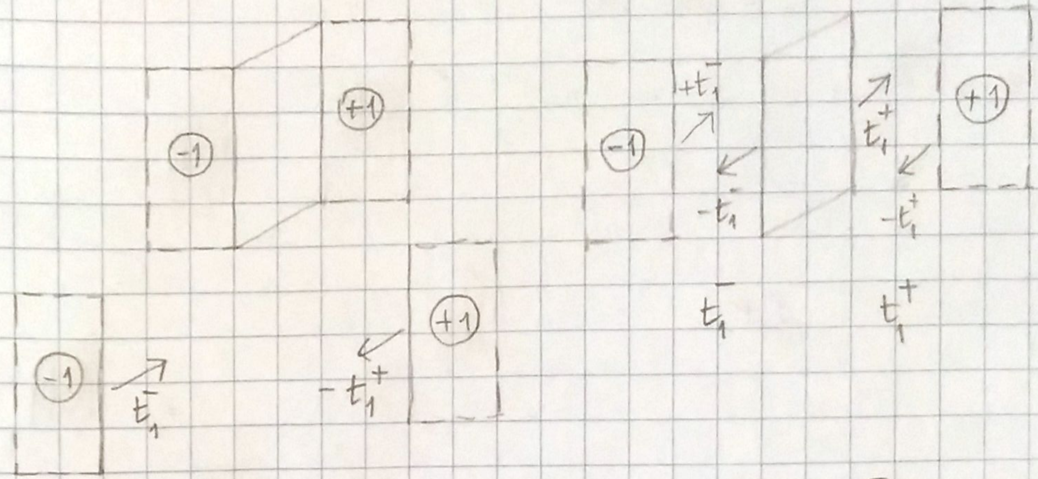
SKIP

Appendix [2016-04-12]

Balanced contiguous bodies



$$t_1^- = T^{(-1)} n_1 + \check t_1^{(-1)} \qquad -t_1^+ = -T^{(+1)} n_1 + \check t_1^{(+1)}$$

$$b^{(-1)} V^{(-1)} + 2\, \check t_1^{(-1)} A_{F_1}^{(-1)} = b^{(-1)} A_{F_1}^{(-1)} h_1^{(-1)} + 2\, \check t_1^{(-1)} A_{F_1}^{(-1)} = 2\, \check t_1^{(-1)} A_{F_1}^{(-1)} + o(h_1^{(-1)})$$

$$b^{(-1)} V^{(-1)} + 2\, \check t_1^{(-1)} A_{F_1}^{(-1)} + 2\, \check t_2^{(-1)} A_{F_2}^{(-1)} + 2\, \check t_3^{(-1)} A_{F_3}^{(-1)} = 0 \qquad \text{balance}$$

$$\lim_{h_1^{(-1)} \to 0} V^{(-1)} = 0 \qquad \lim_{h_1^{(-1)} \to 0} A_{F_2}^{(-1)} = 0 \qquad \lim_{h_1^{(-1)} \to 0} A_{F_3}^{(-1)} = 0$$

$$\text{balance} \Rightarrow \quad \{ h_1^{(-1)} \to 0 \Rightarrow \check t_1^{(-1)} \to 0 \} \equiv \lim_{h_1^{(-1)} \to 0} \check t_1^{(-1)} = 0$$

$$b^{(+1)} V^{(+1)} + 2\, \check t_1^{(+1)} A_{F_1}^{(+1)} + 2\, \check t_2^{(+1)} A_{F_2}^{(+1)} + 2\, \check t_3^{(+1)} A_{F_3}^{(+1)} = 0 \qquad \text{balance}$$

