[2016-04-14]

$$\mathcal{F}^{ext}(v) = \int_{\mathcal{R}} b \cdot v \, dV + \int_{\partial\mathcal{R}} t \cdot v \, dA$$

$$\mathcal{F}^{int}(v) = - \int_{\mathcal{R}} T \cdot \nabla v \, dV$$

$$\mathcal{F}(v) = \mathcal{F}^{ext}(v) + \mathcal{F}^{int}(v)$$

$$\operatorname{div} T \cdot v = \operatorname{div}(T^T v) - T \cdot \nabla v$$

[...]

$$\begin{cases} b + \operatorname{div} T = 0 \\ T n = t \end{cases}$$

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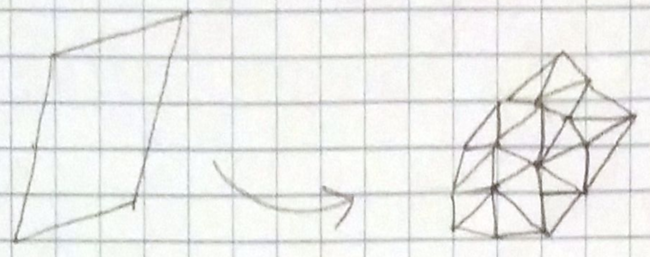
[2016-04-18]



[...]

$$\frac{f}{V_{\mathcal{R}}} = b + \frac{t_1^+ - t_1^-}{h_1} + \frac{t_2^+ - t_2^-}{h_2} + \frac{t_3^+ - t_3^-}{h_3}$$

$$\frac{M}{V_{\mathcal{R}}} n_1 = \frac{t_1^+ + t_1^-}{2} \; ; \; \frac{M}{V_{\mathcal{R}}} n_2 = \frac{t_2^+ + t_2^-}{2} \; ; \; \frac{M}{V_{\mathcal{R}}} n_3 = \frac{t_3^+ + t_3^-}{2}$$

balance $\quad \frac{f}{V_{\mathcal{R}}} = 0 \,, \quad \frac{M}{V_{\mathcal{R}}} = T$

$t_1^+ - t_1^- \to 0 \,, \; h_1 \to 0$

$\downarrow$ $\quad \frac{1}{2}(t_1^+ + t_1^-) \to t$

$$\begin{cases} b + \operatorname{div} T = 0 \\ T n = t \end{cases}$$

$\uparrow$ localization

$$\begin{cases} \mathcal{F}(v) = 0 \\ \operatorname{tr}(\nabla(T^T v)) - \operatorname{tr}(T^T \nabla v) = \operatorname{div} T \cdot v \end{cases}$$

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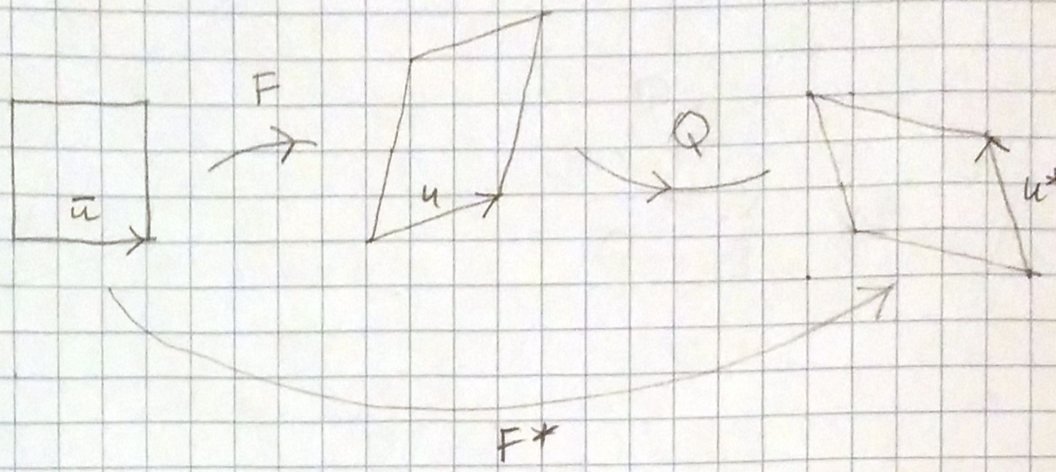
[2016-04-18]

