\,\bar{c}'(0)$$

$$\dot{c}'_t(0) = \underbrace{\dot{F}_t(\bar{p}_0)\,F_t(\bar{p}_0)^{-1}}_{\nabla v}\, c'_t(0)$$

$$\lim_{h \to 0} \frac{v(c(h)) - v(c(0))}{h} = \lim_{h \to 0} \frac{\dot{c}(h) - \dot{c}(0)}{h}$$

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EASTER HOLYDAYS

Thursday 2016/3/24

Tuesday 2016/3/29

Classes will resume on

Wednesday 2016/3/30

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