$(31\text{-}32)_6$ Monday [2014-03-31]

16:00 - 18:00 A1,3

$$\mathcal{F}(v) = \mathcal{F}^{ext}(v) + \mathcal{F}^{int}(v)$$

$\mathcal{F}(v) = 0$ balance principle

$$v(x) = v_o + L(x - p_o)$$

$$\mathcal{F}^{ext} = f \cdot v_o + M \cdot L$$

examples $\longrightarrow$

let us set $\mathcal{F}^{int}(v) = -(z \cdot v_o + T \cdot L)\, V_\mathcal{R}$ $\longleftarrow$ why $V_\mathcal{R}$?

and give the following characterization

$z \cdot v_o + T \cdot L = 0 \quad \forall$ rigid velocity field

$$v(x) = v_o + W(x - p_o)$$

$$\Rightarrow \quad z \cdot v_o + T \cdot W = 0 \quad \forall v_o \; \forall W$$

$$\Downarrow$$

$$z = 0, \quad \operatorname{skw} T = 0$$

$$\mathcal{F}(v) = (f - z V_\mathcal{R}) \cdot v_o + (M - T V_\mathcal{R}) \cdot L$$

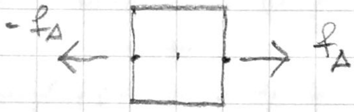
$$= f \cdot v_o + \operatorname{skw} M \cdot W + (\operatorname{sym} M - T V_\mathcal{R}) \cdot D$$

$$\mathcal{F}(v) = 0 \;\forall v \quad \Rightarrow \quad f = 0;\; \operatorname{skw} M = 0;\; \operatorname{sym} M = T V_\mathcal{R}$$

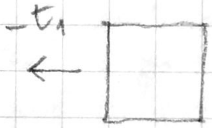
arm force (Gurtin)

$$M_A = (p_A - p_o) \otimes f_A \quad = \quad f_A \otimes (p_A - p_o)$$

force arm



$$\frac{\ell_1}{2} e_1 \otimes (f e_1) = \frac{\ell_1}{2} f \, e_1 \otimes e_1$$



$$\to t_1 = t_{11} e_1 + t_{21} e_2 + t_{31} e_3$$

$$\frac{\ell_1}{2} e_1 \otimes t_1 \qquad \int_{\mathcal{F}_1} t \cdot v \, dA$$

$$e_1 \otimes e_2 = e_2 \otimes e_1$$

$$(e_1 \otimes e_2) e_1 = (e_1 \cdot e_1) e_2$$

$$(e_2 \otimes e_1) e_1 = e_2 (e_1 \cdot e_1)$$

$$[e_1 \otimes e_2] = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$(e_1 \otimes t_1) n_1 = (e_1 \cdot n_1) t_1$$

$$M_A \cdot L = (p_A - p_o) \otimes f_A \cdot L = f_A \cdot L (p_A - p_o) = f_A \cdot (v_A - v_o)$$

$$= f_A \otimes (p_A - p_o) \cdot L = f_A \cdot L (p_A - p_o) = f_A \cdot (v_A - v_o)$$

$$t_1 = t_{11}\, e_1 + t_{21}\, e_2 + t_{31}\, e_3$$

$$\int_{\mathcal{S}_1} t_1(x)\cdot v(x)\, dA = \int_{\mathcal{S}_1} t_1(x)\cdot v_0\, dA + \int_{\mathcal{S}_1} t_1(x)\cdot L\,(x-p_0)\, dA$$

$$= \int_{\mathcal{S}_1} t_1\, dA \cdot v_0 + \int_{\mathcal{S}_1} (x-p_0)\otimes t_1\, dA \cdot L = A_{\mathcal{S}_1}(g_{\mathcal{S}_1}-p_0)\otimes t_1 \cdot L$$

$$\downarrow$$

useless parameterization
$$\begin{cases} x - p_0 = \dfrac{\ell_1}{2}\, e_1 + s_2 e_2 + s_3\, e_3 \\ -\dfrac{\ell_1}{2} \le s_2 \le \dfrac{\ell_2}{2} \qquad -\dfrac{\ell_3}{2} \le s_3 \le \dfrac{\ell_3}{2} \end{cases}$$

$$g_{\mathcal{S}_1} - p_0 = \frac{1}{A_{\mathcal{S}_1}} \int_{\mathcal{S}_1} (x-p_0)\, dA$$

$$\int_{\mathcal{S}_1 \cup \mathcal{S}_{-1}} t(x)\cdot v(x)\, dA = \int_{\mathcal{S}_1} t_1 \cdot v(x)\, dA - \int_{\mathcal{S}_{-1}} t_1 \cdot v(x)\, \cancel{dA}$$

$$= (t_1 A_{\mathcal{S}_1}) \otimes (g_{\mathcal{S}_1} - g_{\mathcal{S}_{-1}}) \cdot L$$