[...] $\qquad \{ h_1^{(+1)} \to 0 \Rightarrow \check t_1^{(+1)} \to 0 \}$

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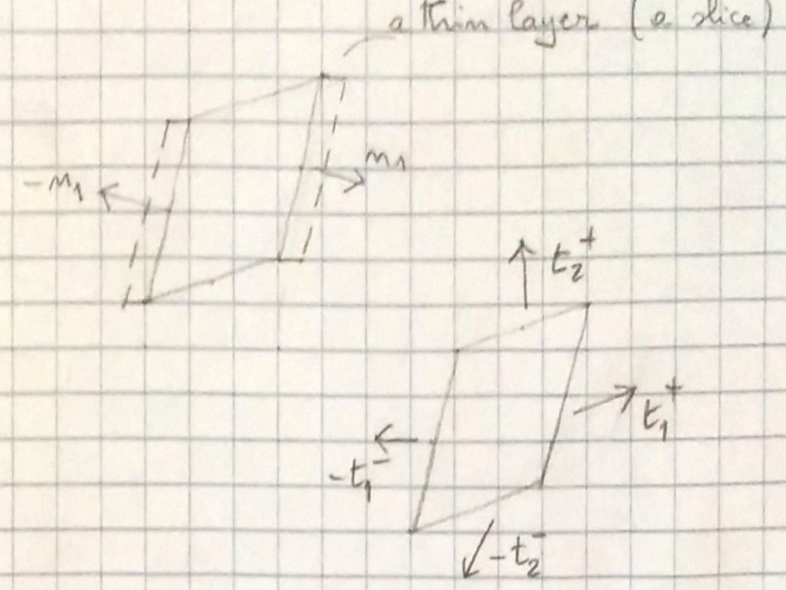

$$t_1^- = T^{(-1)} n_1 + \underbrace{\check{t}_1^{(-1)}}_{\downarrow\, 0} \qquad -t_1^+ = -T^{(+1)} n_1 + \underbrace{\check{t}_1^{(+1)}}_{\downarrow\, 0}$$

$$\Rightarrow \quad t_1^+ - t_1^- = T^{(+1)} n_1 - T^{(-1)} n_1$$

$$\begin{aligned} \frac{1}{V_\delta} f_\partial &= \frac{1}{h_1}\left(t_1^+ - t_1^-\right) + \frac{1}{h_2}\left(t_2^+ - t_2^-\right) + \frac{1}{h_3}\left(t_3^+ - t_3^-\right) \\ &= \frac{1}{h_1}\left(T^{(+1)} n_1 - T^{(-1)} n_1\right) \\ &\quad + \frac{1}{h_2}\left(T^{(+2)} n_2 - T^{(-2)} n_2\right) \\ &\quad + \frac{1}{h_3}\left(T^{(+3)} n_3 - T^{(-3)} n_3\right) \end{aligned}$$

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$$\frac{1}{V_R} M n_1 = \frac{t_1^+ + t_1^-}{2} = \frac{1}{2}\left(T^{(+1)} n_1 + T^{(-1)} n_1\right)$$

$$\frac{1}{V_R} M n_2 = \frac{t_2^+ + t_2^-}{2} = \frac{1}{2}\left(T^{(+2)} n_2 + T^{(-2)} n_2\right)$$

$$\frac{1}{V_R} M n_3 = \frac{t_3^+ + t_3^-}{2} = \frac{1}{2}\left(T^{(+3)} n_3 + T^{(-3)} n_3\right)$$

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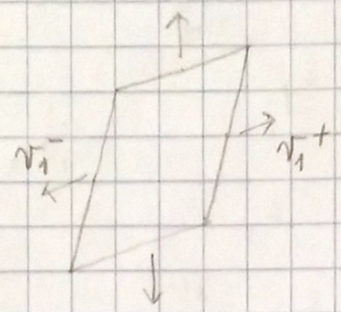
Appendix [2016-04-12]

Discrete forms for divergence and gradient of a vector field (generating a sequence)

$$\int_{\mathcal{R}} \operatorname{div} v \, dV = \int_{\partial\mathcal{R}} v \cdot n \, dA \qquad \text{flux}$$



$$\begin{aligned}
\text{flux}(v) = {} & v_1^+ \cdot n_1 A_{F_1} + v_1^- \cdot (-n_1) A_{F_1} \\
& + v_2^+ \cdot n_2 A_{F_2} + v_2^- \cdot (-n_2) A_{F_2} \\
& + v_3^+ \cdot n_3 A_{F_3} + v_3^- \cdot (-n_3) A_{F_3} \\
= {} & A_{F_1} n_1 \cdot (v_1^+ - v_1^-) \\
& + A_{F_2} n_2 \cdot (v_2^+ - v_2^-) \\
& + A_{F_3} n_3 \cdot (v_3^+ - v_3^-)
\end{aligned}$$

