[2016-05-10]

External dissipation and stability

(balance)

$$M^+ \cdot W \geqslant 0 \quad \text{(dissipation inequality)}$$

$$\mathrm{skw}\, M - M^+ = 0$$

$$M^+ = \eta W \quad \Rightarrow \quad \eta\, W \cdot W \geqslant 0 \quad \Rightarrow \quad \eta \geqslant 0$$

$$\mathrm{skw}\, M - \eta W = 0 \qquad \text{(balance)}$$

$$\frac{\sqrt{2}}{4}\, p\, V_\ell \,(\cos\theta - \sin\theta)\, e_2 \otimes e_1 - \eta\, \dot{\theta}\, e_2 \otimes e_1 = 0$$

$$(\cos\theta - \sin\theta) = -\sqrt{2}\, \sin\left(\theta - \frac{\pi}{4}\right)$$

$$p\, \frac{\sqrt{2}}{4} V_\ell \,(\cos\theta - \sin\theta) = \eta\, \dot{\theta}$$

linearization

$$\cos\theta = \cos\theta_0 - \sin\theta_0 (\theta - \theta_0) + \dots$$

$$\sin\theta = \sin\theta_0 + \cos\theta_0 (\theta - \theta_0) + \dots$$

$$\theta_0 = \frac{\pi}{4} \qquad \cos\theta_0 = \sin\theta_0$$

$$\cos\theta - \sin\theta = 0 - \sqrt{2}\,(\theta - \theta_0) + \dots$$

$$-p\, \frac{\sqrt{2}}{4} V_\ell \sqrt{2}\,(\theta - \theta_0) = \eta\, \dot{\theta} \qquad \text{linearized equation}$$

$$-\alpha\, (\theta - \theta_0) = \eta\, \dot{\theta} \qquad \left(-\alpha\, \tilde{\theta} = \eta\, \dot{\tilde{\theta}}\right)$$

$$(\theta - \theta_0) = \tilde{\theta}_0\, e^{-\frac{\alpha}{\eta} t} \quad \begin{cases} \Rightarrow \dot{\theta} = -\dfrac{\alpha}{\eta}\, \tilde{\theta}_0\, e^{-\frac{\alpha}{\eta} t} \\[2ex] \Rightarrow \dot{\theta} = -\dfrac{\alpha}{\eta}\, (\theta - \theta_0) \end{cases}$$

$$\theta(0) - \theta_0 = \tilde{\theta}_0$$

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$$\theta(t) = \theta_0 + \tilde{\theta}_0 \, e^{-\frac{\alpha}{\eta} t}$$

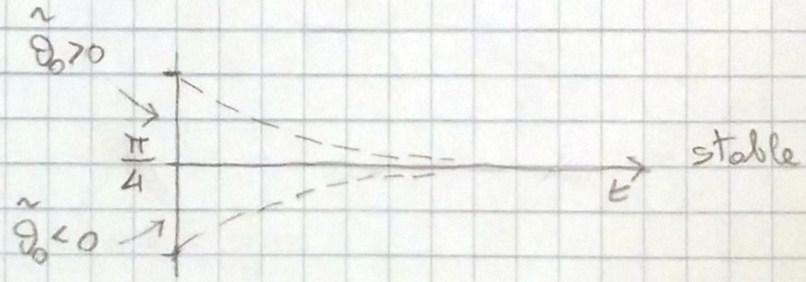
$$-p \, \frac{1}{2} V_k \sin\left(\theta - \frac{\pi}{4}\right) = \eta \, \dot{\theta}$$

linearization around $\theta_0 = \frac{\pi}{4}$

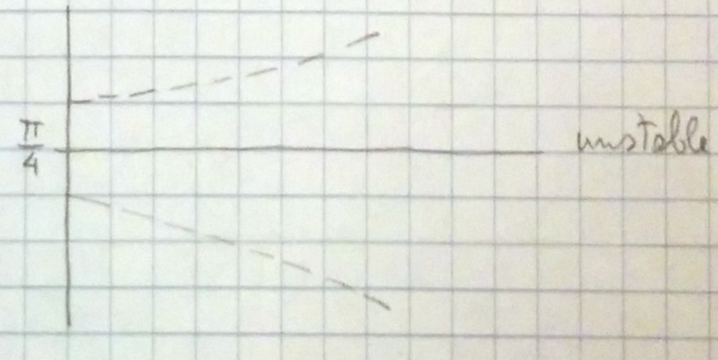
$$\underbrace{-p \, \frac{1}{2} V_k}_{\alpha} \left(\theta - \frac{\pi}{4}\right) = \eta \, \dot{\theta}$$