$$= (t_1\, \ell_2 \ell_3) \otimes (\ell_1 e_1) \cdot L = (t_1 \otimes e_1 \cdot L)\, V_{\mathcal{R}}$$

look at the general formula →

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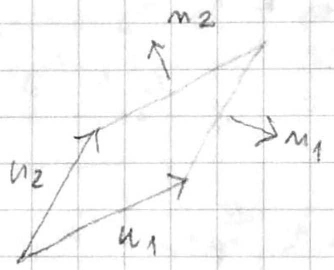
$(33\text{-}34)_6$ Tuesday [2014-04-01] A1.3
9:00 - 11:00



n_1 orthogonal to span$\{u_2, u_3\}$

n_2 orthogonal to span$\{u_3, u_1\}$

n_3 orthogonal to span$\{u_1, u_2\}$

$\mathcal{F}_1$ for face n_1, u_2, u_3

$\mathcal{F}$ $\quad$ ($\underline{\text{not}}$ $\mathfrak{F}$)

(*) $$M = A_{\mathcal{F}_1}\, t_1 \otimes u_1 + A_{\mathcal{F}_2}\, t_2 \otimes u_2 + A_{\mathcal{F}_3}\, t_3 \otimes u_3$$

$$M n_1 = A_{\mathcal{F}_1}\, t_1 \underbrace{(u_1 \cdot n_1)}_{h_1} + A_{\mathcal{F}_2}\, t_2 \underbrace{(u_2 \cdot n_1)}_{0} + A_{\mathcal{F}_3}\, t_3 \underbrace{(u_3 \cdot n_1)}_{0}$$

$$M n_1 = (A_{\mathcal{F}_1} h_1)\, t_1 = V_{\mathcal{R}}\, t_1$$

(*) $$\int_{\mathcal{F}_1} t \cdot v \, dA = \int_{\mathcal{F}_1} t_1 \cdot v_0 \, dA + \int_{\mathcal{F}_1} t_1 \cdot L(x - p_0)\, dA$$

$$= A_{\mathcal{F}_1}\, t_1 \cdot v_0 + \int_{\mathcal{F}_1} t_1 \otimes (x - p_0) \cdot L \, dA = A_{\mathcal{F}_1}\, t_1 \cdot v_0 + A_{\mathcal{F}_1}\, t_1 \otimes \underbrace{(g_{\mathcal{F}_1} - p_0)}_{\frac{1}{2} u_1} \cdot L$$

[...]

Change of observer (frame-invariance)

Galilean transformations (Galilean group)

$$p_A^* = q_o^* + Q(p_A - p_o)$$

This transformation is defined by q_o^* and Q

$q \equiv q_o^*$

$$\dot{p}_A^* = \dot{q}_o^* + \dot{Q}(p_A - p_o) + Q(\dot{p}_A - \dot{p}_o)$$

$$= \dot{q}_o^* + (\dot{Q} + QL)(p_A - p_o) = \dot{q}_o^* + (\dot{Q}Q^T + QLQ^T)(p_A^* - q_o^*)$$

$$v_A^* = v_o^* + L^*(p_A^* - q_o^*) \qquad p_o^* = q_o^*$$

$$\mathcal{F}^*(v^*) = -(z^* \cdot v_o^* + T^* \cdot L^*)\, V_{\mathcal{B}}$$

$$\mathcal{F}^*(v^*) = \mathcal{F}(v)$$

$$z^* \cdot v_o^* + T^* \cdot L^* = z \cdot v_o + T \cdot L$$

where $v_o^* = \dot{q}_o^*$ an arbitrary vector $\dot{q}$

and $L^* = \dot{Q}Q^T + QLQ^T$ with arbitrary $Q, \dot{Q}$

$$\Rightarrow \quad z^* \cdot v_o^* = z \cdot v_o \qquad \forall v_o, \forall \dot{q} \qquad \Rightarrow z = 0, z^* = 0$$

$$\Rightarrow \quad T^* \cdot (\dot{Q}Q^T + QLQ^T) = T \cdot L \qquad \forall L, \forall \dot{Q}$$

$$\dot{Q} = 0 \quad \Rightarrow \quad T^* \cdot QLQ^T = T \cdot L$$

$$z^* \cdot (v_o + w_o^*) = z \cdot v_o \Rightarrow (z^* - z) \cdot v_o = z^* \cdot w^*$$

$$Q^T T^* Q \cdot L = T \cdot L \qquad \forall L$$

$$\Rightarrow \quad Q^T T^* Q = T$$

$$T^* = Q T Q^T$$

$$\dot{Q} \neq 0 \qquad T^* \cdot \dot{Q} Q^T = 0 \qquad \forall \dot{Q} Q^T \quad \Rightarrow \quad \operatorname{skw} T^* = 0$$