Objectivity (frame-indifference)



$$F^* = Q F$$

$$L = \dot{F} F^{-1}$$

$$L^* = \dot{F}^* F^{*-1} = (\dot{Q} F + Q \dot{F}) F^{-1} Q^T$$

$$= \dot{Q} Q^T + Q \dot{F} F^{-1} Q^T$$

objectivity

$$T \cdot L = T^* \cdot L^*$$

$$T \cdot L = T^* \cdot \dot{Q} Q^T + T^* \cdot Q L Q^T$$

$$A \cdot BC = \mathrm{tr}(A^T B C) = \mathrm{tr}((A^T B) C) = (A^T B)^T \cdot C = B^T A \cdot C$$

$$= \mathrm{tr}(A C^T B^T) = A C^T \cdot B$$

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$$T^* \cdot \dot{Q} Q^T = 0 \quad \Rightarrow \quad \text{skw}\, T^* = 0$$

$$T \cdot L = T^* \cdot Q L Q^T$$

$$\downarrow$$

$$Q^T T^* Q \cdot L$$

$$\Rightarrow \quad T = Q^T T^* Q$$

$$Q T Q^T = T^*$$

$$T = \hat{T}(F)$$ elastic material response function

$$T^* = \hat{T}(F^*)$$

objectivity

$$T^* \cdot L^* = T \cdot L \qquad \forall Q, \ \forall L$$

$$\hat{T}(F^*) \cdot L^* = \hat{T}(F) \cdot L$$ condition on the response function

$$\Rightarrow \quad \hat{T}(F^*) = Q \hat{T}(F) Q^T$$

$$Q^T \hat{T}(F^*) Q = \hat{T}(F)$$

$$Q^T \hat{T}(QF) Q = \hat{T}(F)$$

$$R \hat{T}(U) R^T = \hat{T}(F) \qquad \forall F \qquad F = RU$$

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[2016-06-19]

Piola stress

$$T \cdot L \, V_{\mathcal{R}} \qquad \text{stress power}$$

$$T \cdot L \, V_{\mathcal{R}} = T \cdot \nabla v \; V_{\bar{\mathcal{R}}} \det F$$

$$\nabla v \, c' = \nabla v \, F \bar{c}'$$

$$\nabla \bar{v} = \nabla v \, F \quad \Rightarrow \quad \nabla v = \nabla \bar{v} \, F^{-1}$$

$$T \cdot \nabla v \, V_{\mathcal{R}} = \left(T \cdot \nabla \bar{v} \, F^{-1}\right) V_{\bar{\mathcal{R}}} \det F$$

$$= \left(T F^{-T} \cdot \nabla \bar{v}\right) \det F \; V_{\bar{\mathcal{R}}}$$

$$= S \cdot \nabla \bar{v} \; V_{\bar{\mathcal{R}}}$$

$$S := (\det F) \, T F^{-T}$$

$$u_1 \cdot u_2 = F \bar{u}_1 \cdot u_2 = \bar{u}_1 \cdot F^T u_2 = F^{-1} u_1 \cdot F^T u_2$$

$$= u_1 \cdot (F^{-1})^T F^T u_2 \qquad \forall u_1 \; \forall u_2$$

$$\Rightarrow (F^{-1})^T F^T = I \quad \Rightarrow \quad (F^{-1})^T = (F^T)^{-1}$$

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$$A_{\mathcal{F}_1} = \operatorname{vol}(n_1, u_2, u_3) \qquad A_{\bar{\mathcal{F}}_1} = \operatorname{vol}(\bar{n}_1, \bar{u}_2, \bar{u}_3)$$

$$w_1 = u_1 - \underbrace{(u_1 \cdot n_1)}_{h_1} n_1 \quad \Rightarrow \quad w_1 \cdot n_1 = 0$$

$$w_1 \in \operatorname{span}\{u_2, u_3\} \quad \Rightarrow \quad \operatorname{vol}(u_1, u_2, u_3) = h_1 \operatorname{vol}(n_1, u_2, u_3)$$

$$V_{\mathcal{R}} = h_1 A_{\mathcal{F}_1} \qquad V_{\bar{\mathcal{R}}} = \bar{h}_1 A_{\bar{\mathcal{F}}_1}$$

$$h_1 := u_1 \cdot n_1 \qquad u_2 \cdot n_1 = 0 \qquad u_3 \cdot n_1 = 0$$

$$\bar{h}_1 := \bar{u}_1 \cdot \bar{n}_1 \qquad \bar{u}_2 \cdot \bar{n}_1 = 0 \qquad \bar{u}_3 \cdot \bar{n}_1 = 0$$

$$\Rightarrow \quad h_1 = F\bar{u}_1 \cdot n_1 = \bar{u}_1 \cdot F^T n_1, \quad F\bar{u}_2 \cdot n_1 = 0, \quad F\bar{u}_3 \cdot n_1 = 0$$

$$\Rightarrow \quad \bar{u}_2 \cdot F^T n_1 = 0, \quad \bar{u}_3 \cdot F^T n_1 = 0$$