$$-\alpha \, \tilde{\theta}_0 \, e^{-\frac{\alpha}{\eta} t} = \eta \left(-\frac{\alpha}{\eta}\right) \tilde{\theta}_0 \, e^{-\frac{\alpha}{\eta} t}$$

$$p > 0 \Leftrightarrow \alpha > 0$$




$$p < 0 \Leftrightarrow \alpha < 0$$

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Linearization around

$$\theta_0 = \frac{\pi}{4} + \pi = \frac{5}{4}\pi$$

$$\sin\left(\theta - \frac{\pi}{4}\right) = \sin\left(\theta_0 - \frac{\pi}{4}\right) + \cos\left(\theta_0 - \frac{\pi}{4}\right)(\theta - \theta_0) + \dots$$

$$= \sin(\pi) + \cos(\pi)(\theta - \theta_0) + \dots$$

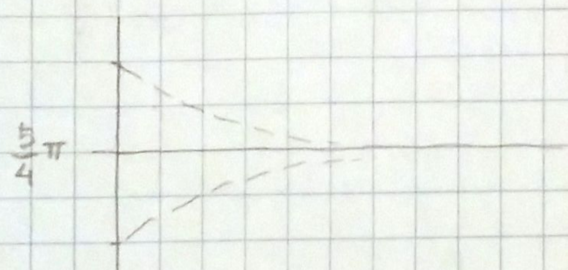
$$= -(\theta - \theta_0)$$

$$\cos\theta - \sin\theta = -\sqrt{2}\sin\left(\theta - \frac{\pi}{4}\right) = +\sqrt{2}\left(\theta - \frac{5}{4}\pi\right) + \dots$$

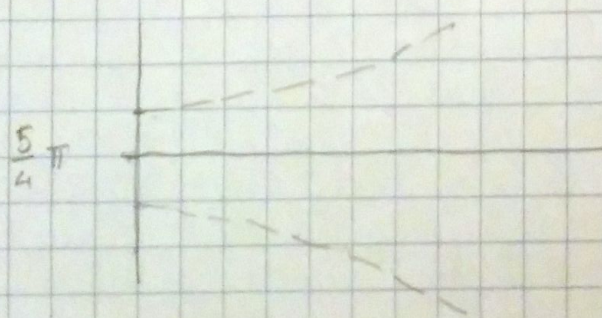
$$p\,\frac{1}{2}\,V_g\,(\theta - \theta_0) = \eta\,\dot\theta \qquad \alpha(\theta - \theta_0) = \eta\,\dot\theta$$