$$0 = \operatorname{skw} T^* = \frac{1}{2}\left(Q T Q^T - Q T^T Q^T\right) = \frac{1}{2} Q \left(T - T^T\right) Q^T$$

$$\Rightarrow \quad T = T^T$$

$(35-36)_6$ Wednesday [2014-04-02] A1.3
11:00 - 13:00

Summarizing affine motions

(affine bodies: bodies undergoing affine motions)

$$v(x) = v_0 + L(x - p_0)$$

$$\mathcal{F}^{ext}(v) = f \cdot v_0 + M \cdot L$$

$$\mathcal{F}^{int}(v) = -(z \cdot v_0 + T \cdot L) V_{\mathcal{R}}$$

Balance $\quad \mathcal{F}(v) = 0 \quad \forall v$

Objectivity $\quad z = 0, \quad T = T^T$

$$\Rightarrow \quad f = 0$$

$$\operatorname{skw} M = 0$$

$$\operatorname{sym} M = T V_{\mathcal{R}}$$

with $\quad T m_1 = t_1 \quad$ [...]

Balance equations in general

$$\mathcal{F}^{ext}(v) = \int_{\mathcal{R}} b(x)\cdot v(x)\, dV + \int_{\partial\mathcal{R}} t(x)\cdot v(x)\, dA$$

$$\mathcal{F}^{int}(v) = -\int_{\mathcal{R}} T(x)\cdot \underbrace{L(x)}_{\nabla v}\, dV$$

$$\mathcal{F}(v) = \mathcal{F}^{ext}(v) + \mathcal{F}^{int}(v) = \int_{\mathcal{R}} b\cdot v\, dV + \int_{\partial\mathcal{R}} t\cdot v\, dA - \int_{\mathcal{R}} T\cdot\nabla v\, dV$$

$\operatorname{div} v = \operatorname{tr}\nabla v$ $\qquad$ ∇v already defined

$$\operatorname{div} T\cdot a = \operatorname{div}(T^T a) = \operatorname{tr}\nabla(T^T a)$$

$$\operatorname{div}(T^T v) = \operatorname{tr}\nabla(T^T v) = \operatorname{div} T\cdot v + T\cdot\nabla v$$

$$\nabla(T^T v)e = \lim_{h\to 0}\frac{T^T(x+he)\,v(x+he) - T^T(x)\,v(x)}{h}$$

$$= \lim_{h\to 0}\frac{T^T(x+he)\,v(x) - T^T(x)\,v(x)}{h} + T^T(x)\nabla v(x)e$$

$(21\text{-}22)_4 \rightarrow \quad v(c(h)) = v(c(0)) + \nabla v(c(0))\big(c(h)-c(0)\big) + o(h)$

$c(h) = x + he \Rightarrow v(x+he) = v(x) + \nabla v(x)(he) + o(h)$

$$w(x+he) := T^T(x+he)\, v(x)$$

$$\nabla(T^T v)e = \lim_{h\to 0} \frac{w(x+he) - w(x)}{h} + T^T(x)\nabla v(x)\, e$$

$$= \nabla w(x)\, e + T^T(x)\nabla v(x)\, e$$

$$\operatorname{tr}\nabla(T^T v) = \operatorname{tr}\nabla w + \operatorname{tr}(T^T \nabla v)$$

$$\operatorname{tr}\nabla w = \operatorname{div} T \cdot v$$ since $\operatorname{tr}\nabla w$ is linear in $v(x)$

by the representation theorem

$$\operatorname{div}(T^T v) = \operatorname{tr}\nabla(T^T v) = \operatorname{div} T \cdot v + \operatorname{tr}(T^T \nabla v)$$

$$\operatorname{div} T \cdot v = \operatorname{div}(T^T v) - T \cdot \nabla v$$

$$-\int_{\mathcal{R}} T \cdot \nabla v \, dV = \int_{\mathcal{R}} \operatorname{div} T \cdot v \, dV - \int_{\mathcal{R}} \operatorname{div}(T^T v)\, dV$$

$$\int_{\mathcal{R}} \operatorname{div} T \cdot v \, dV - \int_{\partial\mathcal{R}} T^T v \cdot n \, dA$$

$$= \int_{\mathcal{R}} \operatorname{div} T \cdot v \, dV - \int_{\partial\mathcal{R}} Tn \cdot v \, dA$$

$$\mathcal{F}(v) = \int_{\mathcal{R}} b \cdot v \, dV + \int_{\partial\mathcal{R}} t \cdot v \, dA + \int_{\mathcal{R}} \operatorname{div} T \cdot v \, dV - \int_{\partial\mathcal{R}} Tn \cdot v \, dA = 0$$

CAUCHY BALANCE EQUATIONS
$$\begin{cases} b + \operatorname{div} T = 0 & \mathcal{R} \\ t - Tn = 0 & \partial\mathcal{R} \end{cases}$$