$$\Rightarrow \quad k_1 F^T n_1 = \bar{n}_1$$

$$\Rightarrow \quad F^{-T} \bar{n}_1 = k_1 n_1$$

$$\bar{h}_1 = \bar{u}_1 \cdot \bar{n}_1 = \bar{u}_1 \cdot (k_1 F^T n_1) = k_1 \bar{u}_1 \cdot F^T n_1 = k_1 u_1 \cdot n_1 = k_1 h_1$$

$$\Rightarrow \quad k_1 = \frac{\bar{h}_1}{h_1} = \frac{V_{\bar{\mathcal{R}}}}{A_{\bar{\mathcal{F}}_1}} \frac{A_{\mathcal{F}_1}}{V_{\mathcal{R}}} = \frac{A_{\mathcal{F}_1}}{A_{\bar{\mathcal{F}}_1}} \frac{1}{\det F}$$

$$(\det F) F^{-T} \bar{n}_1 = \frac{A_{\mathcal{F}_1}}{A_{\bar{\mathcal{F}}_1}} n_1$$

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Cofactor of F

$$\operatorname{cof} F := (\det F)\, F^{-T}$$

$$F^c$$

$$F^c \bar{n}_1 = n_1 \frac{A_{\mathcal{F}_1}}{A_{\bar{\mathcal{F}}_1}}$$

$$\bar{t}_1 = S\bar{n}_1 = T(\operatorname{cof} F)\bar{n}_1 = T n_1 \frac{A_{\mathcal{F}_1}}{A_{\bar{\mathcal{F}}_1}}$$

$$\bar{t}_1 = t_1 \frac{A_{\mathcal{F}_1}}{A_{\bar{\mathcal{F}}_1}}$$

$$\int_{\mathcal{F}_1} t_1\, dA = \int_{\bar{\mathcal{F}}_1} t_1 \left(\frac{A_{\mathcal{F}_1}}{A_{\bar{\mathcal{F}}}}\right) dA$$

$$t_1 = T n_1 \,,\quad \bar{t}_1 = S\bar{n}_1 \;\Rightarrow\; \bar{t}_1 = T(\operatorname{cof} F)\bar{n}_1$$

$$\int_{\mathcal{F}_1} t_1\, dA = \int_{\bar{\mathcal{F}}_1} \bar{t}_1\, dA \;\Rightarrow\; \bar{t}_1 = t_1 a_1 = a_1 T n_1$$

$$\Rightarrow\; (\operatorname{cof} F)\bar{n}_1 = a_1 n_1 \;\Rightarrow\; \|(\operatorname{cof} F)\bar{n}_1\| = a_1$$

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Cofactor matrix

$$(\operatorname{cof} F)e_1 = (\det F)F^{-T}e_1 = n_1 \frac{A_{\mathcal{F}_1}}{A_{\bar{\mathcal{F}}_1}}$$

$\{e_1, e_2, e_3\}$ orthonormal basis

$$A_{\bar{\mathcal{F}}_1} = 1,\ A_{\bar{\mathcal{F}}_2} = 1,\ A_{\bar{\mathcal{F}}_3} = 1,\ V_{\bar{\mathcal{R}}} = 1$$

$$(\operatorname{cof} F)e_1 = n_1 A_{\mathcal{F}_1} \qquad n_1 \cdot Fe_2 = 0,\ n_1 \cdot Fe_3 = 0,\ n_1 \cdot n_1 = 1$$

$$(\operatorname{cof} F)e_1 \cdot e_i = (n_1 \cdot e_i) A_{\mathcal{F}_1} = (n_1 \cdot e_i)\operatorname{vol}(n_1, Fe_2, Fe_3)$$

$$w_{1i} := e_i - (e_i \cdot n_1) n_1 \quad \Rightarrow w_{1i} \cdot n_1 = 0$$

$$w_{1i} \in \operatorname{span}\{Fe_2, Fe_3\}$$

$$(n_1 \cdot e_i) n_1 = e_i - w_{1i}$$

$$(\operatorname{cof} F)e_1 \cdot e_i = \operatorname{vol}(e_i, Fe_2, Fe_3)$$

[...]

$$(\operatorname{cof} F)e_2 \cdot e_i = \operatorname{vol}(Fe_1, e_i, Fe_3)$$

$$(\operatorname{cof} F)e_3 \cdot e_i = \operatorname{vol}(Fe_1, Fe_2, e_i)$$

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Restriction of F to a 2-dim subspace