$$\theta(t) = \frac{5}{4}\pi + \tilde\theta_0\, e^{\frac{\alpha}{\eta}t}$$

$$p < 0 \Leftrightarrow \alpha < 0$$




$$p > 0 \Leftrightarrow \alpha > 0$$

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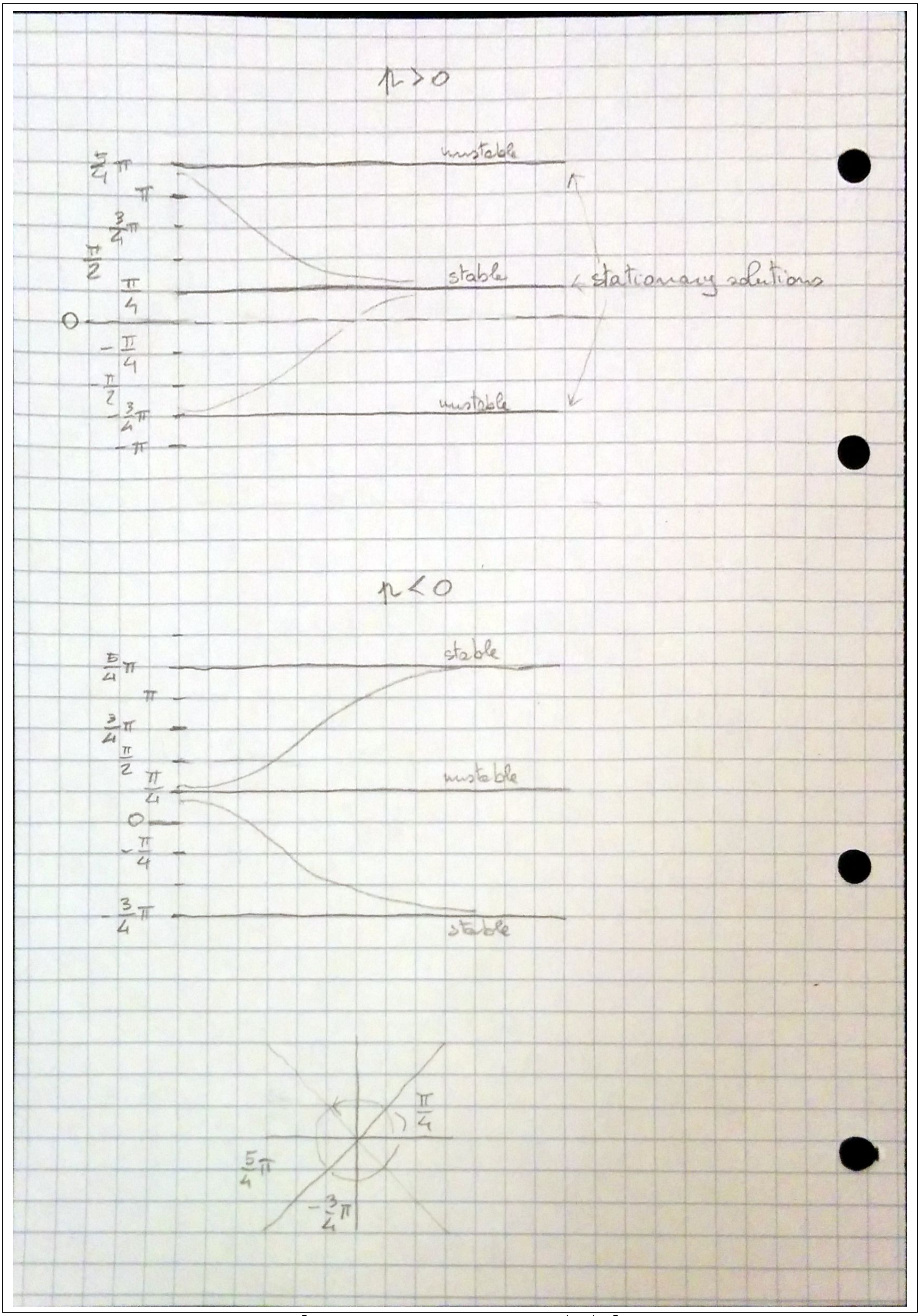


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# Potential energy

$$\operatorname{skw} M \cdot \operatorname{skw} \nabla v$$

$$= \frac{\sqrt{2}}{2}\, p\, V_R \left(\cos\vartheta - \sin\vartheta\right)\dot{\vartheta}$$

(just the skewsymmetric part?)