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$$\text{flux}(v) = V_R \left(n_1 \cdot \frac{v_1^+ - v_1^-}{h_1} + n_2 \cdot \frac{v_2^+ - v_2^-}{h_2} + n_3 \cdot \frac{v_3^+ - v_3^-}{h_3} \right)$$

$$\frac{1}{V_R} \text{flux}(T^T v) = n_1 \cdot \frac{1}{h_1} \left((T^T v)_1^+ - (T^T v)_1^- \right) + \dots$$

$$(T^T v)_1^+ := (T_1^+)^T v_1^+$$

$$(T^T v)_1^- := (T_1^-)^T v_1^-$$

$$n_1 \cdot (T_1^+)^T v_1^+ = T_1^+ n_1 \cdot v_1^+$$

$$n_1 \cdot (T_1^-)^T v_1^- = T_1^- n_1 \cdot v_1^-$$

$$n_1 \cdot \frac{1}{h_1} \left((T^T v)_1^+ - (T^T v)_1^- \right)$$

$$= \frac{1}{h_1} \left(T_1^+ n_1 \cdot v_1^+ - T_1^- n_1 \cdot v_1^- \right)$$

$$= \frac{1}{h_1} \left(T_1^+ n_1 \cdot (v_0 + \tfrac{1}{2} L u_1) - T_1^- n_1 \cdot (v_0 - \tfrac{1}{2} L u_1) \right)$$

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$$n_1 \cdot \frac{1}{h_1}\left((T^T v)^+_1 - (T^T v)^-_1 \right)$$

$$= \frac{1}{h_1}\left(T^+_1 n_1 - T^-_1 n_1 \right) \cdot v_0$$

$$+ \frac{1}{h_1}\,\frac{1}{2}\left(T^+_1 n_1 \otimes u_1 + T^-_1 n_1 \otimes u_1 \right) \cdot L$$

$$= \frac{1}{h_1}\left(T^+_1 - T^-_1 \right) n_1 \cdot v_0$$

$$+ \frac{1}{h_1}\,\frac{1}{2}\left((T^+_1 + T^-_1) n_1 \otimes u_1 \right) \cdot L$$

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$$\frac{1}{V_R}\,\text{flux}(T^T v) =$$

$$\left(\frac{1}{h_1}\left(T_1^+ n_1 - T_1^- n_1\right) + \frac{1}{h_2}\left(T_2^+ n_2 - T_2^- n_2\right) + \frac{1}{h_3}\left(T_3^+ n_3 - T_3^- n_3\right)\right)\cdot v_0$$

$$+\left(\frac{1}{h_1}\frac{1}{2}\left(T_1^+ n_1 + T_1^- n_1\right)\otimes u_1\right.$$

$$+\frac{1}{h_2}\frac{1}{2}\left(T_2^+ n_2 + T_2^- n_2\right)\otimes u_2$$

$$\left.+\frac{1}{h_3}\frac{1}{2}\left(T_3^+ n_3 + T_3^- n_3\right)\otimes u_3\right)\cdot L$$

What we defined as the flux of $(T^T v)$ turns out to be made up of the sum of two parts

Let us set (as the limit value)

$$\text{div}\,T = \left(\frac{1}{h_1}\left(T_1^+ n_1 - T_1^- n_1\right) + \dots\right)$$

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$$\frac{1}{h_1}(m_1 \otimes u_1)\, m_1 = m_1$$

$$\frac{1}{h_1}(m_1 \otimes u_1)\, m_2 = \frac{1}{h_1} m_1 (u_1 \cdot m_2) = 0$$

$$\frac{1}{h_1}(m_1 \otimes u_1)\, m_3 = \frac{1}{h_1} m_1 (u_1 \cdot m_3) = 0$$

Let us set (as the limit value)

$$T m_1 = \frac{1}{2}(T_1^+ m_1 + T_1^- m_1) = \frac{1}{2}(T_1^+ + T_1^-) m_1$$

$$T m_2 = \frac{1}{2}(T_2^+ m_2 + T_2^- m_2) = \frac{1}{2}(T_2^+ + T_2^-) m_2$$

$$T m_3 = \frac{1}{2}(T_3^+ m_3 + T_3^- m_3) = \frac{1}{2}(T_3^+ + T_3^-) m_3$$

the second part will turn into

$$\left(\frac{1}{h_1} T m_1 \otimes u_1 + \frac{1}{h_2} T m_2 \otimes u_2 + \frac{1}{h_3} T m_3 \otimes u_3\right) \cdot L$$

$$= \left(T \underbrace{\left((m_1 \otimes u_1)\frac{1}{h_1} + (m_2 \otimes u_2)\frac{1}{h_2} + (m_3 \otimes u_3)\frac{1}{h_3} \right)}_{I} \right) \cdot L$$

$$= T \cdot L$$

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$$\lim \frac{1}{V_\Omega} \operatorname{flux}(\mathsf{T}^T v) = \operatorname{div}\mathsf{T} \cdot v + \mathsf{T} \cdot \nabla v$$

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