$\forall u \in \mathrm{span}\{e_2, e_3\} \qquad u = \alpha_2 e_2 + \alpha_3 e_3$

$$\tilde F_1 u = \alpha_2 F e_2 + \alpha_3 F e_3 = F e_2 (e_2 \cdot u) + F e_3 (e_3 \cdot u)$$

$$= F\big((e_2 \otimes e_2) + (e_3 \otimes e_3)\big) u = F P_1 u$$

$\tilde F_1 e_2 = F e_2 \qquad\qquad P_1 e_2 = e_2$

$\tilde F_1 e_3 = F e_3 \qquad\qquad P_1 e_3 = e_3$

(orthogonal) projection $\underset{(2\times 3)}{P_1^T} : \mathrm{span}\{e_1, e_2, e_3\} \rightleftarrows \mathrm{span}\{e_2, e_3\} : \underset{(3\times 2)}{P_1}$ (canonical) embedding

$$P_1^T P_1 = \underset{(2\times 2)}{\tilde I}$$

$$\tilde C = \tilde F_1^T \tilde F_1 = P_1^T F^T F P_1 = P_1^T C P_1$$

$$\tilde U := P_1^T U P_1$$

$$(P_1 \tilde U)^T P_1 \tilde U = \tilde U P_1^T P_1 \tilde U = \tilde C$$

$$\underset{3\times 2}{\tilde F_1} = \underset{3\times 3}{R_1} \underset{3\times 2}{P_1} \underset{2\times 2}{\tilde U}$$

$$\Downarrow$$

$$\tilde F_1^T \tilde F_1 = \tilde U P_1^T \underbrace{R_1^T R_1}_{I} P_1 \tilde U = \tilde C = P_1^T F^T F P_1$$

$$\left.\begin{array}{l} R_1^T \tilde F_1 = P_1 \tilde U \\ \tilde R_1 := R_1 P_1 \end{array}\right\} \Rightarrow \quad \tilde F_1 = \tilde R_1 \tilde U$$

$$\tilde R_1^T \tilde R_1 = P_1^T R^T R P_1 = \tilde I$$

↑ isometric embedding

[2016-05-15]

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$$A_{\mathcal{S}_1} = \mathrm{vol}(n_1, Fe_2, Fe_3)$$

$$A_{\bar{\mathcal{S}}_1} = \mathrm{vol}(e_1, e_2, e_3)$$

$$w_1 = Fe_1 - \underbrace{(Fe_1 \cdot n_1)}_{h_1} n_1$$

$$\Rightarrow \quad w_1 \cdot n_1 = 0 \quad \Rightarrow \quad w_1 \in \mathrm{span}\{Fe_2, Fe_3\}$$

$$h_1 n_1 = Fe_1 - w_1$$

$$h_1 A_{\mathcal{S}_1} = h_1 \mathrm{vol}(n_1, Fe_2, Fe_3)$$
$$= \mathrm{vol}(Fe_1, Fe_2, Fe_3) = \det F \, \mathrm{vol}(e_1, e_2, e_3)$$

$$A_{\mathcal{S}_1} = \mathrm{vol}(n_1, Fe_2, Fe_3) = \mathrm{vol}(n_1, \tilde{F}e_2, \tilde{F}e_3)$$
$$= \mathrm{vol}(n_1, R_1 P_1 \tilde{U} e_2, R_1 P_1 \tilde{U} e_3)$$

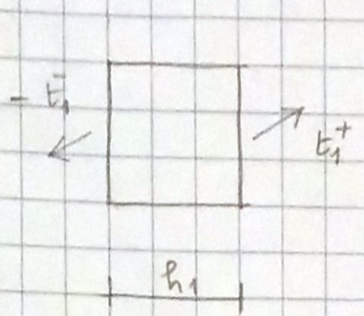
$$= \mathrm{vol}(R_1^T n_1, P_1 \tilde{U} e_2, P_1 \tilde{U} e_3)$$

$$= \det \tilde{U} \, \mathrm{vol}(\underbrace{R_1^T n_1}_{e_1}, e_2, e_3)$$

$$\Rightarrow \qquad \det F = h_1 \det \tilde{U}$$

[2016-05-15]

[Notebook page scanned on 2016/06/30]



$$t_1^+ \mapsto t_1 \qquad t_1^- \mapsto t_1$$

$$t_1^+ - t_1^- \to 0 \qquad \frac{t_1^+ - t_1^-}{h_1} \neq 0$$

$$t_1^+ + t_1^- \to 2t_1$$

$$t_1 = T m_1 = T e_1 = \sigma_{11}\, e_1 + \sigma_{21}\, e_2 + \sigma_{31}\, e_3$$

$$t_2 = \qquad = T e_2 = \sigma_{12}\, e_1 + \sigma_{22}\, e_2 + \sigma_{32}\, e_3$$

$$t_3 = \qquad = T e_3 = \sigma_{13}\, e_1 + \sigma_{23}\, e_2 + \sigma_{33}\, e_3$$

$$b_1 + (\sigma_{11,1} + \sigma_{12,2} + \sigma_{13,3}) = 0$$

$$b_2 + (\sigma_{21,1} + \sigma_{22,2} + \sigma_{23,3}) = 0$$

$$b_3 + (\sigma_{31,1} + \sigma_{32,2} + \sigma_{33,3}) = 0$$

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