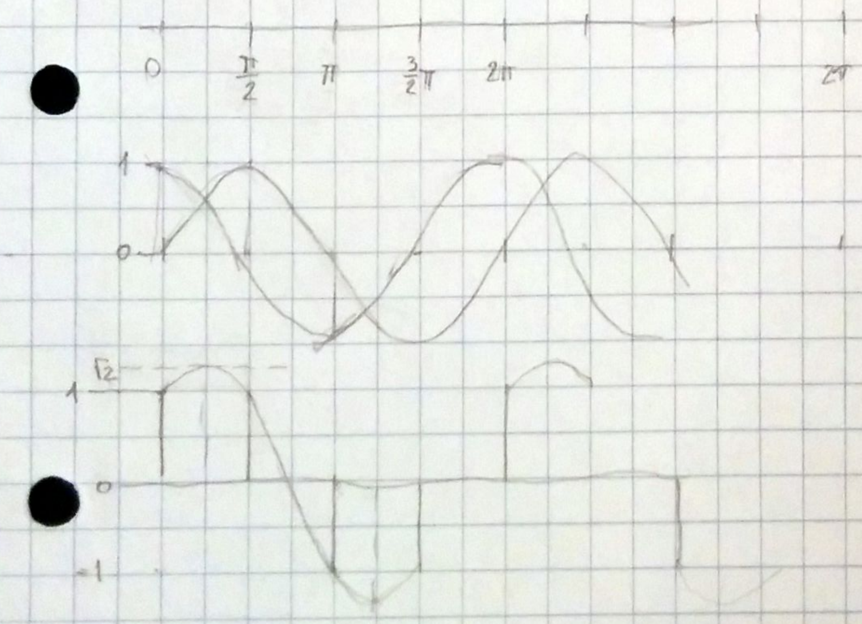
This is the power of the external force distribution.

Is there any potential?

$$\frac{\sqrt{2}}{2}\, p\, V_R \left(\cos\vartheta - \sin\vartheta\right)\dot{\vartheta} = \frac{d}{dt}\frac{\sqrt{2}}{2}\, p\, V_R \left(\sin\vartheta + \cos\vartheta\right)$$

$$\cos\vartheta\,\dot{\vartheta} = \frac{d}{dt}\sin\vartheta \quad ; \quad -\sin\vartheta\,\dot{\vartheta} = \frac{d}{dt}\cos\vartheta$$

We define conventionally the "potential energy" as the opposite of that expression.

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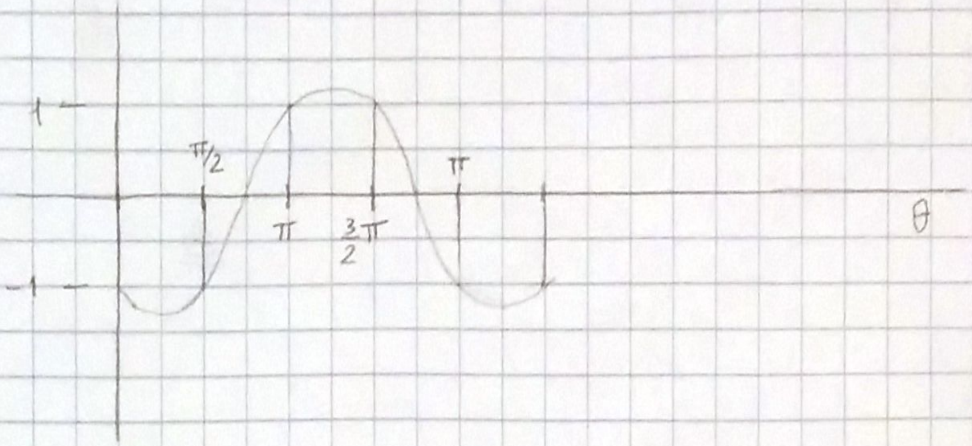
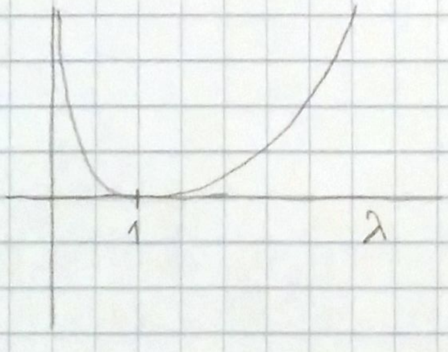
The graph of the potential energy turns out to be




According to the previous stability analysis the minima of this graph denote stable shapes, while maxima denote unstable shapes.

The strain energy, as a function of $\lambda$, is described by the graph




$$\varphi(\lambda) = c\left(\lambda^2 + \frac{2}{\lambda} - 3\right)$$

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